# Cascading Behavior in Networks

CS 322: (Social and Information) Network Analysis
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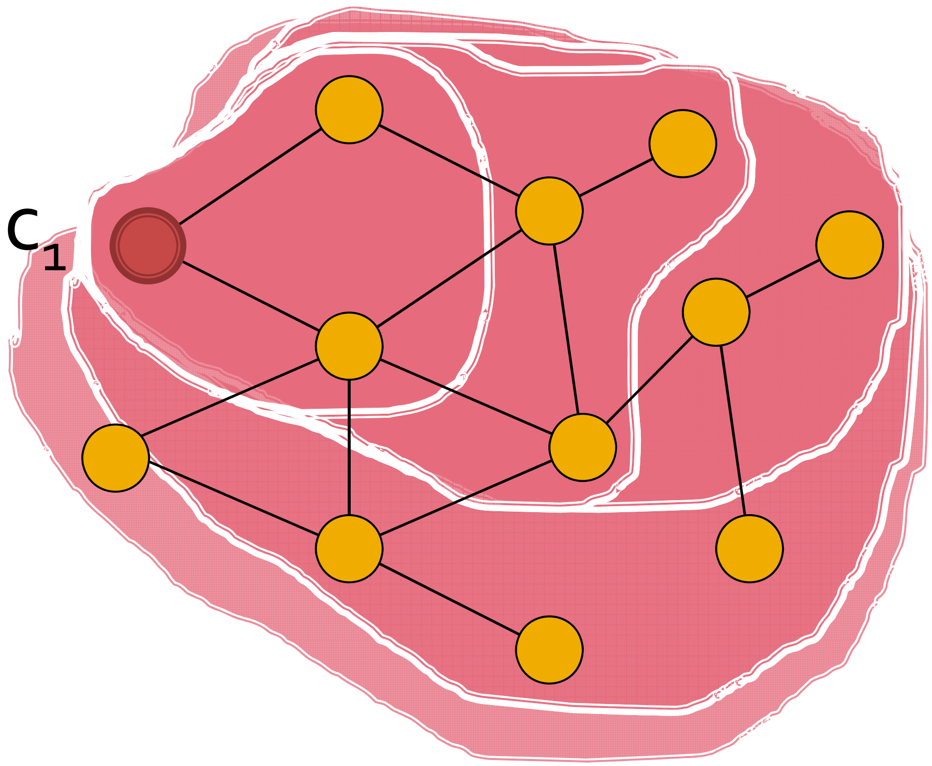
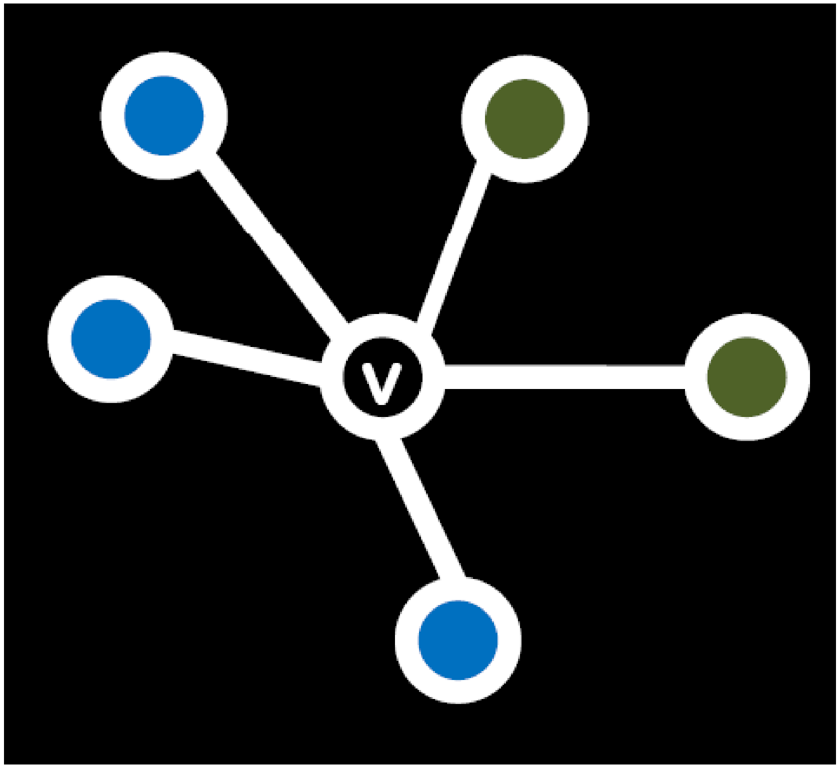
# Spread of diseases



# Game theoretic model of diffusion

- Based on 2 player coordination game
- 2 players – each chooses technology A or B

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# Diffusion of inovation

1. Each person can only adopt one “behavior”

2. You gain more if your friends have adopted the same behavior as you

# The model for two nodes

- If both v and w adopt behavior A, they each get payoff a>0


- If v and w adopt behavior B, they reach get payoff b>0

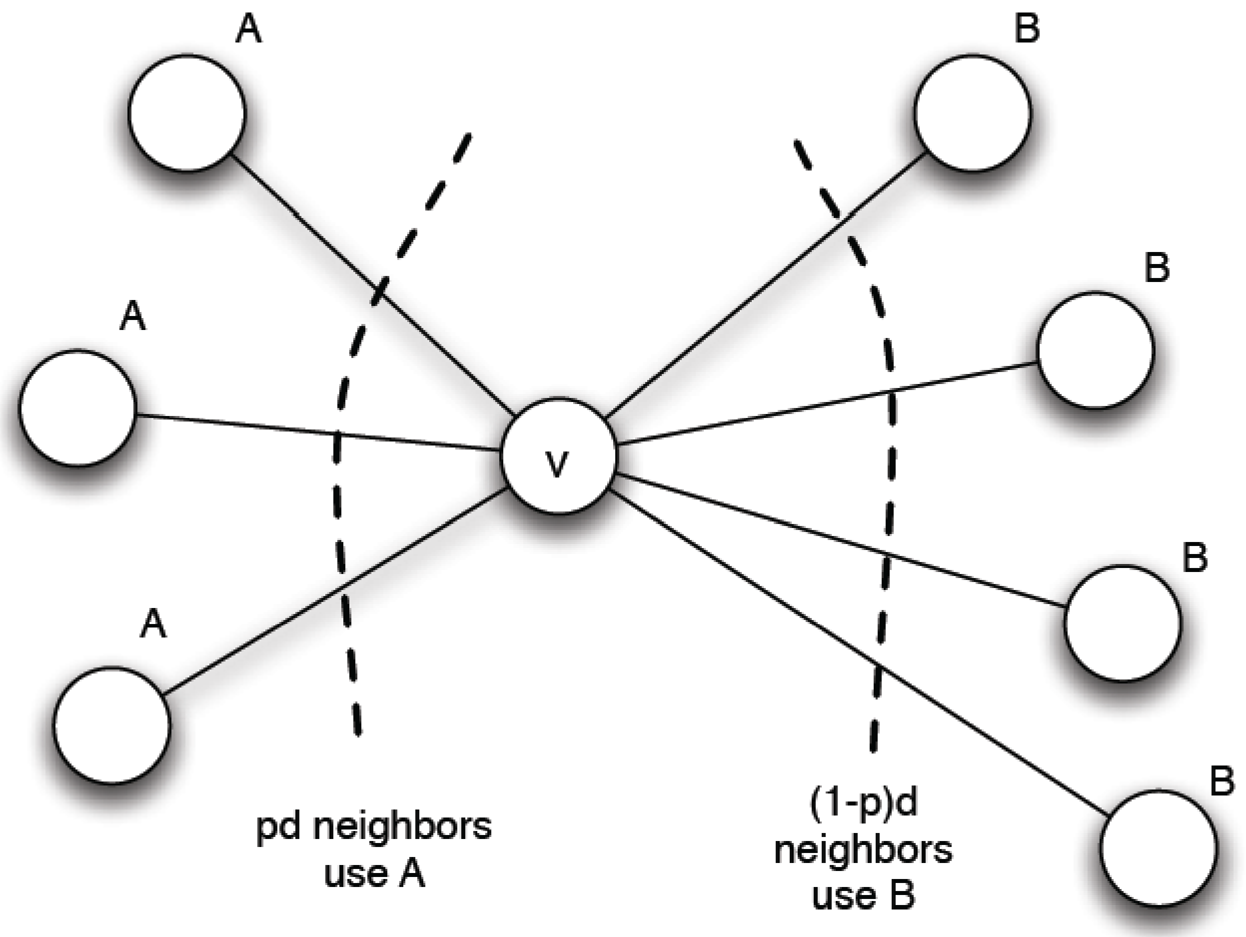

- If v and w adopt the opposite behaviors, they each get 0

# Payoff matrix

| | | $w$: $A$ | $w$: $B$ |
|---|---|---|---|
| $v$ | $A$ | $a, a$ | $0, 0$ |
| | $B$ | $0, 0$ | $b, b$ |

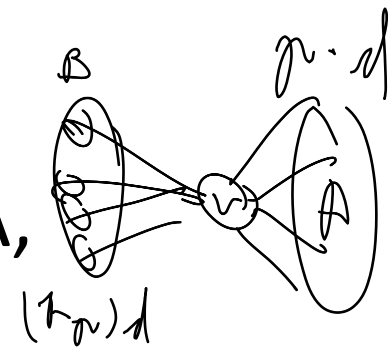
- In some large network:
  - Each node v is playing a copy of this game with each of its neighbors
  - Payoff = sum of payoffs per game

# v's calculation



# v's calculation

- Let v have d neighbors
- If a fraction p of its neighbors adopt A, then:



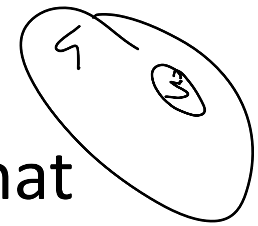

- *Payoff*$_v$ $= a \cdot p \cdot d$ if v chooses A
  $= b \cdot (1-p) \cdot d$ if v chooses B

- *v* chooses A if: $a \cdot p \cdot d > b \cdot (1-p) \cdot d$

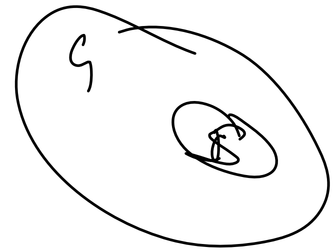
$$p > \frac{b}{a+b} = q$$

# Scenario

- <u>Scenario:</u> graph where everyone starts with B. Small set S of early adopters of A
  - hard wire S – they keep using A no matter what payoffs tell them to do



# Monotonic spreading



- Observation:
  - The use of A spreads monotonically (nodes only switch from B to A, and never back to B)
- Why?

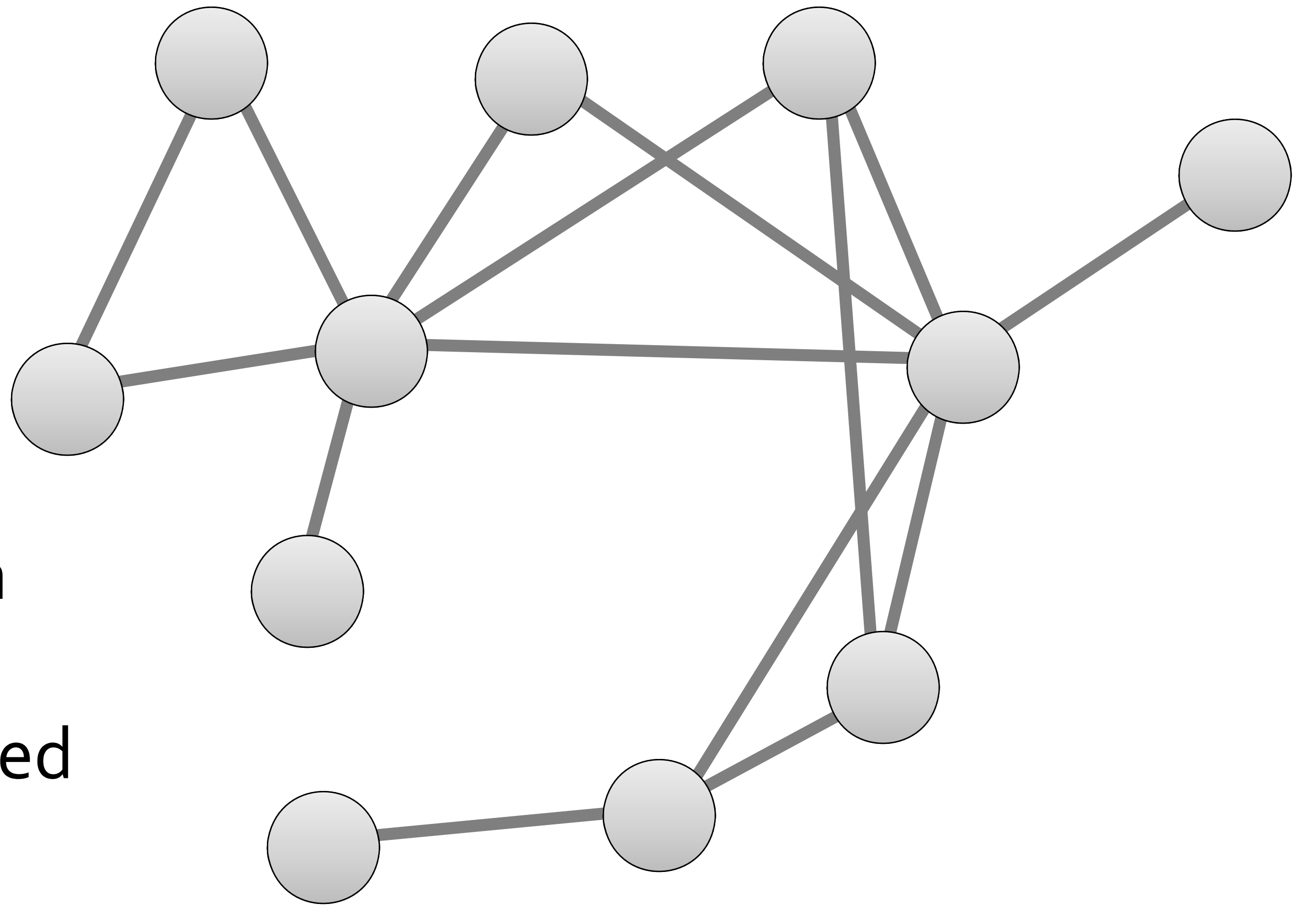
v A → B at time t

at t' v did B → A

t' < t

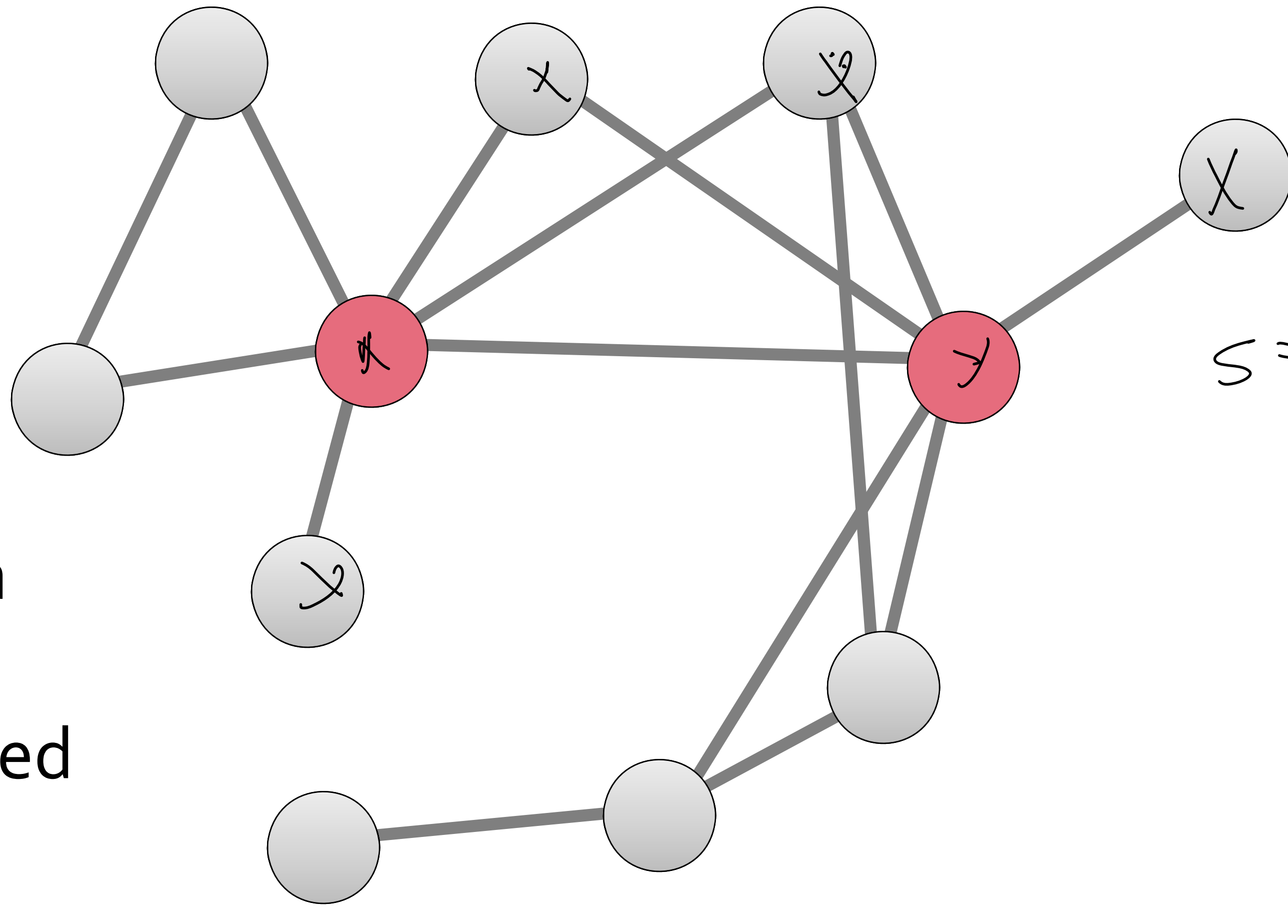
→ ←

# Example



If more than 50% of my friends are red I'll be red

# Example



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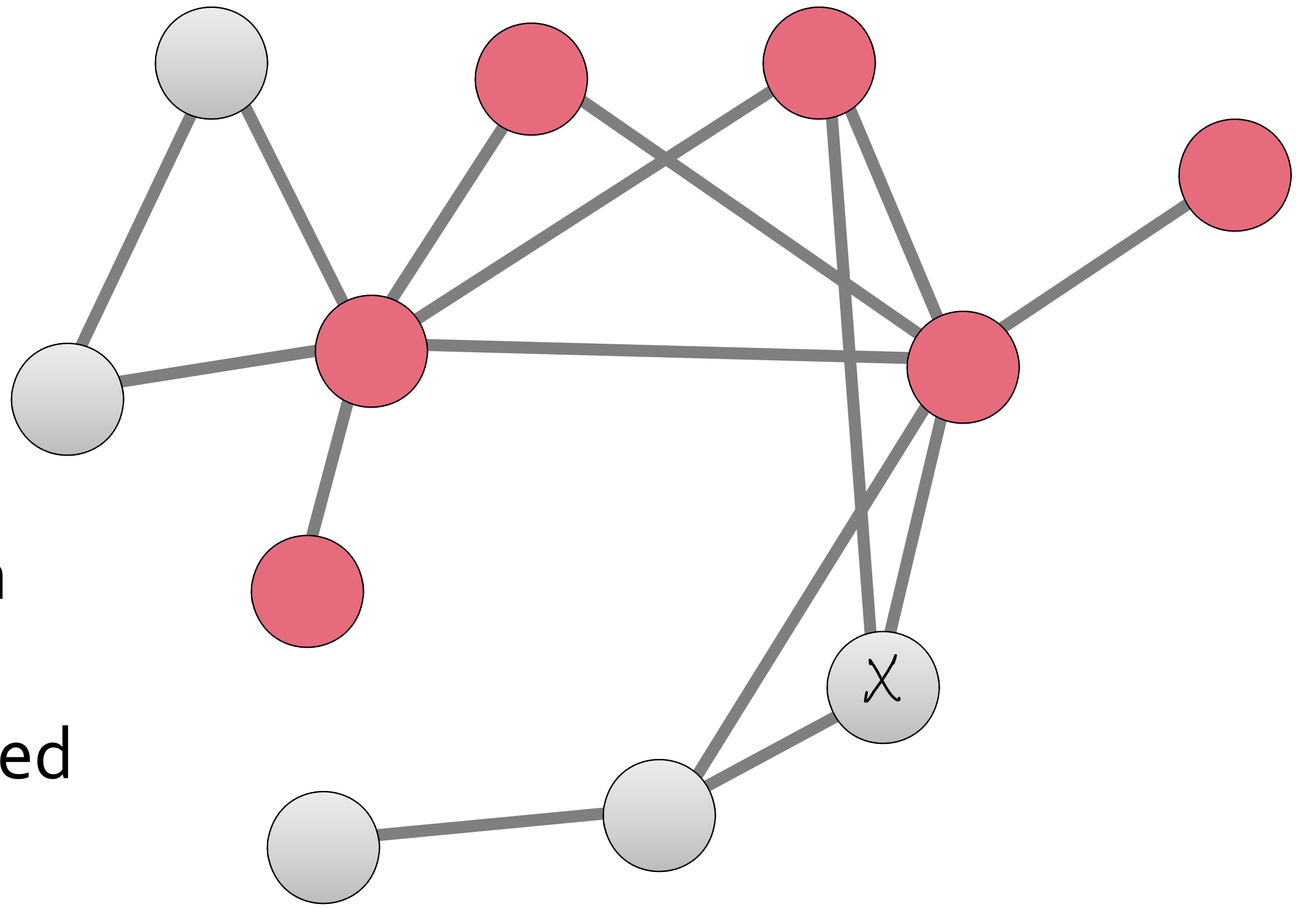


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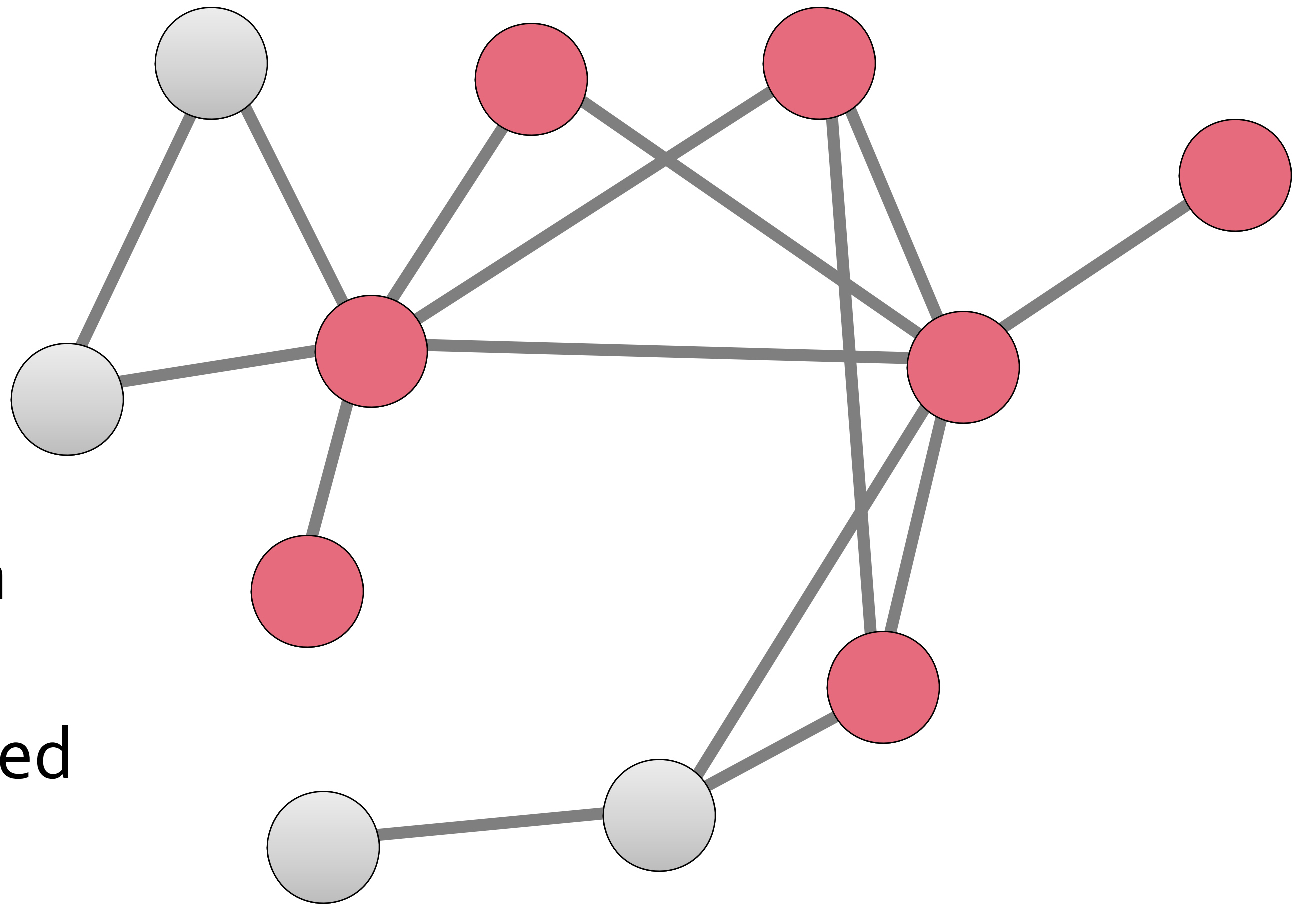

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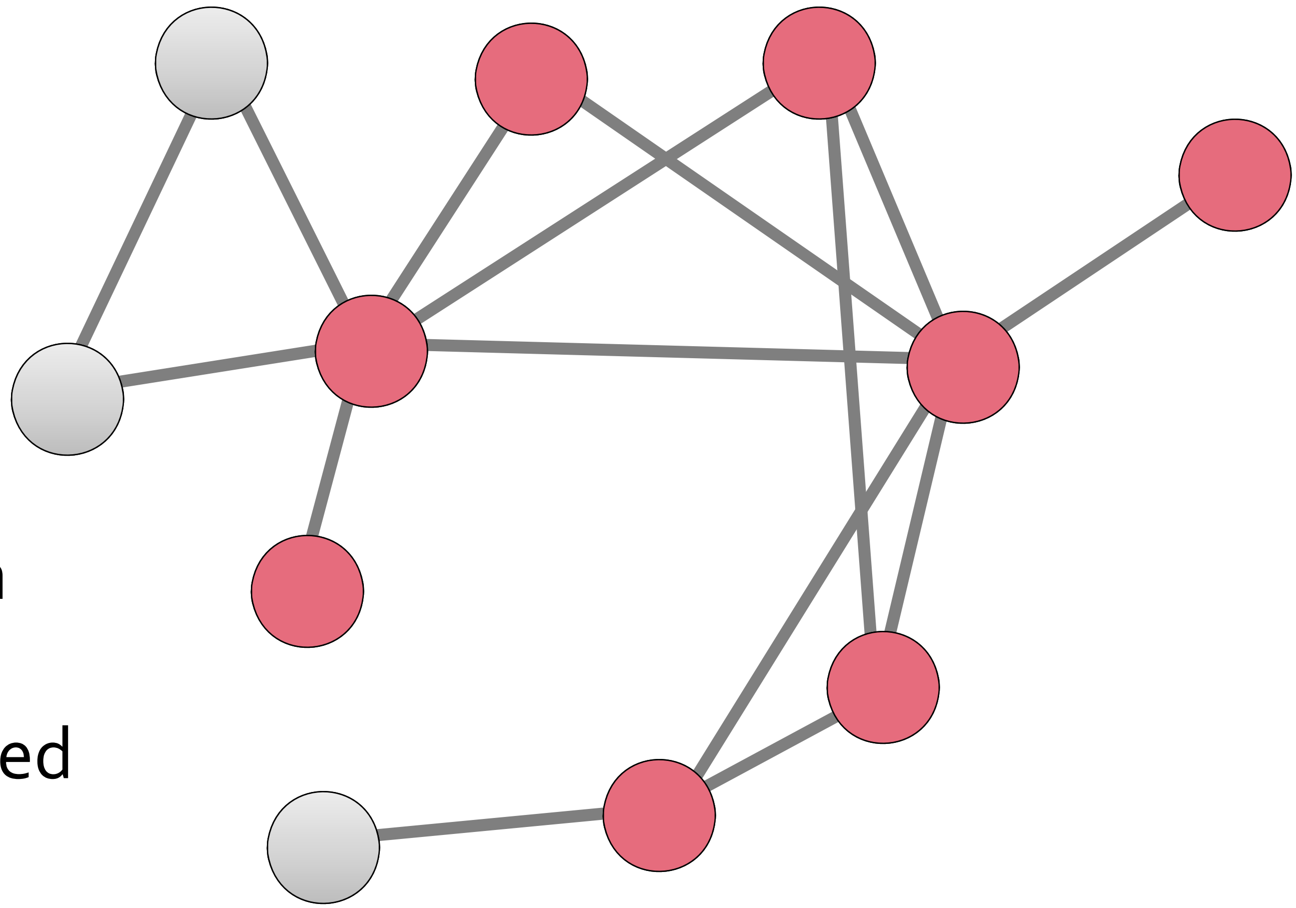

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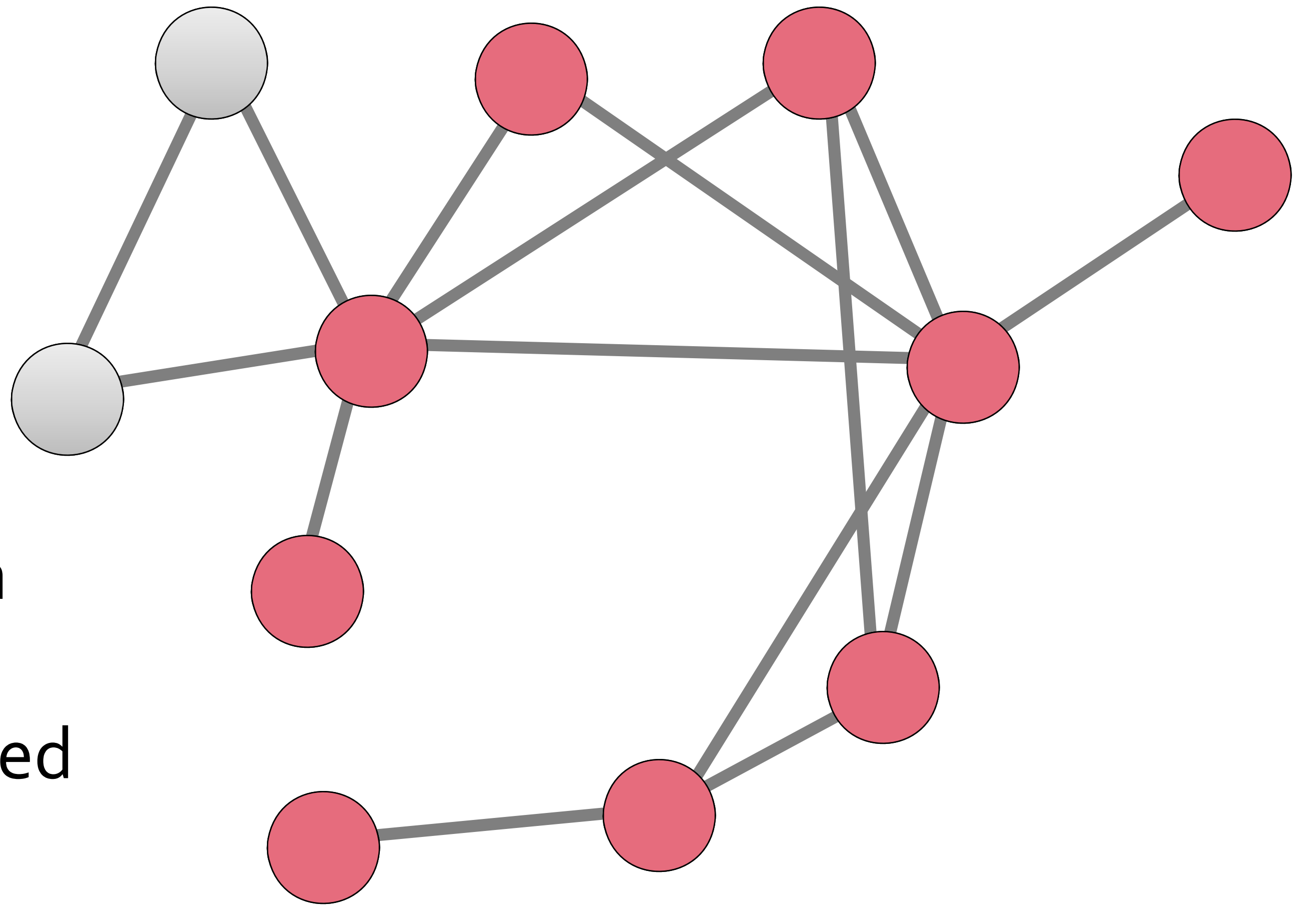

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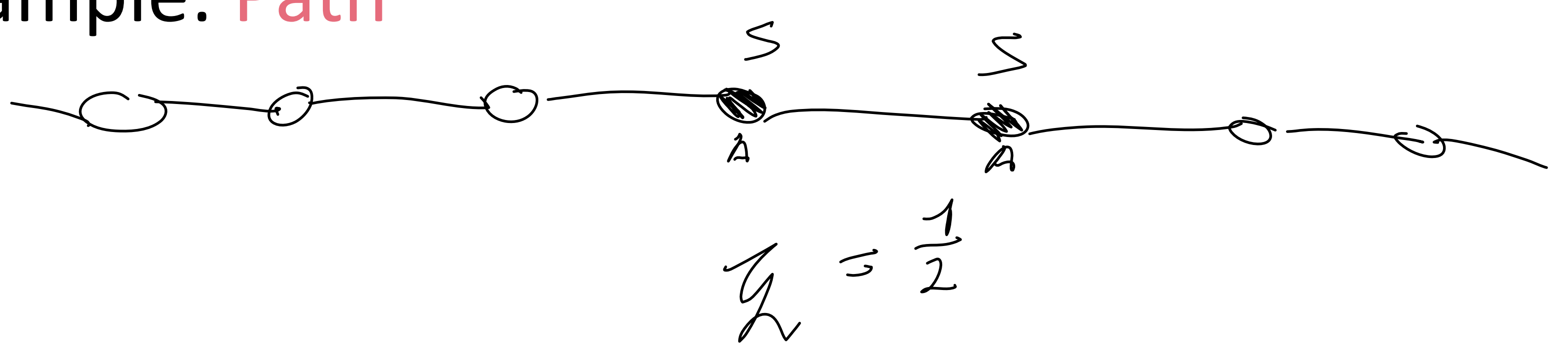
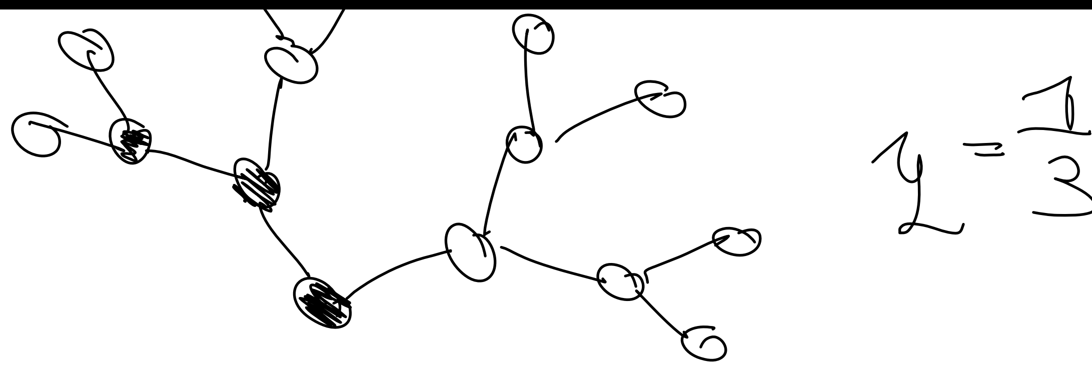
# Infinite graphs

- Consider infinite graph *G* (but each node has finite number of neighbors)
- We say that a finite set *S* causes a cascade in *G* with threshold *q* if when *S* adopts *A* eventually every node adopts *A*.
- Example: Path

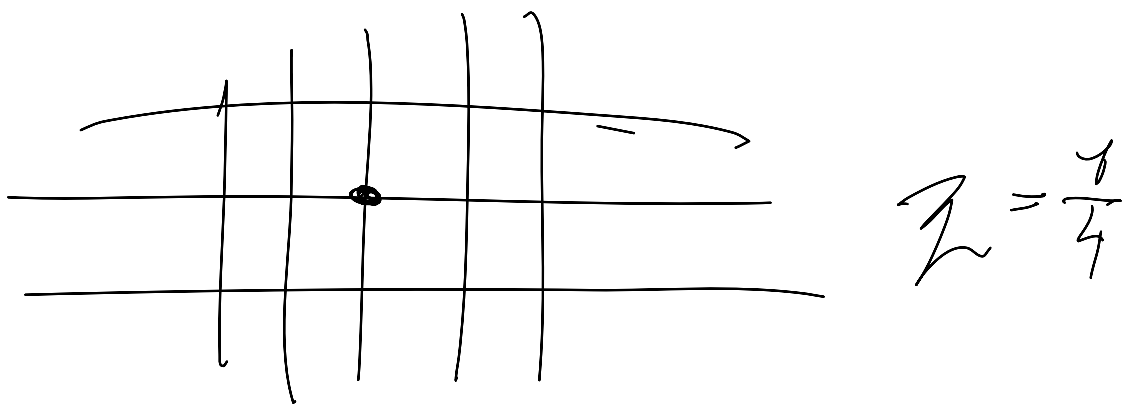


# Infinite graphs

- Tree:



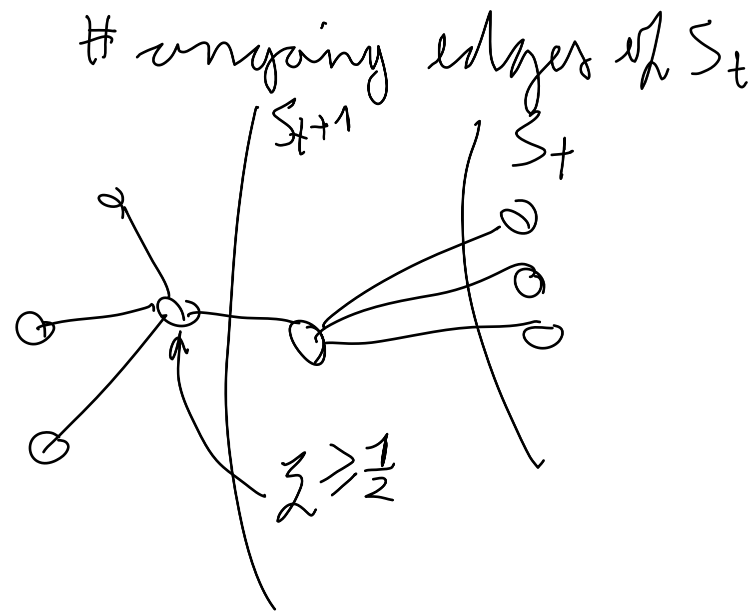
- Grid:



# Cascade threshold

- Def: The cascade threshold of a graph *G* is the largest *q* for which some finite set S can cause a cascade
- Fact: There is no *G* where cascade threshold is greater than ½

$$\text{Energy} = |\delta^+(S)|$$

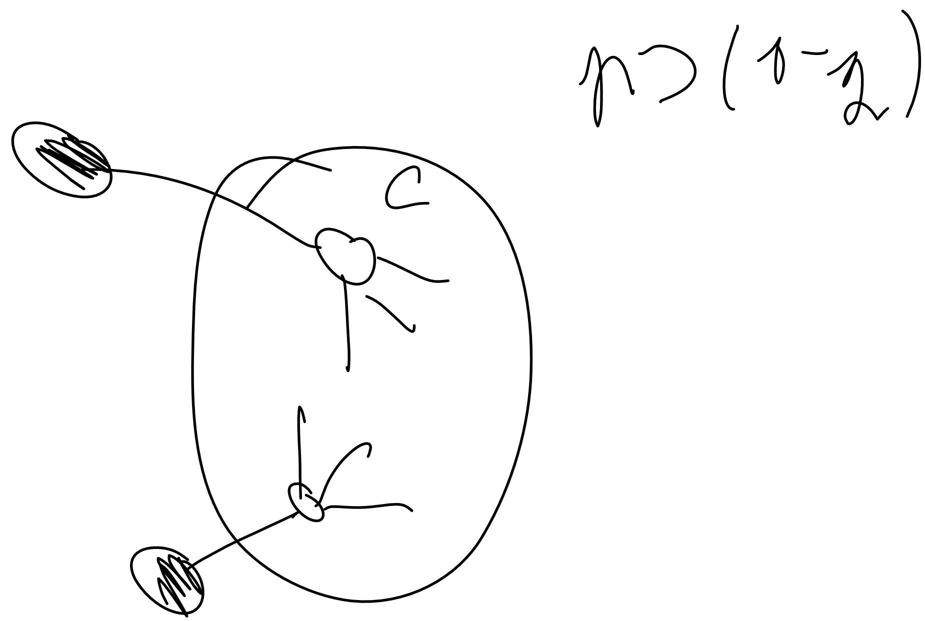


# Stopping cascades

- What prevents cascades from spreading?
- Def: cluster of density $p$ is a set of nodes $C$ where each node in the set has at least $p$ fraction of edges in $C$.

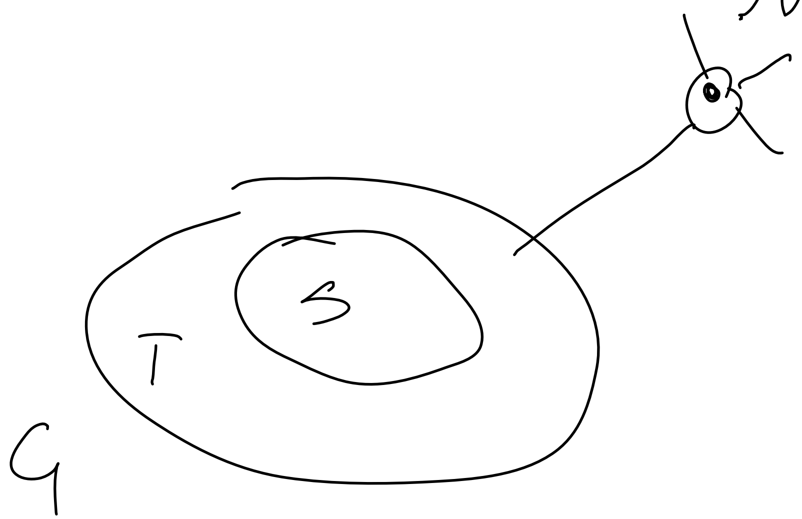
# Stopping cascades

- Fact: If *G\S* contains a cluster of density *>(1-q)* then *S* can not cause a cascade



# Stopping cascades

- <u>Fact:</u> If S fails to create a cascade, then there is a cluster of density >(1-q) in G\S

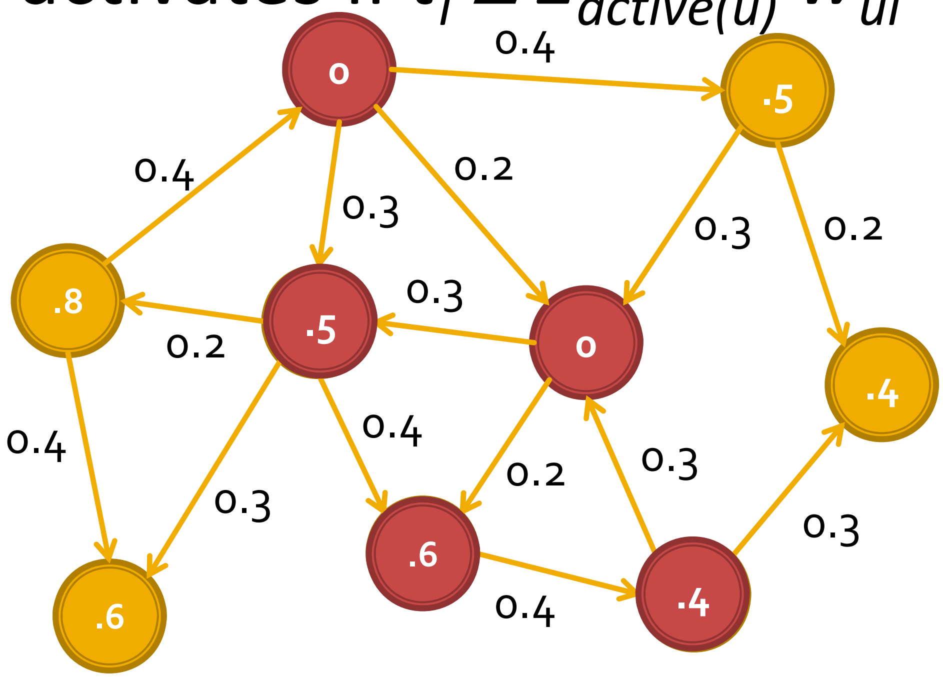



$v \in G \setminus T$, $<q$-frac of its nbrs not in T

$\Rightarrow >(1-q)$ edges of v have to be $G \setminus T$

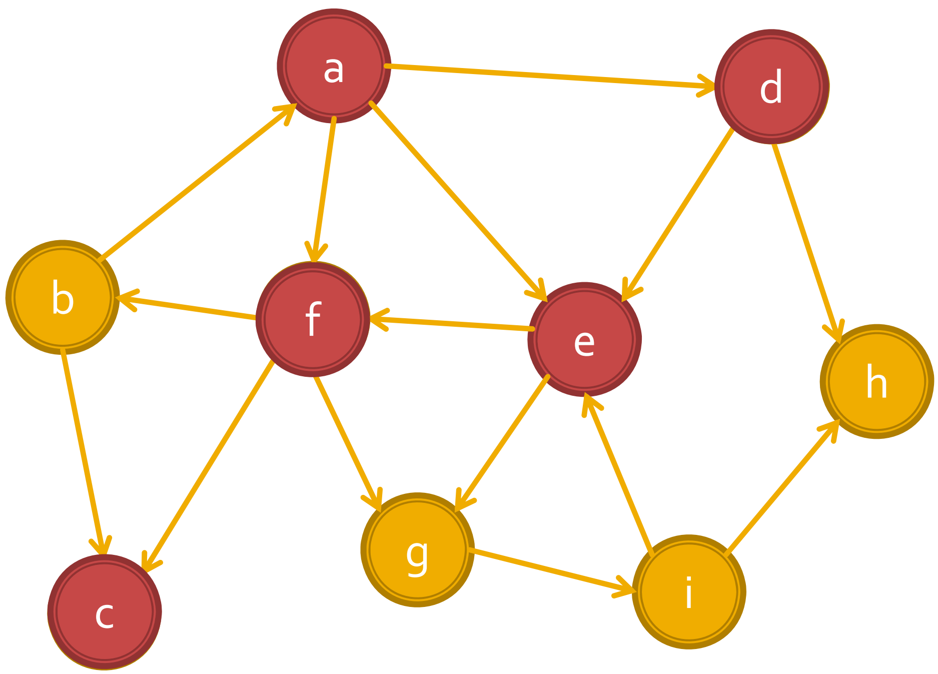
# Generalization

- Initially some nodes are active
- Each edge *(u,v)* has weight $w_{uv}$
- Each node i has a threshold $t_i$
- Node i activates if $t_i \leq \Sigma_{active(u)}\ w_{ui}$



# Threshold Model

- Initially some nodes are active
- Active nodes spread their influence on the other nodes, and so on …

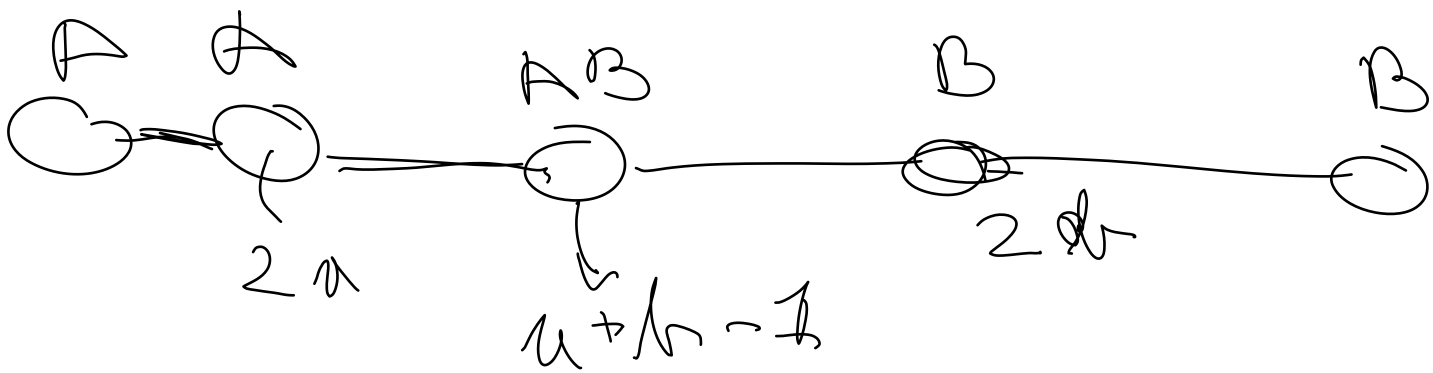


# Cascades & Compatibility

- So far:
  - behaviors *A* and *B* compete
  - Can only get utility from neighbors of same behavior: *A-A* get *a*, *B-B* get *b*, *A-B* get *0*
- Let's add extra strategy "*A-B*" – can choose both
  - *AB-A*: gets *a*
  - *AB-B*: gets *b*
  - *AB-AB*: gets max(*a*, *b*)
  - Also: some cost *c* for the effort of maintaining both strategies (summed over all interactions)

# Model

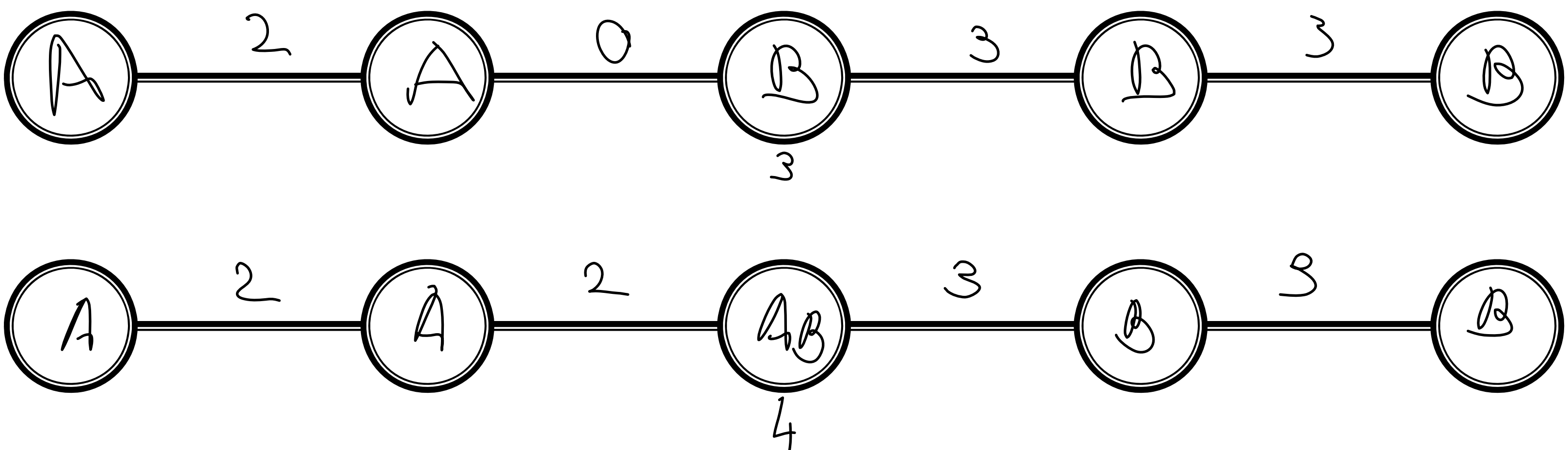
- Every node in an infinite network starts with *B*
- Then a finite set *S* initially adopts *A*
- Run the model for *t=1,2,3,…*
  - Each node selects behavior that will optimize payoff (given what neighbors did in at time *t-1*)




- How will nodes switch from *B* to *A* or *AB*?

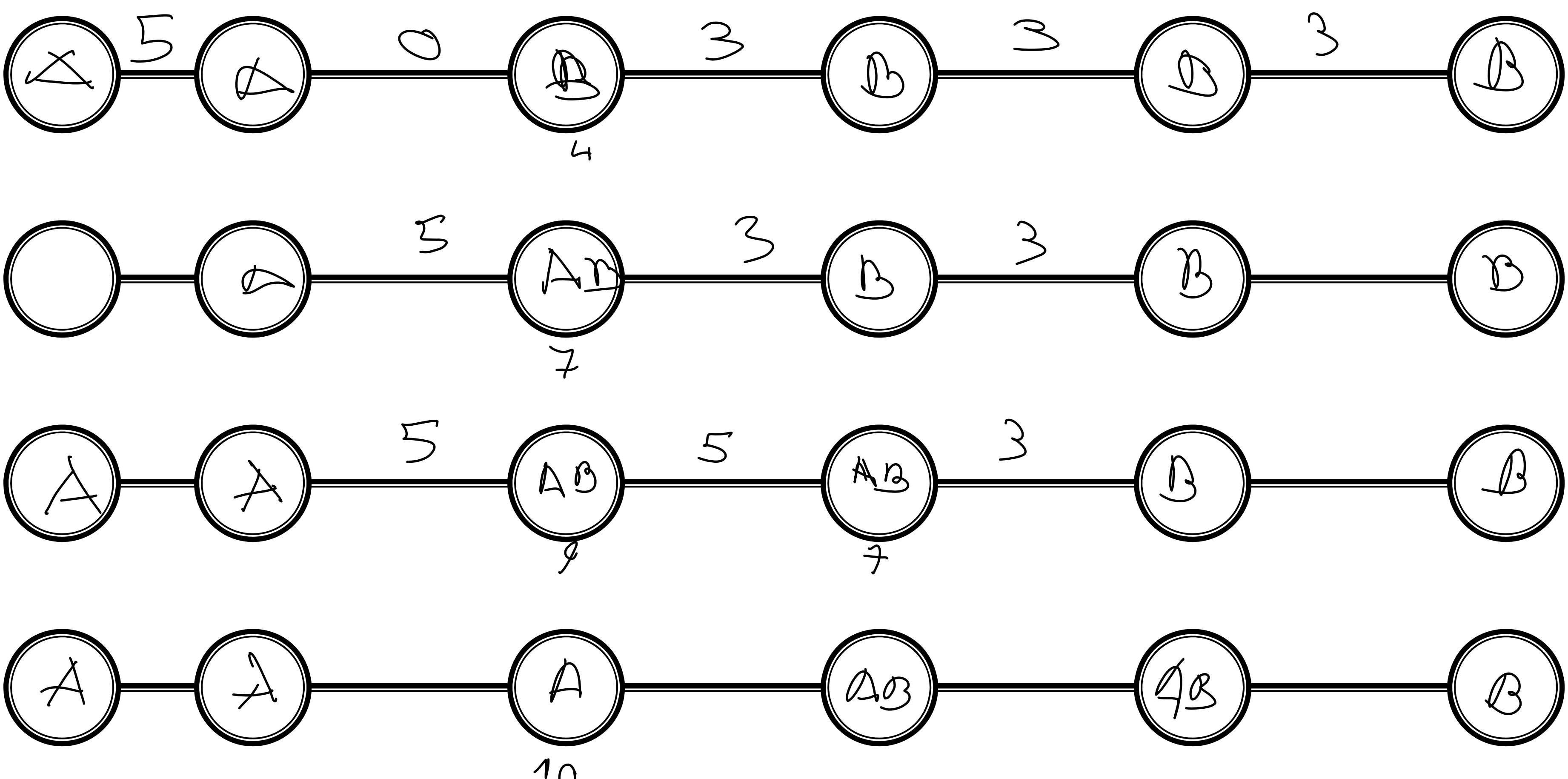
# Example

- Path: start with all Bs, a>b (A is better)
- One node switches to A – what happens?
  - With just A, B: A spreads if $b \leq a$
  - With A, B, AB: Does A spread?
- Let a=2, b=3, c=1



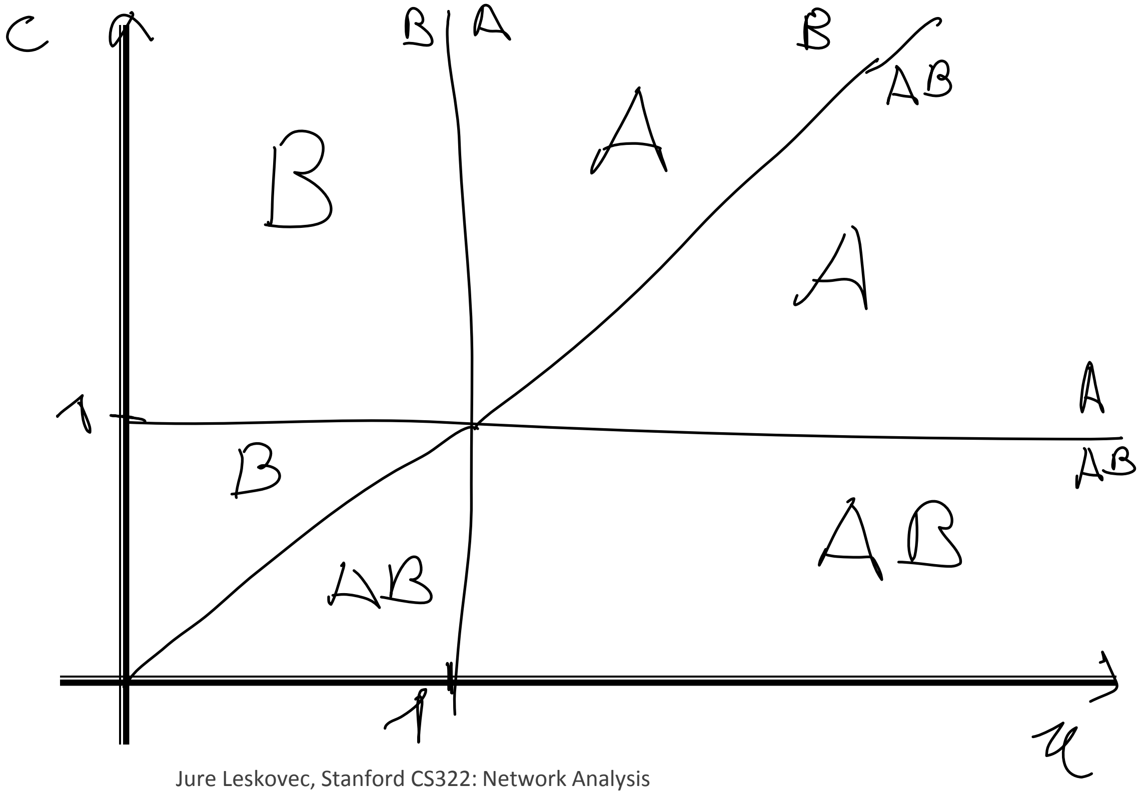
# Example

- Let a=5, b=3, c=1



# For what pairs (c,a) does A spread?

- Assume infinite path
- Payoffs: A:a, B:1, AB:a+1-c
- What does w in A-w-B do?

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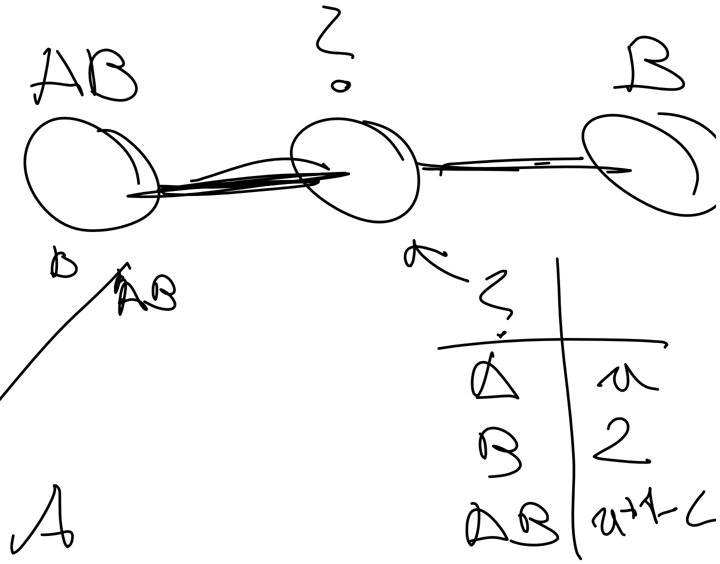


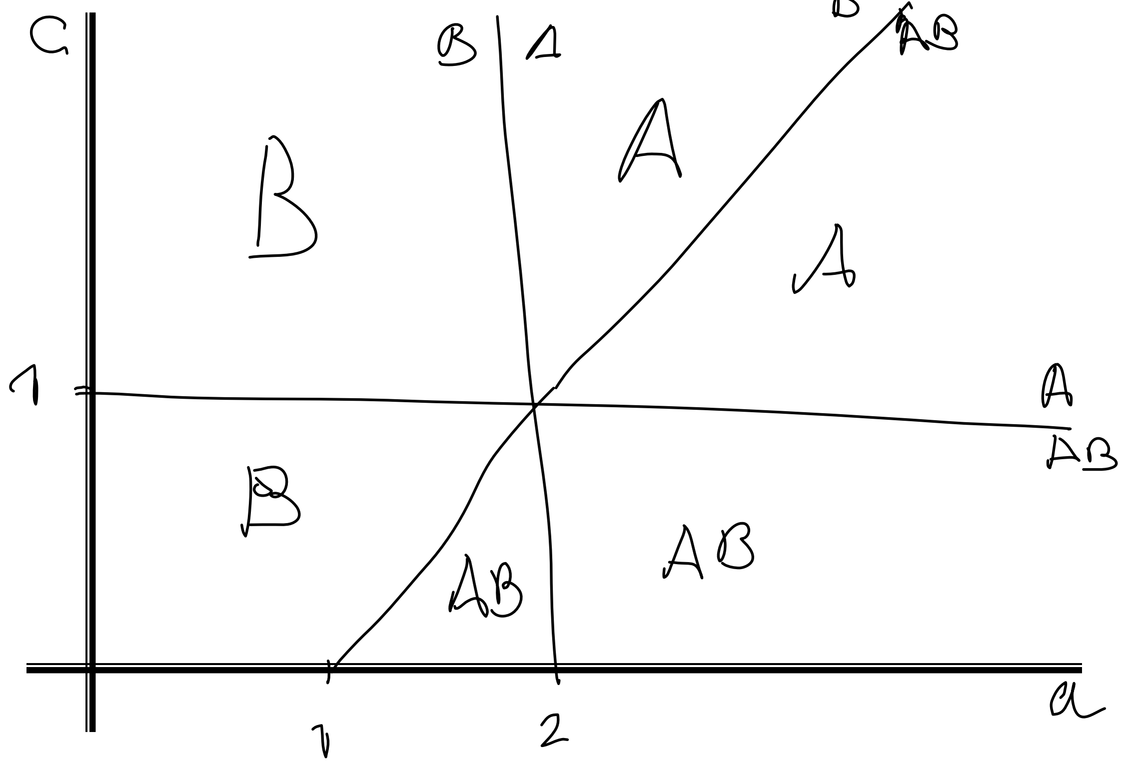

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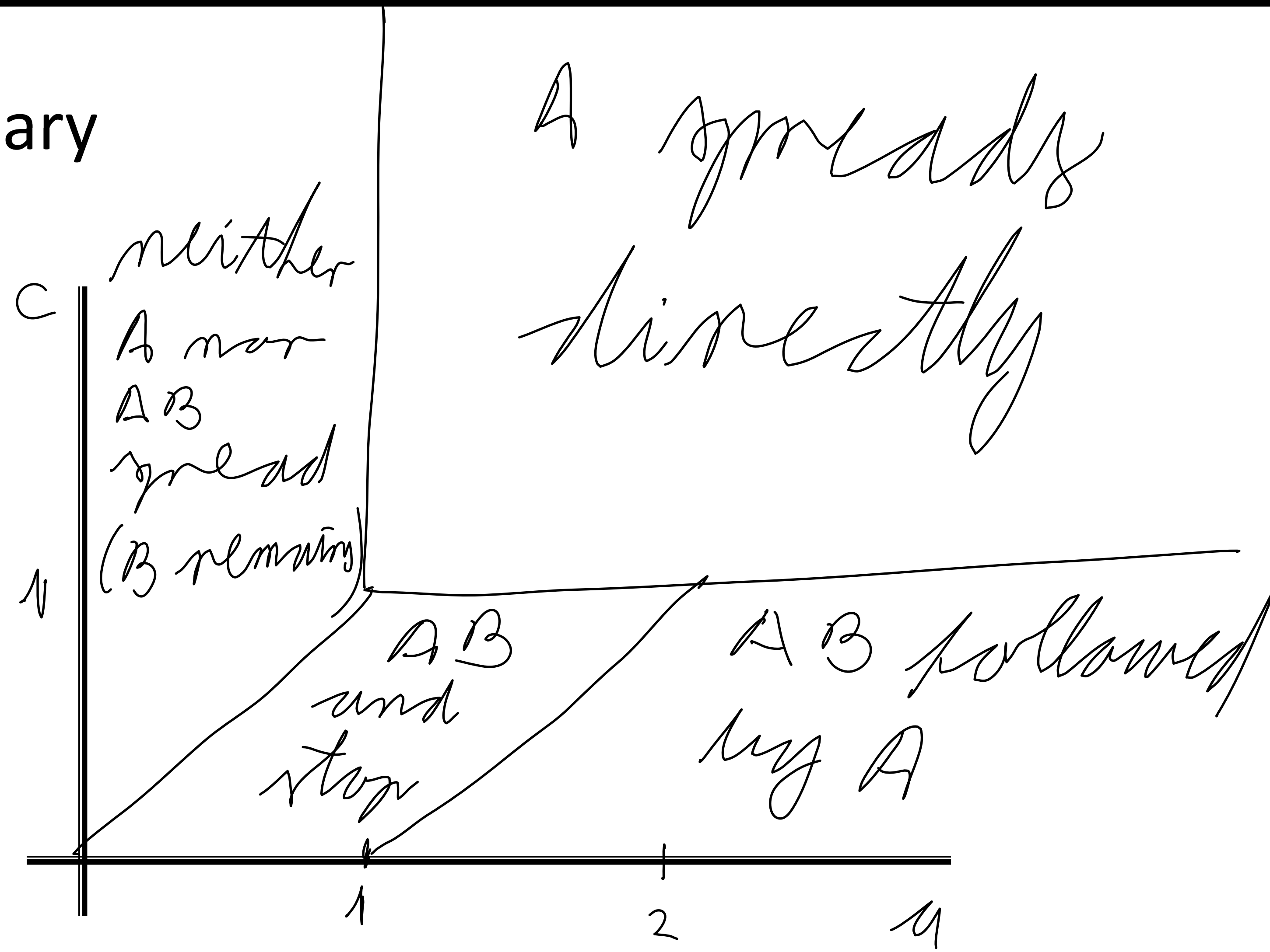
- Summary



# Lesson

- You manufacture default B and new/better A comes along:
  - If you make B too compatible then people will take on both and then drop the worse one (B).
  - If A makes itself not compatible – people on the border must choose. They pick the better one (A)
  - If you choose an optimal level then you keep a "buffer" between A and B