Announcements

Late period deadline for HW4 will be 11:59 AM on Monday so that we can release solutions prior to the exam.

Extend the exam period to Monday 2 PM - Wednesday 2PM

Gradient Boosted Decision Trees

- **Prediction at round t is:** $\hat{y}_i^{(t)} = \hat{y}_i^{(t-1)} + f_t(x_i)$
- **Goal: Find tree $f_t(\cdot)$ that minimizes:**

$$\mathrm{obj}^{(t)} = \sum_{i=1}^{n} l\left(y_i, \hat{y}_i^{(t)}\right) + \omega\left(f_t\right)$$

- The optimal objective is:

$$\mathrm{obj}^* = -\frac{1}{2}\sum_{j=1}^{T}\frac{G_j^2}{H_j+\lambda} + \gamma T$$

  - G_j, H_j depend on the loss function, T= # of leaves.

In principle we could:

- Enumerate possible tree structures f and take the one that minimizes obj

How to find a single tree f_t

- **In practice we grow tree greedily:**
  - Start with tree with depth 0
  - For each leaf node in the tree, try to add a split
  - The change of the objective after adding a split is:

$$Gain = \frac{1}{2}\left[\frac{G_L^2}{H_L+\lambda} + \frac{G_R^2}{H_R+\lambda} - \frac{(G_L+G_R)^2}{H_L+H_R+\lambda}\right] - \gamma$$

Score of left child — Score of right child — Score if we do not split

  - Take the split that gives best gain
- **Next: How to find the best split?**

How to Find the Best Split?

- **For each node, enumerate over all features**
  - For each feature, sort the instances by feature value
  - Use a linear scan to decide the best split along that feature
  - Take the best split solution along all the features
- **Pre-stopping:**
  - Stop split if the best split have negative gain
  - But maybe a split can benefit future splits.
- **Post-Prunning:**
  - Grow a tree to maximum depth, recursively prune all the leaf splits with negative gain.

Summary: GBDT Algorithm

- **Add a new tree $f_t(x)$ in each iteration**
  - Compute necessary statistics for our objective

$$g_i = \partial_{\hat{y}^{(t-1)}} l(y_i, \hat{y}^{(t-1)}), \quad h_i = \partial^2_{\hat{y}^{(t-1)}} l(y_i, \hat{y}^{(t-1)})$$

  - Greedily grow the tree that minimizes the objective:

$$Obj = -\frac{1}{2} \sum_{j=1}^{T} \frac{G_j^2}{H_j + \lambda} + \gamma T$$

- **Add $f_t(x)$ to our ensemble model**

$$y^{(t)} = y^{(t-1)} + \epsilon f_t(x_i)$$

ϵ is called step-size or shrinkage, usually set around 0.1

Goal: prevent overfitting

- **Repeat until we user M ensemble of trees**

XGBoost

- XGBoost: eXtreme **G**radient **B**oosting
  - A highly scalable implementation of gradient boosted decision trees with regularization

  Widely used by data scientists and provides state-of-the-art results on many problems!

- System optimizations:
  - Parallel tree constructions using column block structure
  - Distributed Computing for training very large models using a cluster of machines.
  - Out-of-Core Computing for very large datasets that don’t fit into memory.

Advertising on the Web

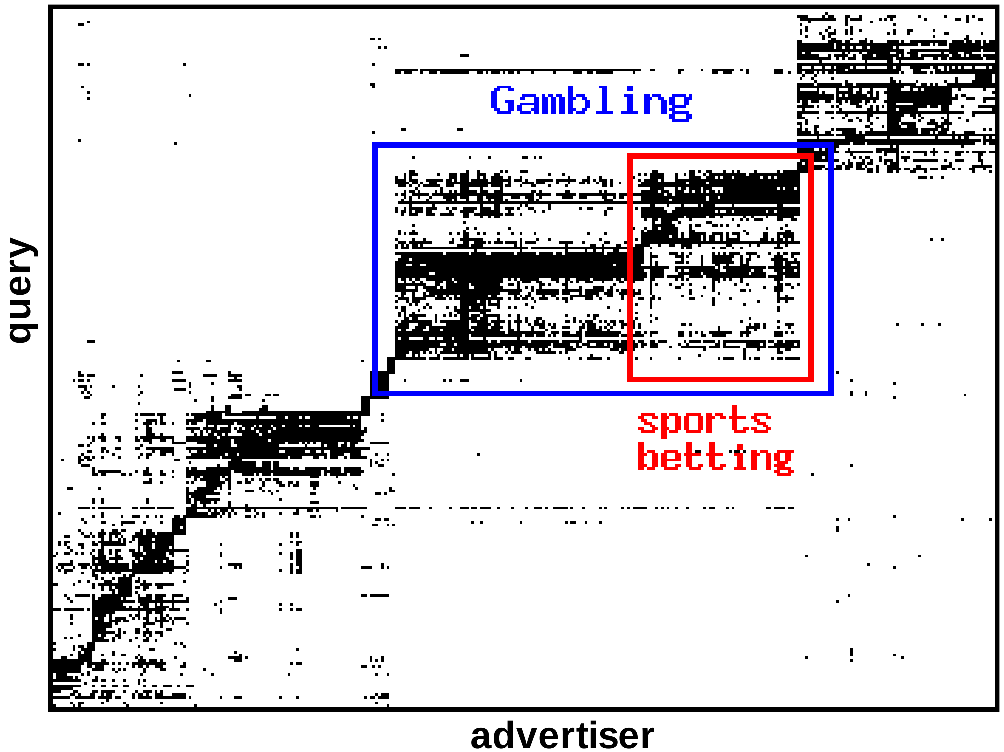
CS246: Mining Massive Datasets
Jure Leskovec, Stanford University
Charilaos Kanatsoulis, Stanford University
http://cs246.stanford.edu

Online Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, "**offline** algorithm"
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

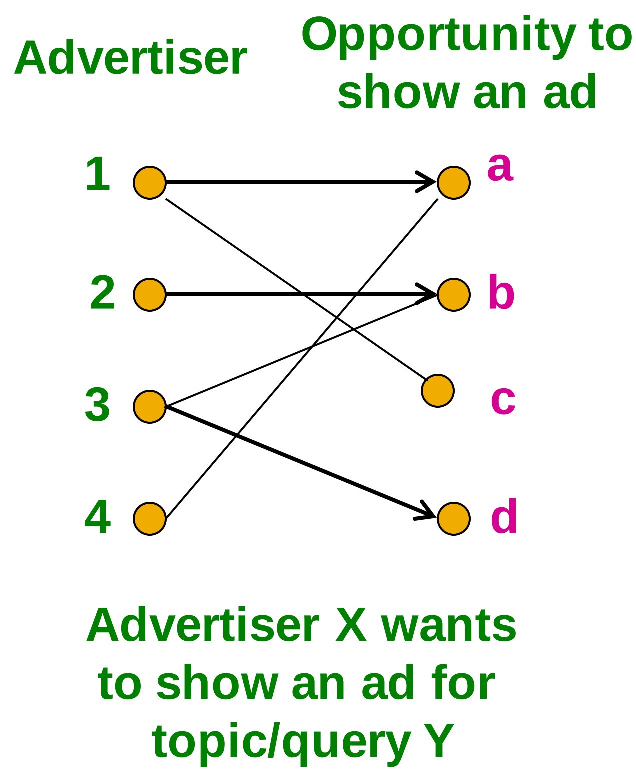
Sponsored Search: Ads

- **Query-to-advertiser graph:**



[Andersen, Lang: Communities from seed sets, 2006]

Graph Matching for Advertising



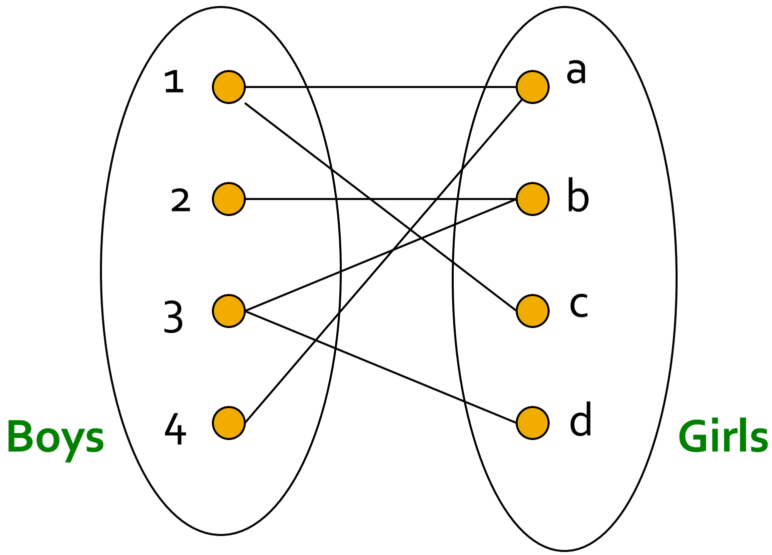
Which advertiser gets picked

(1,a)
(2,b)
(3,d)

This is an online problem: We have to make decisions as queries/topics show up. We do not know what topics will show up in the future.

Online Bipartite Matching

Example: Bipartite Matching

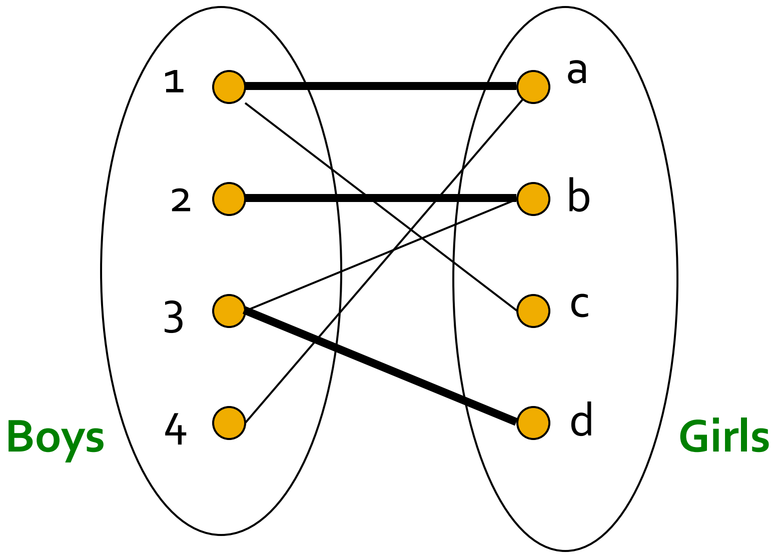


Nodes: Boys and Girls; Links: Preferences

Goal: Match boys to girls so that the most preferences are satisfied

Note: edges are only preferences with no weight or order.

Example: Bipartite Matching

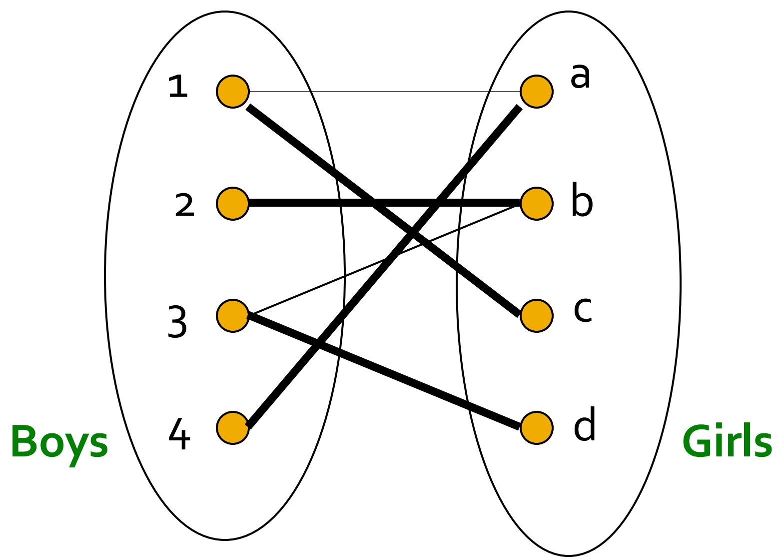


M = {(1,a),(2,b),(3,d)} is a **matching**

Cardinality of matching = |M| = 3

Matching means that we are not using any vertex twice

Example: Bipartite Matching



M = {(1,c),(2,b),(3,d),(4,a)} is a **perfect matching**

Perfect matching … all vertices of the graph are matched
Maximum matching … matching that contains the largest possible number of matches

Matching Algorithm

- **Problem: Find a maximum matching for a given bipartite graph**
  - A perfect one if it exists

- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)

- **But what if we do not know the entire graph upfront?**

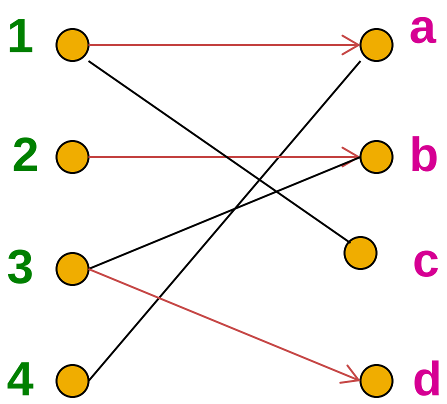
Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
  - That is, the girl's **edges** are revealed
- **At that time, we have to decide to either:**
  - Pair the **girl** with a **boy**
  - Do not pair the **girl** with any **boy**
- **Example of application:**

  Assigning tasks to servers

Note: Matching means that we are not using any girl or boy twice

Online Graph Matching: Example



(1,a)
(2,b)
(3,d)

Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair the girl

- **How good is the algorithm?**

Competitive Ratio

- For input ***I***, suppose greedy produces matching $\boldsymbol{M_{greedy}}$ while an optimal matching is $\boldsymbol{M_{opt}}$

Competitive ratio =

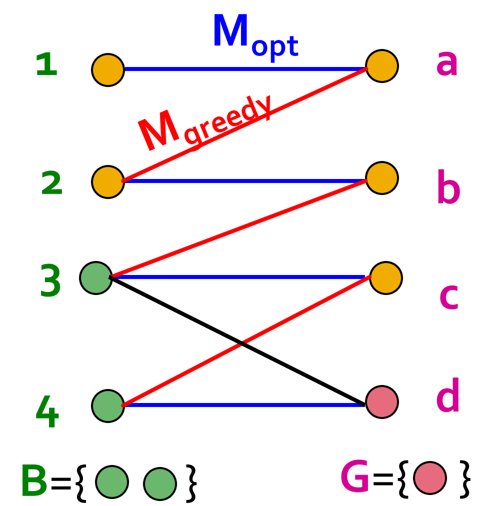
$$\min_{\text{all possible inputs } I} \left(|M_{greedy}| / |M_{opt}|\right)$$

(what is greedy's <u>worst</u> performance <u>over all possible</u> inputs *I*)

Analyzing the Greedy Algorithm

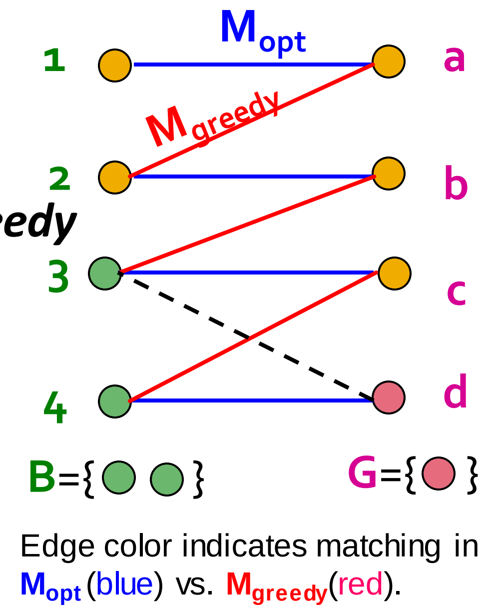
- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set G of girls matched in M_{opt} but not in M_{greedy}
- **(1)** By definition of G:
  $|M_{opt}| \leq |M_{greedy}| + |G|$
- **(2)** Define set ***B of boys*** linked to girls in G
  - Notice boys in B are already matched in M_{greedy}. Why?
    - If there would exist such non-matched (by M_{greedy}) boy adjacent to a non-matched girl then greedy would have matched them

So: $|M_{greedy}| \geq |B|$



Edge color indicates matching in M_{opt} (blue) vs. M_{greedy} (red).

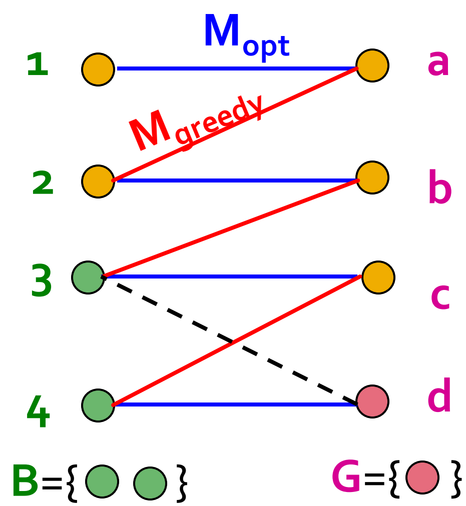
Analyzing the Greedy Algorithm



- **Summary so far:**
  - Girls G matched in M_{opt} but not in M_{greedy}
  - Boys B adjacent to girls in G
  - **(1)** $|M_{opt}| \leq |M_{greedy}| + |G|$
  - **(2)** $|M_{greedy}| \geq |B|$
- Optimal matches all girls in G to (some) boys in B
  - **(3)** $|G| \leq |B|$
- Combining **(2)** and **(3)**:
  - **(4)** $|G| \leq |B| \leq |M_{greedy}|$

Analyzing the Greedy Algorithm

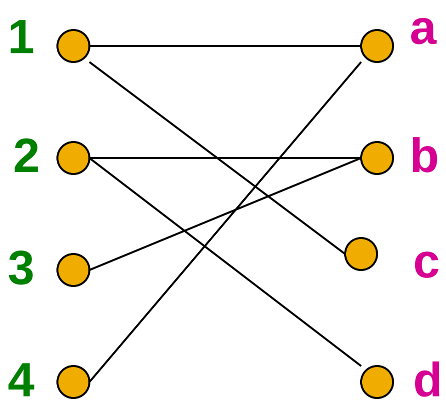
- **So we have:**
  - **(1)** $|M_{opt}| \leq |M_{greedy}| + |G|$
  - **(4)** $|G| \leq |B| \leq |M_{greedy}|$



Edge color indicates matching in M_{opt} (blue) vs. M_{greedy} (red).

- Combining **(1)** and **(4)**:
  - Worst case is when $|G| = |B| = |M_{greedy}|$
  - $|M_{opt}| \leq |M_{greedy}| + |M_{greedy}|$
  - **Then** $|M_{greedy}|/|M_{opt}| \geq 1/2$

Worst-case Scenario



(1,a)
(2,b)

Web Advertising

History of Web Advertising

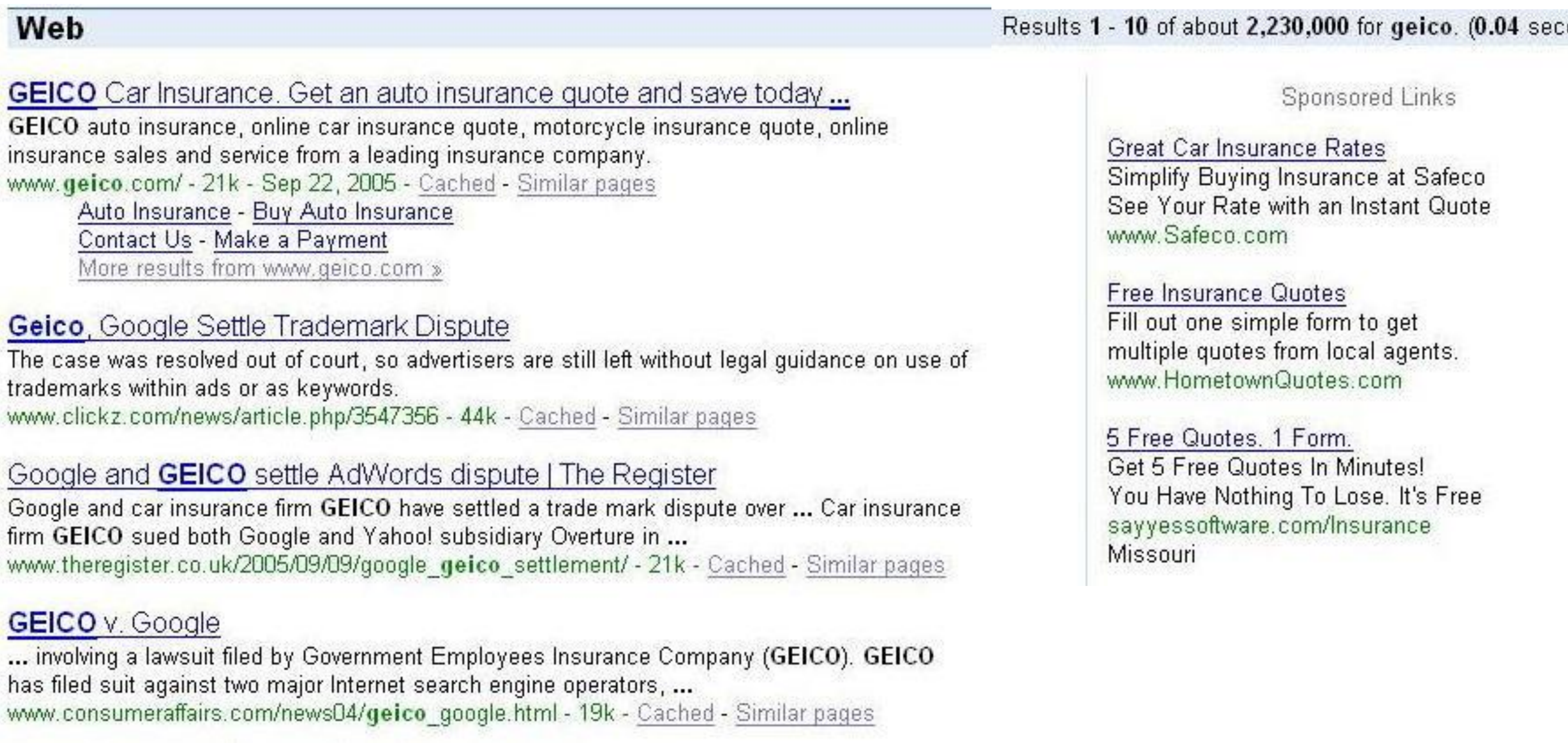
- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged $X for every 1,000 "impressions" of the ad
    - Called "**CPM**" rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI for advertisers

CPM…cost per *mille*
Mille…thousand in Latin

Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

Ads vs. Search Results



Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry
- **Interesting problem:**
  Which ads to show for a given query?
  - (Today's lecture)
- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (Not focus of today's lecture)

AdWords Problem

- A stream of queries arrives at the search engine: $q_1, q_2, \ldots$
- Several advertisers bid on each query
- When query q_i arrives, search engine must pick a subset of advertisers to show their ads

- Goal: **Maximize search engine's revenues**
  - Simple solution: Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.25 cents

Click through rate

Expected revenue

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.25 cents
A	$1.00	1%	1 cent

Instead of sorting advertisers by bid, sort by expected revenue

Limitations of Simple Algorithm

Instead of sorting advertisers by bid, sort by expected revenue

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.25 cents
A	$1.00	1%	1 cent

Challenges:

- CTR of an ad is unknown
- Advertisers have limited budgets and bid on multiple queries

Complications: Budget

- **Two complications:**
  - **Budget**
  - **CTR of an ad is unknown**

1) Budget: Each advertiser has a limited budget

- **Search engine guarantees that the advertiser will not be charged more than their daily budget**

Complications: CTR

- **2) CTR (Click-Through Rate): Each ad-query pair has a different likelihood of being clicked**
  - **Advertiser 1** bids \$2 on query A, click probability = 0.1
  - **Advertiser 2** bids \$1 on query B, click probability = 0.5
- **CTR** is predicted or measured historically
  - Averaged over a time period
- **Some complications we will <u>not</u> cover:**
  - **1) CTR is position dependent:**
    - Ad #1 is clicked more than Ad #2

Complications: CTR

- **Some complications we will cover:**
  - **2) Exploration vs. exploitation**

    Exploit: Should we keep showing an ad for which we have good estimates of click-through rate?

    or

    Explore: Shall we show a brand new ad to get a better sense of its click-through rate?

Online Algorithms
The BALANCE Algorithm

Adwords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query
- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Bid value of each ad is the same (=$**1**)
- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

Bad Scenario for Greedy

- **Two advertisers A and B**
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** **_ _ _ _**
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the <u>largest unspent budget</u>**
    - Break ties arbitrarily (**but in a deterministic way**)

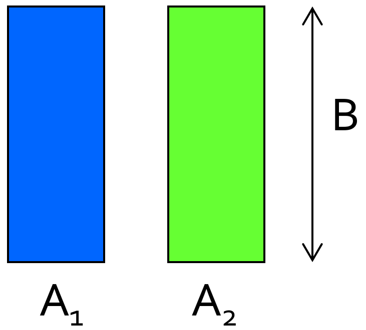
Example: BALANCE

- Two advertisers A and B
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- Query stream: ***x x x x y y y y***
- BALANCE choice: **A B A B B B _ _**
  - Optimal: **A A A A B B B B**
- **In general:** For **BALANCE** on **2** advertisers
  Competitive ratio = ¾

Analyzing BALANCE

- **Consider simple case (w.l.o.g.):**
  - **2** advertisers, $\mathbf{A_1}$ and $\mathbf{A_2}$, each with budget **B** ($\geq$1)
  - Optimal solution exhausts both advertisers' budgets
- **BALANCE must exhaust at least one budget:**
  - **If not, we can allocate more queries**
    - Whenever BALANCE makes a mistake (both advertisers bid on the query), advertiser's unspent budget only decreases
    - Since optimal exhausts both budgets, one will for sure get exhausted
  - Assume BALANCE exhausts A_2's budget, but allocates x queries fewer than the optimal
    - **So revenue of** ***BALANCE = 2B – x*** **(where OPT is 2B)**
  - **Let's work out what x is!**

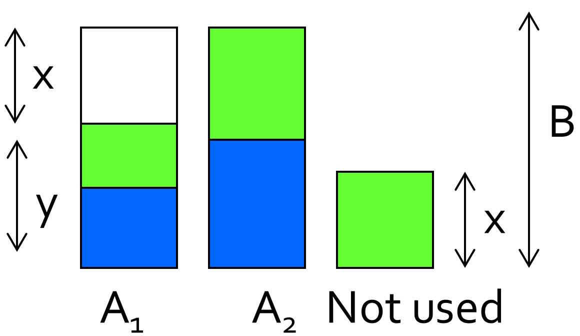
Analyzing Balance



Queries allocated to A_1 in optimal solution

Queries allocated to A_2 in optimal solution

Opt revenue = 2B

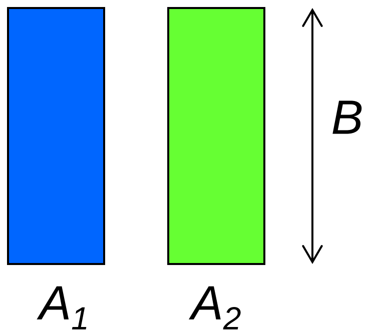


Balance allocation

Balance revenue = 2B-x = B+y

We claim y ≥ x (next slide).
Balance revenue is minimum for x=y=B/2.
Minimum Balance revenue = 3B/2.
Competitive Ratio = 3/4.

Analyzing BALANCE: What's x?



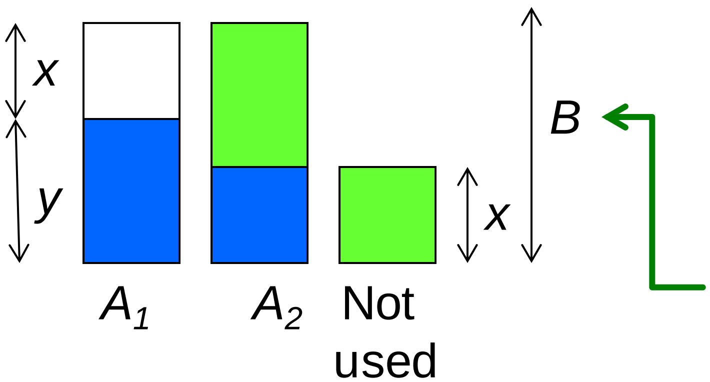
Queries allocated to A_1 in the optimal solution

Queries allocated to A_2 in the optimal solution

Optimal revenue = **2B**

Assume Balance gives revenue **= 2B-x = B+y**

Assume we exhausted A_2's budget



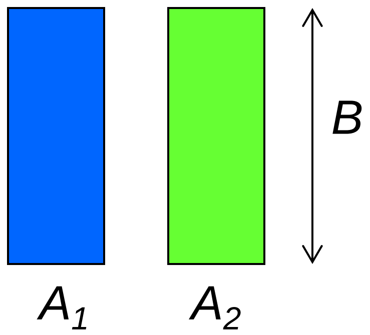
Notice: Unassigned queries should be assigned to A_2 (since if we could assign to A_1 we would since we still have the budget)

Goal: Show we have $\mathbf{y \geq B/2}$

Case 1) BALANCE assigns $\geq$B/2 blue queries to A_1.

Then trivially, $y \geq B/2$

Analyzing BALANCE: What's x?



Queries allocated to A_1 in the optimal solution

Queries allocated to A_2 in the optimal solution

Optimal revenue = **2B**

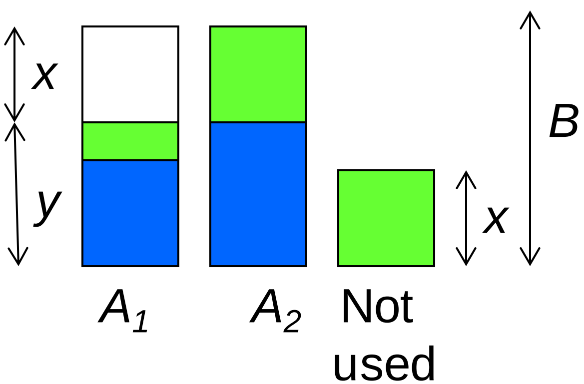
Assume Balance gives revenue **= 2B-x = B+y**

Assume we exhausted A_2's budget

Unassigned queries should be assigned to A_2

(if we could assign to A_1 we would since we still have the budget)

Goal: Show we have **$y \geq B/2$**



Balance revenue is minimum for $x = y = B/2$

Minimum Balance revenue = $\mathbf{3B/2}$

Competitive Ratio: BAL/OPT = 3/4

Case 2) BALANCE assigns >B/2 blue queries to A_2.

Consider the last blue query assigned to A_2.

At that time, A_2's unspent budget must have been at least as big as A_1's.

That means at least as many queries have been assigned to A_1 as to A_2.

At this point, we have already assigned at least B/2 queries to A_2.

So, $x \leq B/2$ **and** $x + y = B$ **then** $y > B/2$

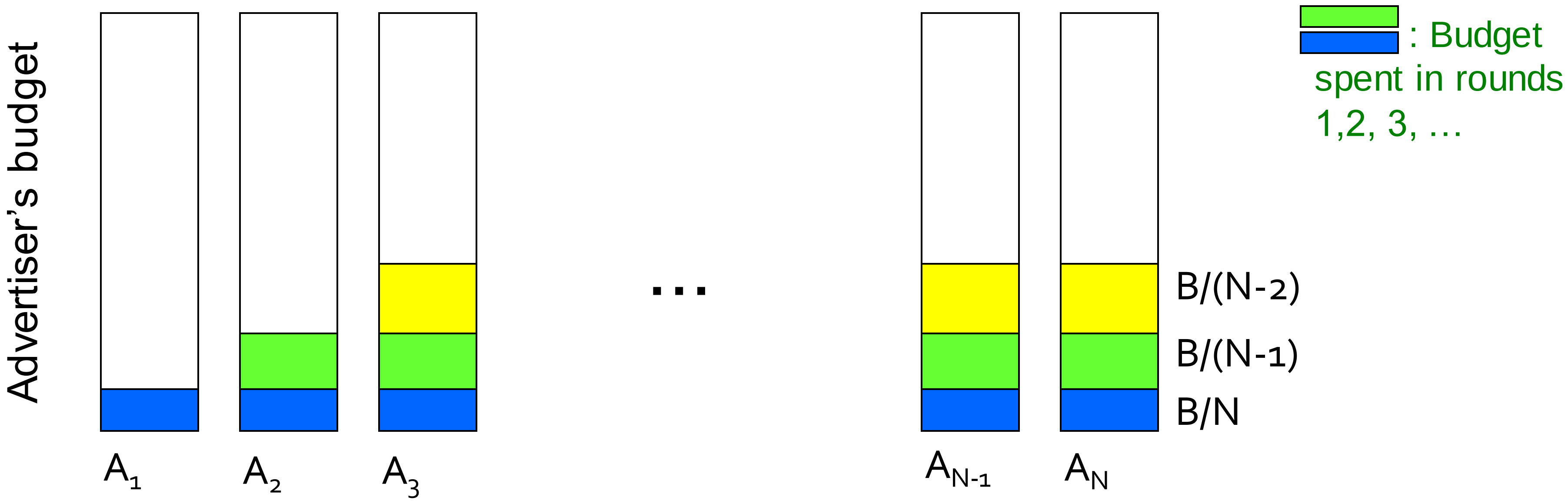
BALANCE: General Result

- **In the general case, worst competitive ratio of BALANCE is 1–1/e = approx. 0.63**
  - ***e*** **= 2.7182**
  - Interestingly, no online algorithm has a better competitive ratio!
- **Let's see the worst case example that gives this ratio**

Worst case for BALANCE

- ***N* advertisers:** $A_1, A_2, \ldots A_N$
  - Each with budget $B > N$
- **Queries:**
  - $N \cdot B$ queries appear in N rounds of B queries each
- **Bidding:**
  - Round **1** queries: bidders $A_1, A_2, \ldots, A_N$
  - Round **2** queries: bidders $A_2, A_3, \ldots, A_N$
  - Round ***i*** queries: bidders $A_i, \ldots, A_N$
- **Optimum allocation:**
  Allocate all round ***i*** queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE Allocation



BALANCE assigns each of the queries in round 1 to **N** advertisers. After ***k*** rounds, sum of allocations S_k to each of advertisers $A_k,\dots,A_N$ is $S_k = S_{k+1} = \cdots = S_N = \sum_{i=1}^{k} \frac{B}{N-(i-1)}$

If we find the smallest *k* such that $S_k \geq B$, then after *k* rounds we cannot allocate any queries to any advertiser

BALANCE: Analysis

B/1 B/2 B/3 … B/(N-(k-1)) … B/(N-1) B/N

S_1

S_2

S_k **= B**

Can divide everything by B:

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

S_1

S_2

S_k **= 1**

BALANCE: Analysis

- **Fact:** $H_n = \sum_{i=1}^{n} 1/i \approx \ln(n)$ for large n
  - Result due to Euler

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

$\xleftrightarrow{\hspace{10em}}$ ln(N)

ln(N)-1 $\longleftrightarrow$ $\longleftrightarrow$ $S_k = 1$

- $S_k = 1$ implies: $H_{N-k} = ln(N) - 1 = ln(\frac{N}{e})$
- We also know: $H_{N-k} = ln(N-k)$
- **So:** $N - k = \frac{N}{e}$
- **Then:** $k = N(1 - \frac{1}{e})$

N terms sum to ln(*N*).
Last *k* terms sum to 1.
First *N*-*k* terms sum
to ln(*N*-*k*) but also to ln(*N*)-1

BALANCE: Analysis

- So after the first **k=N(1-1/e)** rounds, we cannot allocate a query to any advertiser
- **Revenue = B·N (1-1/e)**
- **Competitive ratio = 1-1/e**
- **Note: So far we assumed:**
  - **All advertisers have the same budget B**
  - **All advertisers bid 1 for the ad**
  - **(but each advertiser can bid on any subset of ads)**

General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query $\boldsymbol{q}$, advertiser $\boldsymbol{i}$
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$
  - $\boldsymbol{A_1}$: $\boldsymbol{x_1 = 1}$, $\boldsymbol{b_1 = 110}$
  - $\boldsymbol{A_2}$: $\boldsymbol{x_2 = 10}$, $\boldsymbol{b_2 = 100}$
  - Consider we see **10** instances of **q**
  - BALANCE always selects $\boldsymbol{A_1}$ and earns **10**
  - Optimal earns **100**

Generalized BALANCE

- **Arbitrary bids:** consider query $\boldsymbol{q}$, bidder $\boldsymbol{i}$
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
  - Amount spent so far = $\boldsymbol{m_i}$
  - Fraction of budget left over $\boldsymbol{f_i = 1-m_i/b_i}$
  - Define $\boldsymbol{\psi_i(q) = x_i(1-e^{-f_i})}$
- Allocate query $\boldsymbol{q}$ to bidder $\boldsymbol{i}$ with largest value of $\boldsymbol{\psi_i(q)}$
- **Same competitive ratio (1-1/*e*) = 0.63**