PageRank

Random Surfers on the Web
Transition Matrix of the Web
Dead Ends and Spider Traps
Topic-Specific PageRank
Hubs and Authorities

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Intuition – (1)

- Web pages are important if people visit them a lot.
- But we can’t watch everybody using the Web.
- A good surrogate for visiting pages is to assume people follow links randomly.
- Leads to *random surfer* model:
  - Start at a random page and follow random out-links repeatedly, from whatever page you are at.
  - *PageRank* = limiting probability of being at a page.

Intuition – (2)

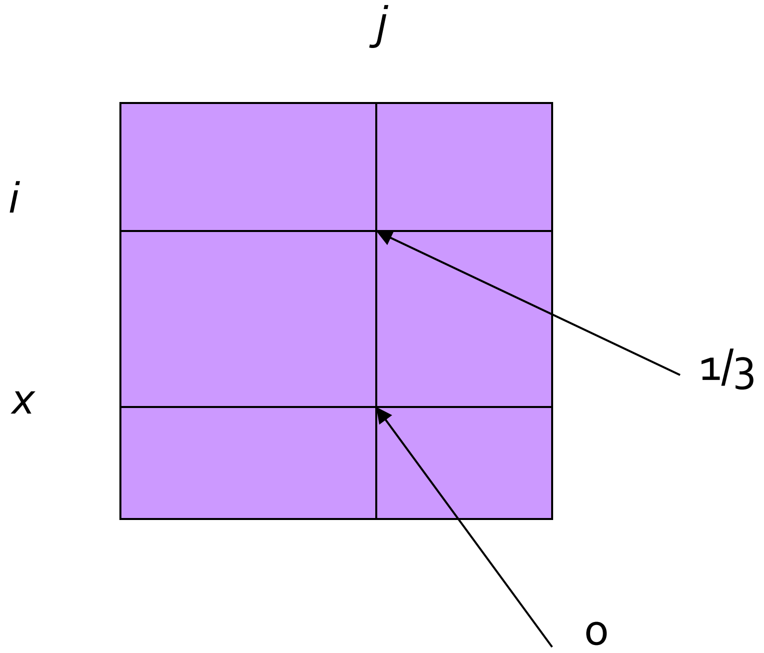
- Solve the recursive equations: "importance of a page = its share of the importance of each of its predecessor pages."
  - Equivalent to the random-surfer definition of PageRank.
- Technically, *importance* = the principal eigenvector of the transition matrix of the Web.
  - A few fixups needed.

Transition Matrix of the Web

- Number the pages 1, 2,… .
  - Page i corresponds to row and column i.
- $M[i, j] = 1/n$ if page j links to n pages, including page i ; 0 if j does not link to i.
  - $M[i, j]$ is the probability a surfer will next be at page i if it is now at page j.
  - Or it is the share of j's importance that i receives.

Example: Transition Matrix

Suppose page *j* links to 3 pages, including *i* but not *x.*



Called a *stochastic matrix* =
"all columns sum to 1."

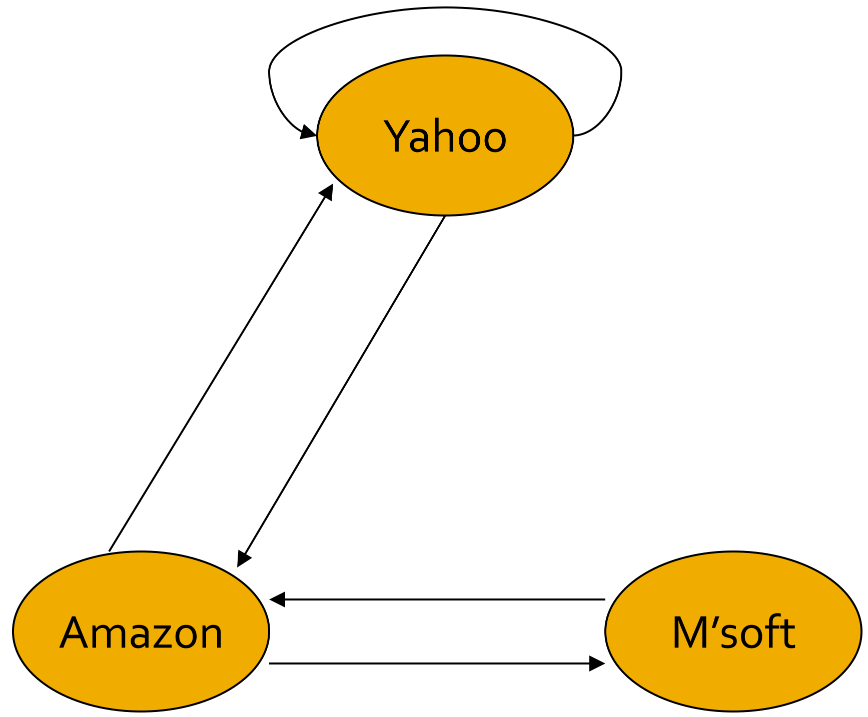
Random Walks on the Web

- Suppose **v** is a vector whose i^{th} component is the probability that a random surfer is at page *i* at a certain time.
- If a surfer chooses a successor page from page *i* at random, the probability distribution for surfers is then given by the vector $M\mathbf{v}$.

Random Walks – (2)

- Starting from any vector **u**, the limit *M* (*M* (…*M* (*M* **u**) …)) is the long-term distribution of the surfers.
- The math: limiting distribution = principal eigenvector of *M* = PageRank.
  - Note: If **v** is the limit of MM…M**u**, then **v** satisfies the equation **v** = M**v**, so **v** is an eigenvector of M with eigenvalue 1.

Example: The Web in 1839



	y	a	m
y	1/2	1/2	0
a	1/2	0	1
m	0	1/2	0

Solving The Equations

- Because there are no constant terms, the equations $\mathbf{v} = M\mathbf{v}$ do not have a unique solution.
  - Example: doubling each component of solution $\mathbf{v}$ yields another solution.
- In Web-sized examples, we cannot solve by Gaussian elimination anyway; we need to use *relaxation* (= iterative solution).

Simulating a Random Walk

- Start with the vector **u** = [1, 1,…, 1] representing the idea that each Web page is given one unit of *importance*.
  - Note: it is more common to start with each vector element = 1/N, where N is the number of Web pages and to keep the sum of the elements at 1.
  - Question for thought: Why such small values?
- Repeatedly apply the matrix *M* to **u**, allowing the importance to flow like a random walk.
- About 50 iterations is sufficient to estimate the limiting solution.

Example: Iterating Equations

- Equations $\mathbf{v} = M\mathbf{v}$:

$y = y/2 + a/2$

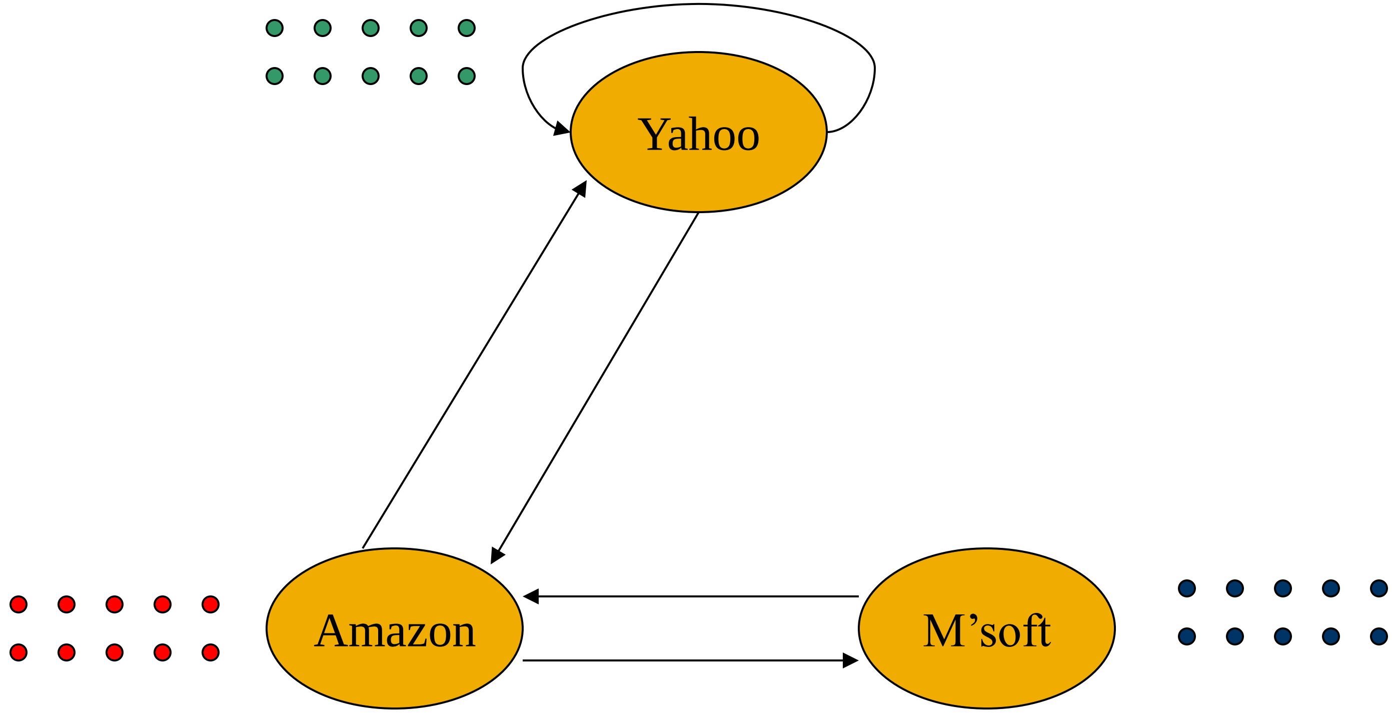
$a = y/2 + m$

$m = a/2$

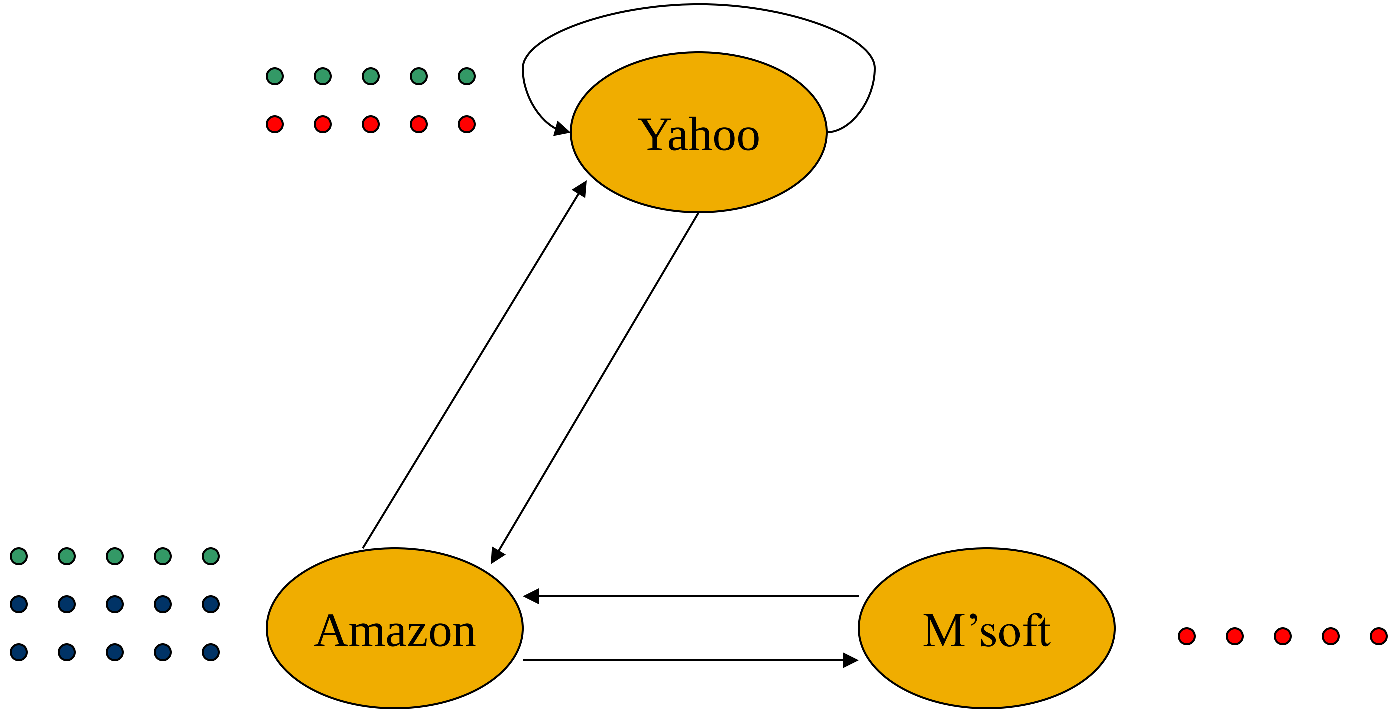
Note: "=" is really "assignment."

y		1	1	5/4	9/8		6/5
a	=	1	3/2	1	11/8	. . .	6/5
m		1	1/2	3/4	1/2		3/5

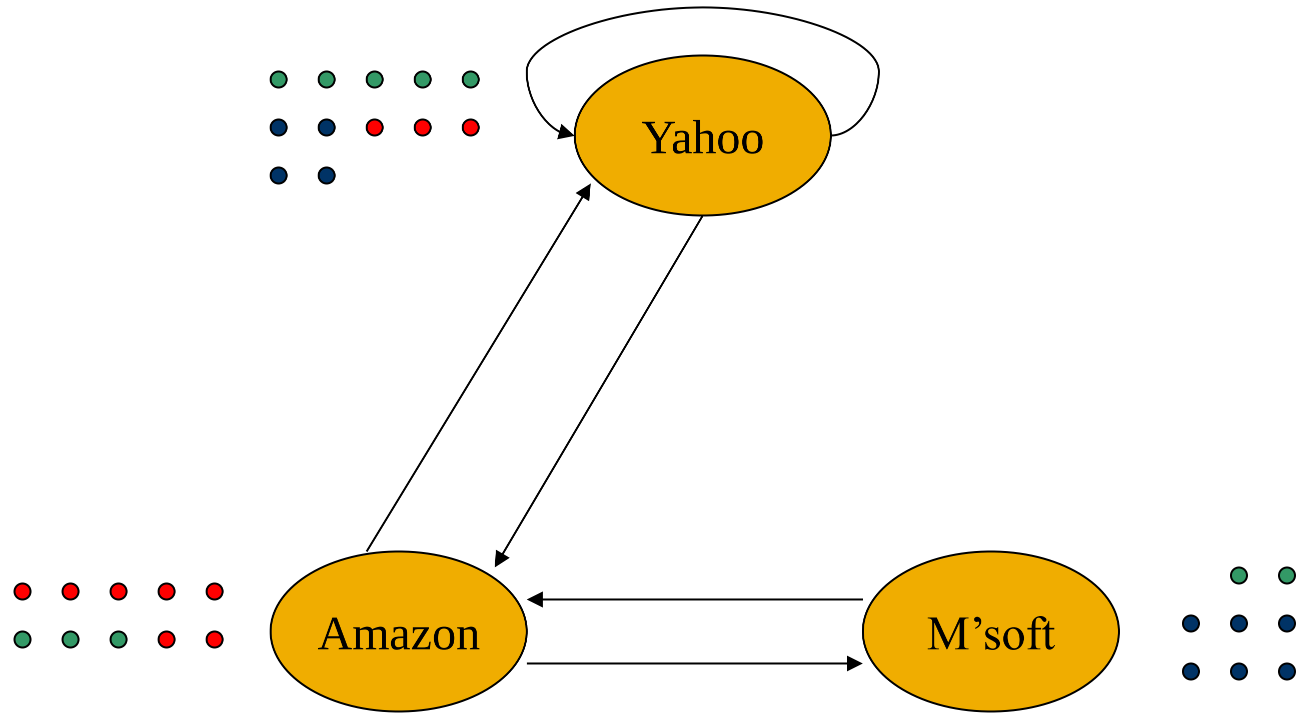
The Surfers



The Surfers



The Surfers



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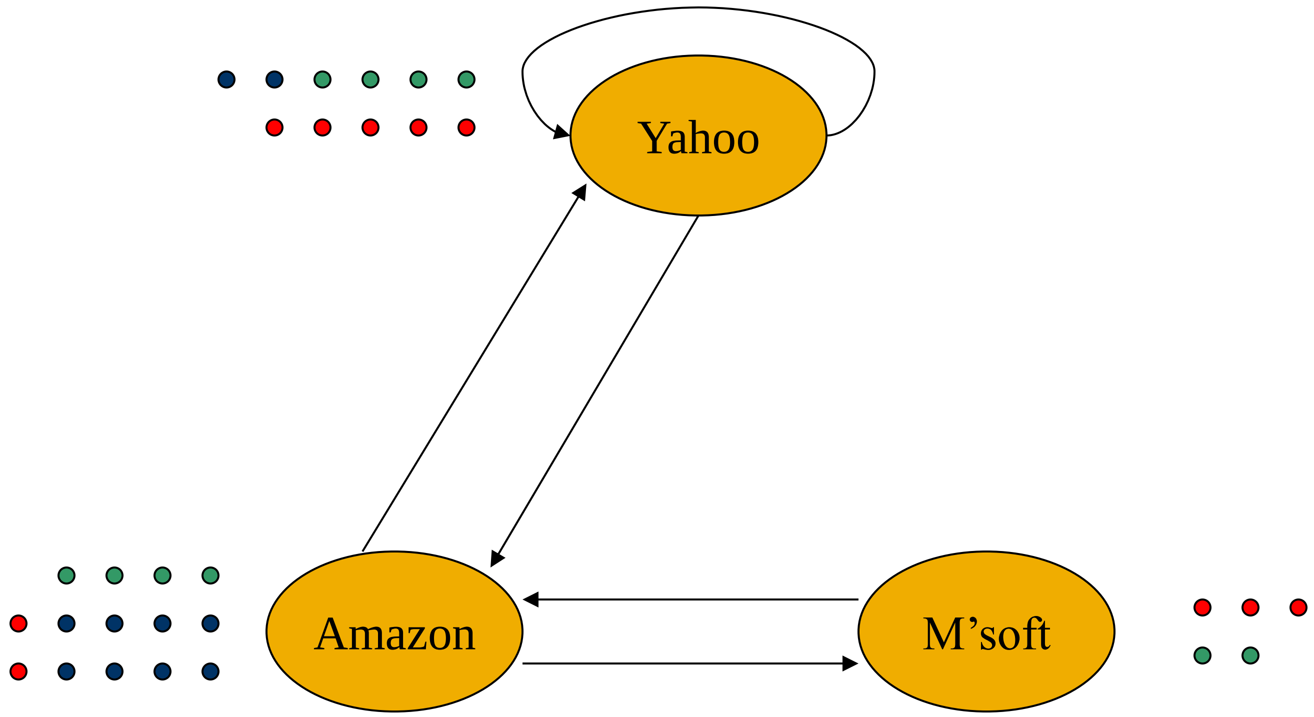
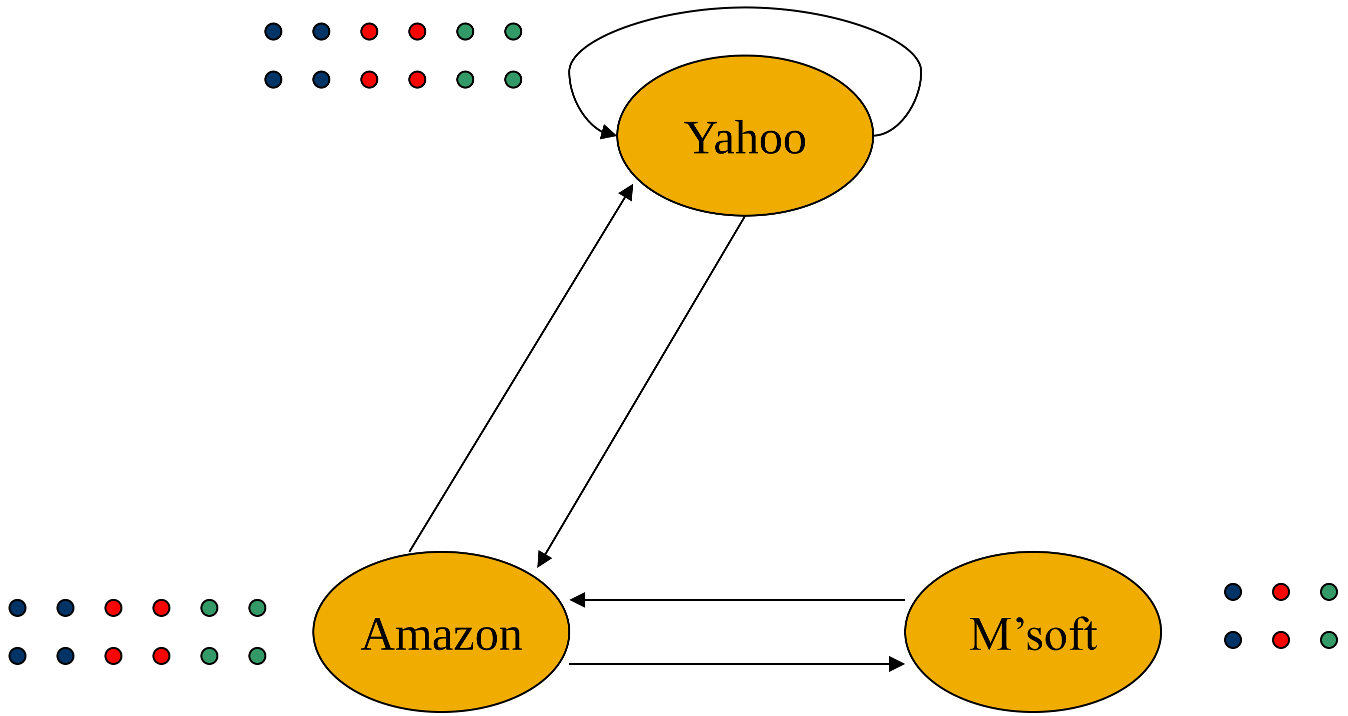


In the Limit …



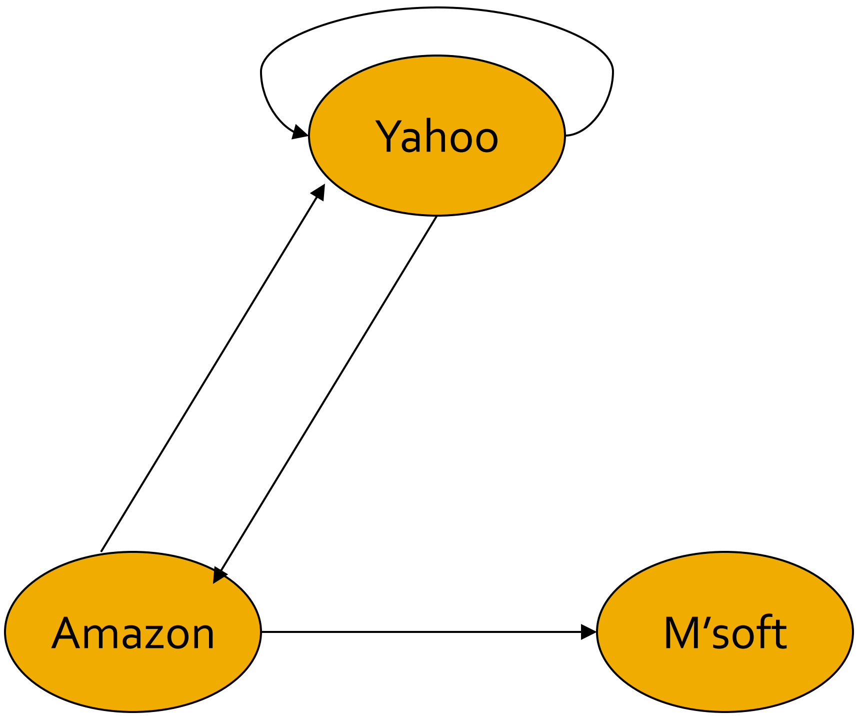
The Web Is More Complex Than That

Dead Ends
Spider Traps
Taxation Policies

Real-World Problems

- Some pages are *dead ends* (have no links out).
  - Such a page causes importance to leak out, or surfers to disappear.
- Other groups of pages are *spider traps* (all out-links are within the group).
  - Eventually spider traps absorb all importance; all surfers get stuck in the trap.

Microsoft Becomes Dead End



$$
\begin{array}{c|ccc|}
 & y & a & m \\
\hline
y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 0 \\
m & 0 & 1/2 & 0 \\
\hline
\end{array}
$$

A *substochastic matrix* =
"all columns sum to at most 1."

Example: Effect of Dead Ends

- Equations $\mathbf{v} = M\mathbf{v}$:

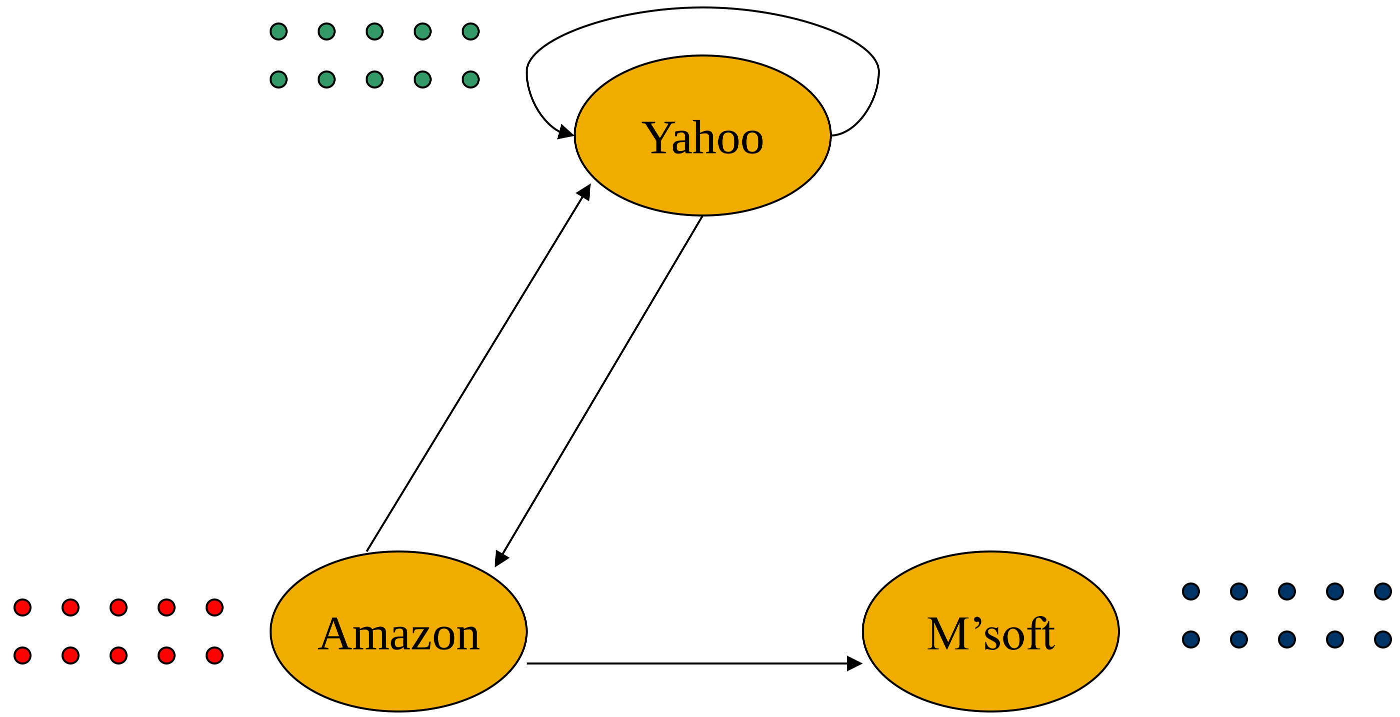
$y = y/2 + a/2$

$a = y/2$

$m = a/2$

$$\begin{array}{l} y \\ a \\ m \end{array} = \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \quad \begin{array}{c} 1 \\ 1/2 \\ 1/2 \end{array} \quad \begin{array}{c} 3/4 \\ 1/2 \\ 1/4 \end{array} \quad \begin{array}{c} 5/8 \\ 3/8 \\ 1/4 \end{array} \quad \ldots \quad \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

Microsoft Becomes a Dead End



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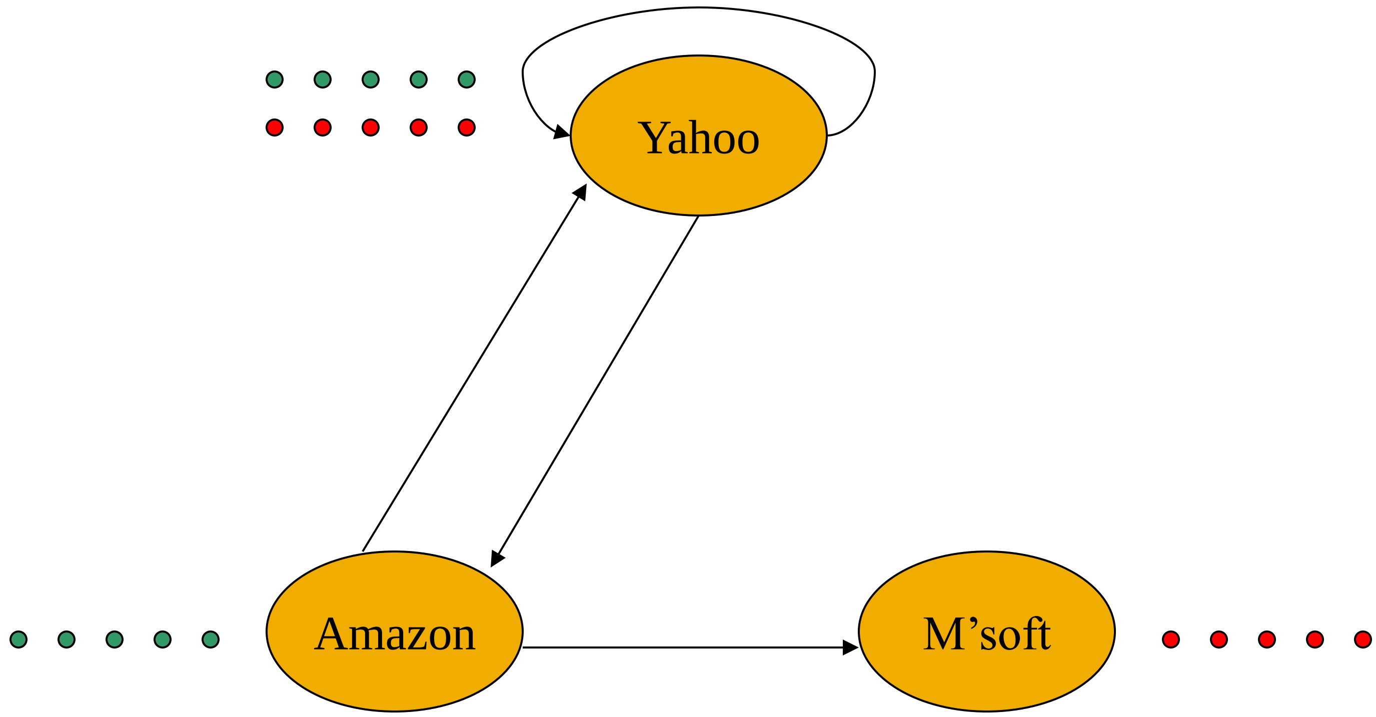

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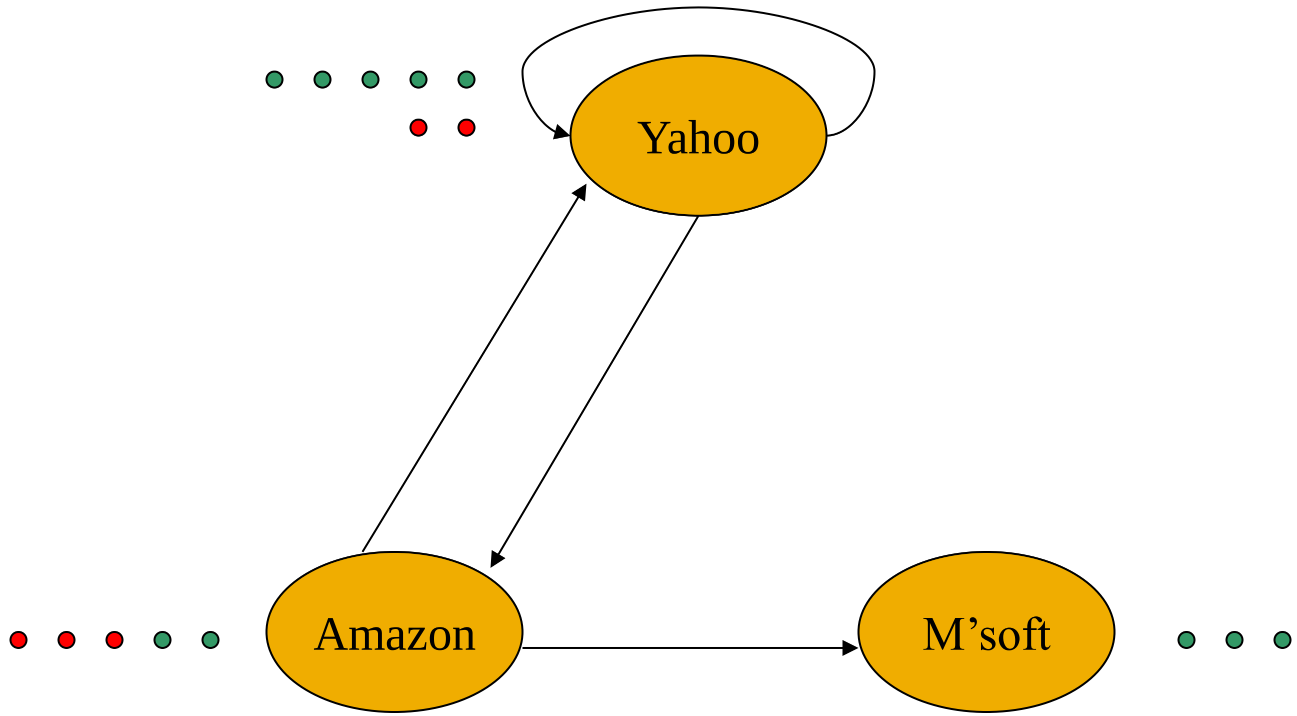

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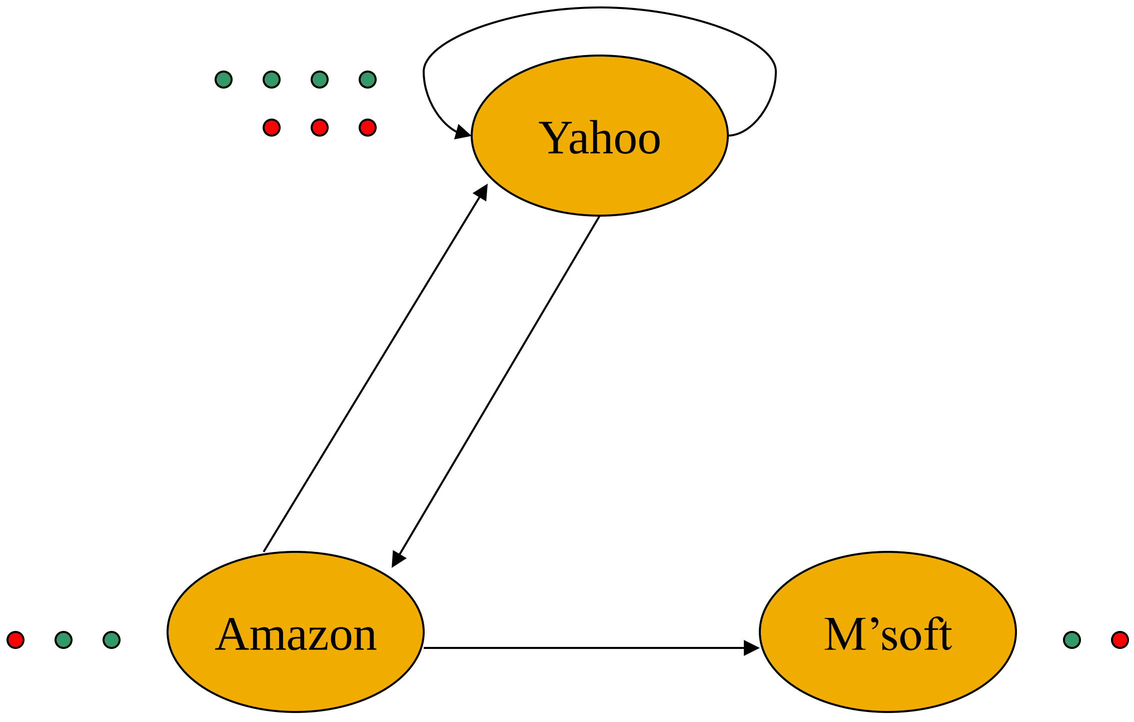

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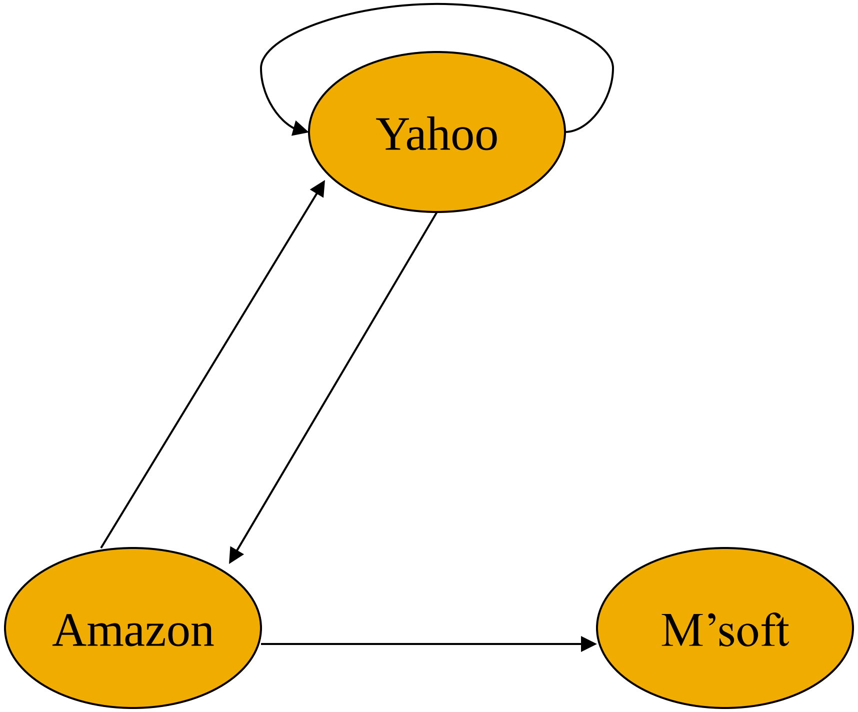
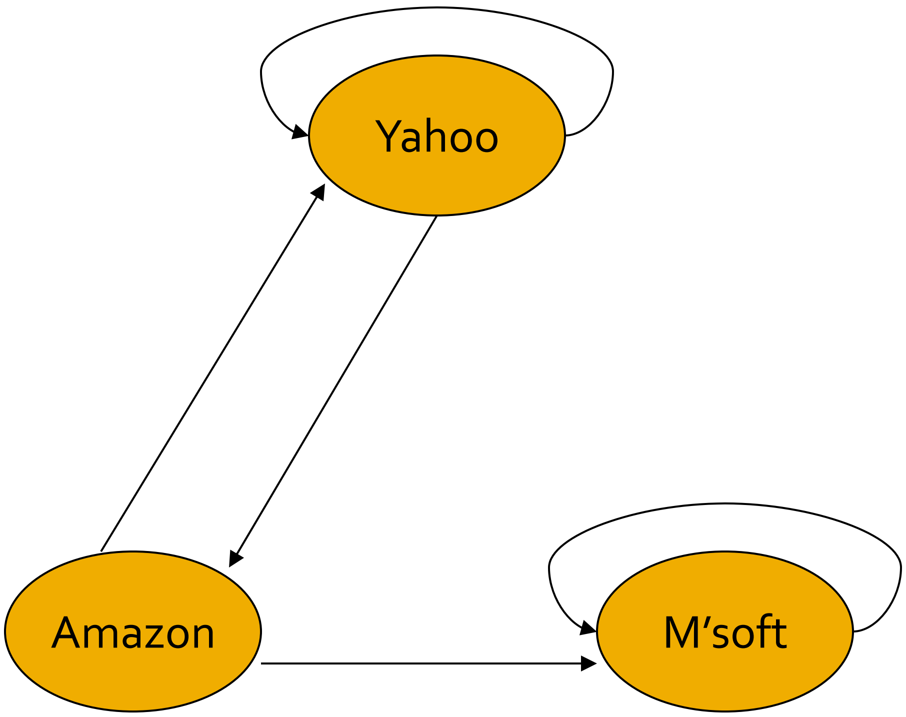


M'soft Becomes Spider Trap



$$\begin{array}{c|ccc|}
 & y & a & m \\
\hline
y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 0 \\
m & 0 & 1/2 & 1 \\
\hline
\end{array}$$

Example: Effect of Spider Trap

- Equations **v** = *M***v**:

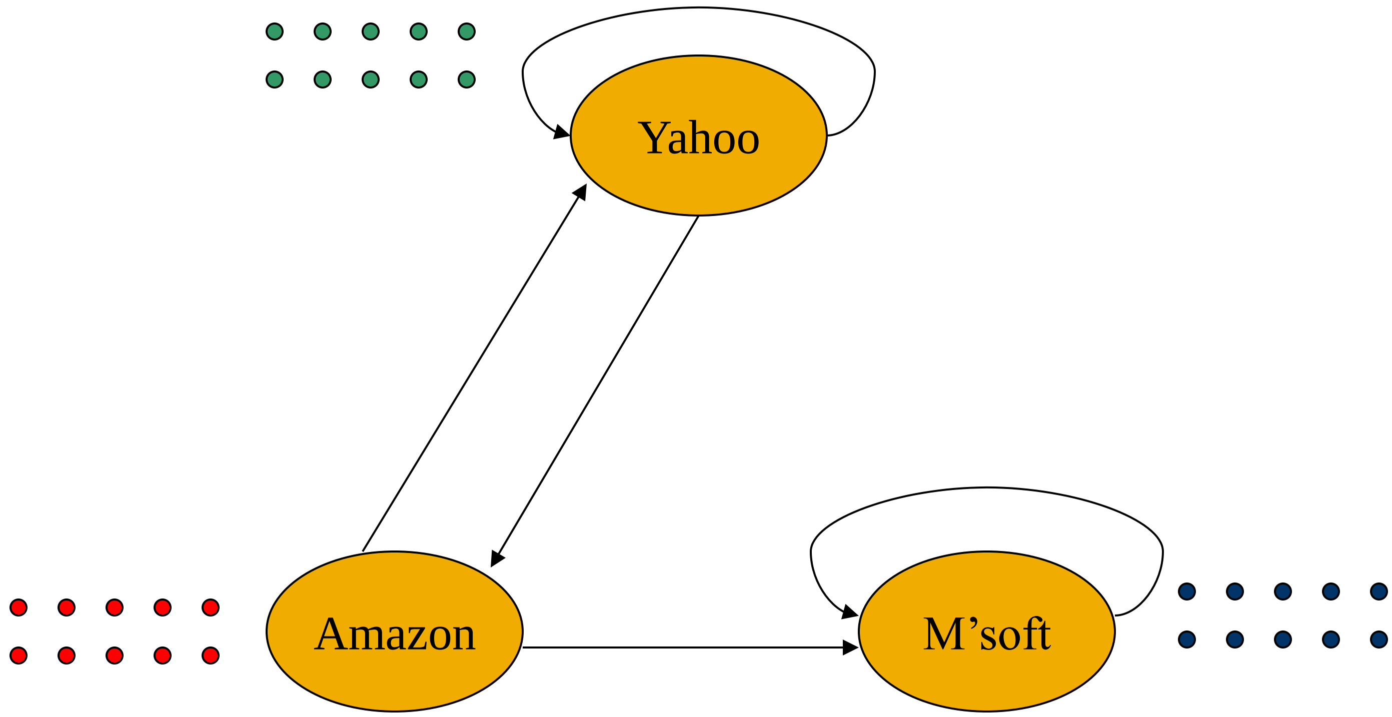
$y = y/2 + a/2$

$a = y/2$

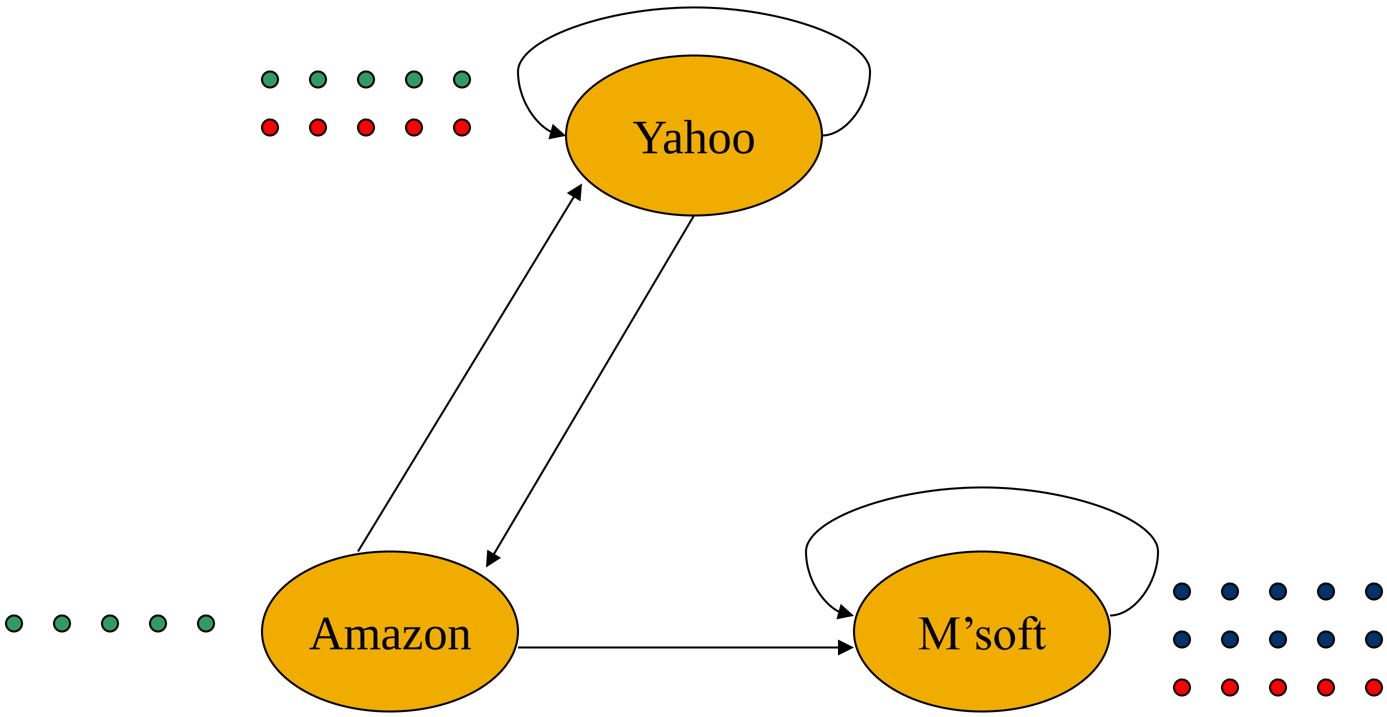
$m = a/2 + m$

y		1	1	3/4	5/8		0
a	=	1	1/2	1/2	3/8	. . .	0
m		1	3/2	7/4	2		3

Microsoft Becomes a Spider Trap



Microsoft Becomes a Spider Trap



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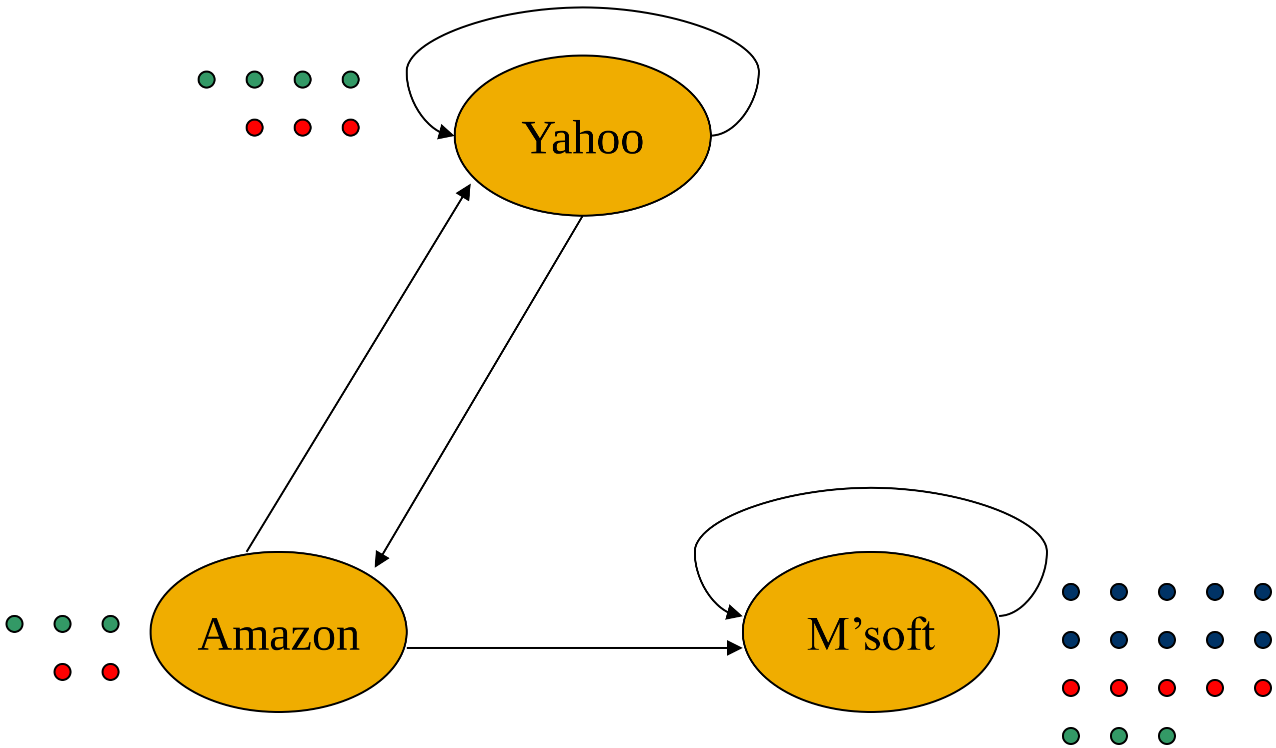
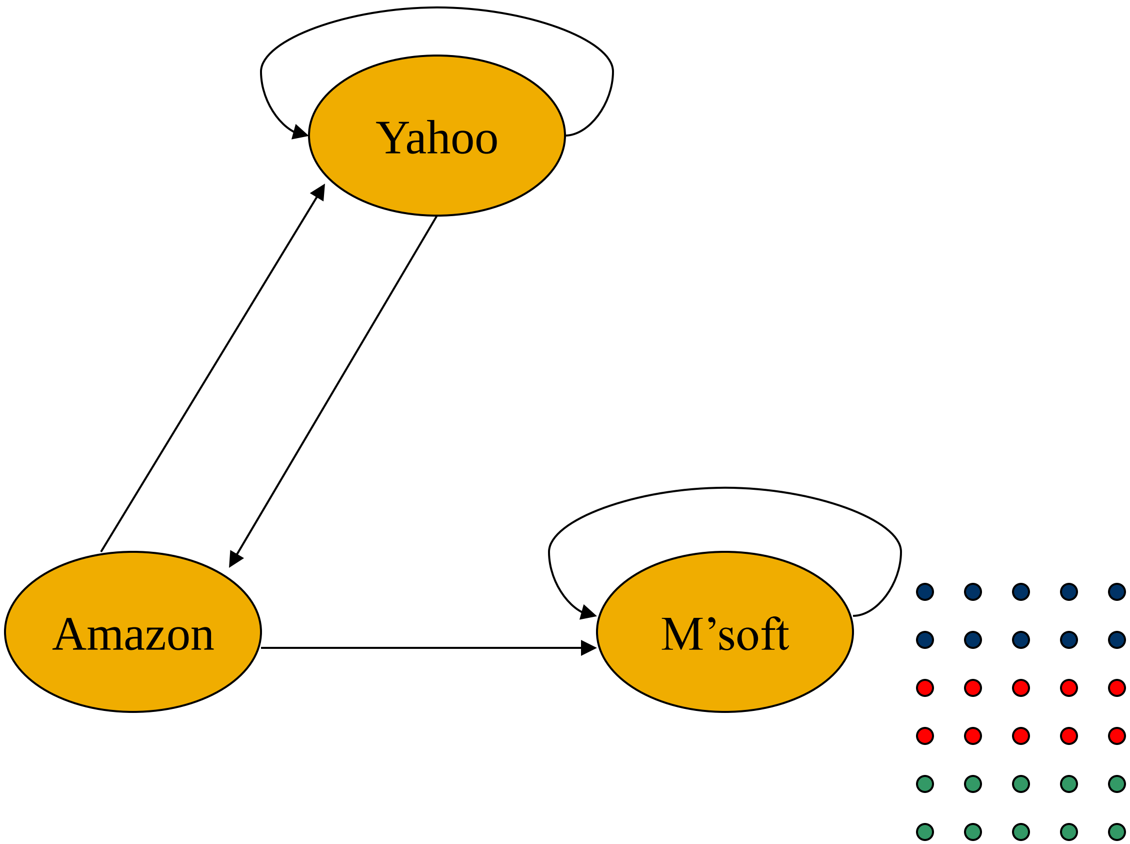


In the Limit …



PageRank Solution to Traps, Etc.

- "Tax" each page a fixed percentage at each iteration.
- Add a fixed constant to all pages.
  - Optional but useful: add exactly enough to balance the loss (tax + PageRank of dead ends).
- Models a random walk with a fixed probability of leaving the system, and a fixed number of new surfers injected into the system at each step.
  - Divided equally among all pages.

Example: Microsoft is a Spider Trap; 20% Tax

- Equations $\mathbf{v} = 0.8(M\mathbf{v}) + \mathbf{0.2}$:

$$y = 0.8(y/2 + a/2) + 0.2$$

$$a = 0.8(y/2) + 0.2$$

$$m = 0.8(a/2 + m) + 0.2$$

Note: amount injected is chosen to balance the tax. If we started with 1/3 for each rather than 1, the 0.2 would be replaced by 0.0667.

y		1	1.00	0.84	0.776		7/11
a	=	1	0.60	0.60	0.536	. . .	5/11
m		1	1.40	1.56	1.688		21/11

Topic-Specific PageRank

Focusing on Specific Pages
Teleport Sets
Interpretation as a Random Walk

Topic-Specific Page Rank

- Goal: Evaluate Web pages not just by popularity, but also by relevance to a particular topic, e.g. "sports" or "history."
- Allows search queries to be answered based on interests of the user.
- Example: Search query [jaguar] wants different pages depending on whether you are interested in automobiles, nature, or sports.
  - Might discover interests by browsing history, bookmarks, e.g.

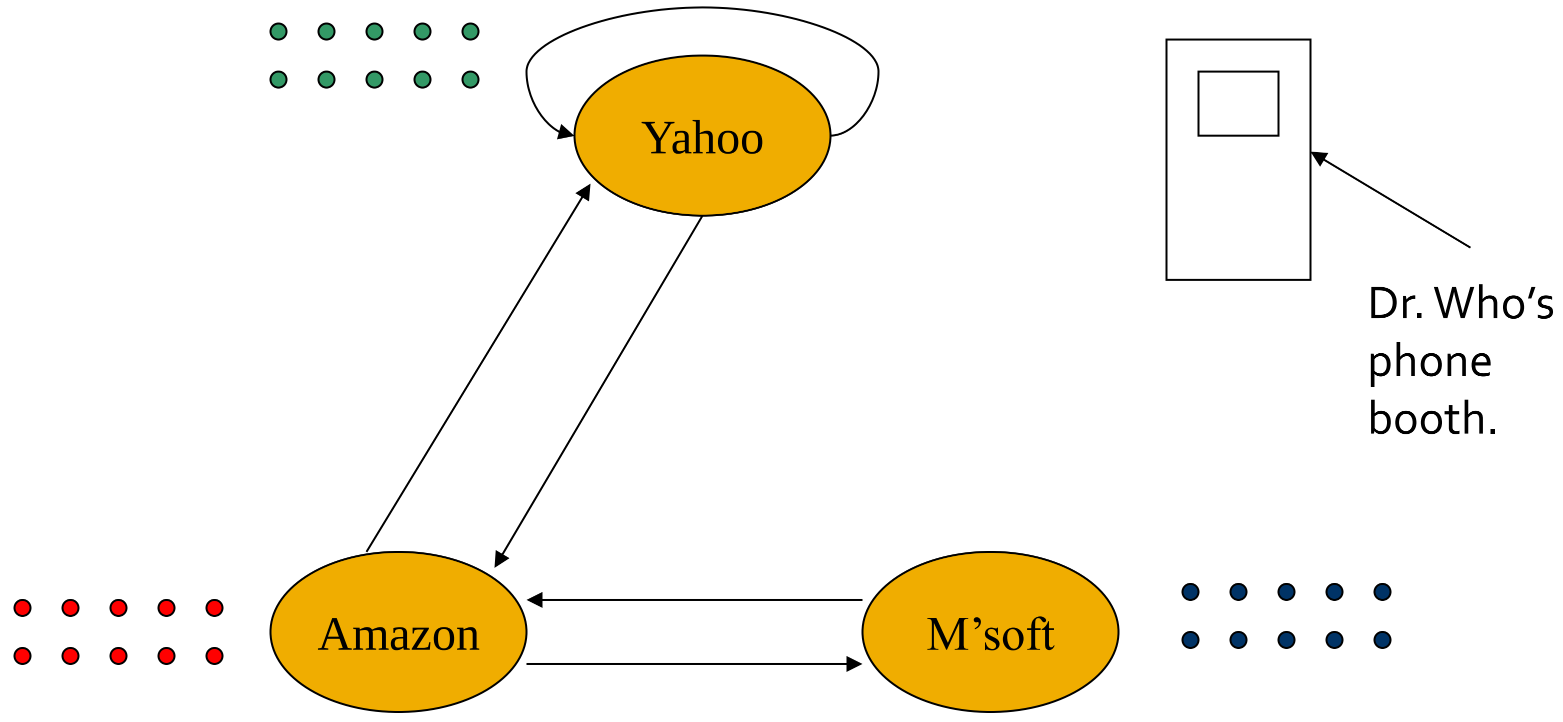
Teleport Sets

- Assume each surfer has a small probability of "teleporting" at any tick.
- Teleport can go to:
  1. Any page with equal probability.
     - As in the "taxation" scheme.
  2. A set of "relevant" pages (*teleport set*).
     - For *topic-specific* PageRank.
- Note: can also inject surfers to compensate for surfers lost at dead ends.
  - Or imagine a surfer always teleports from a dead end.

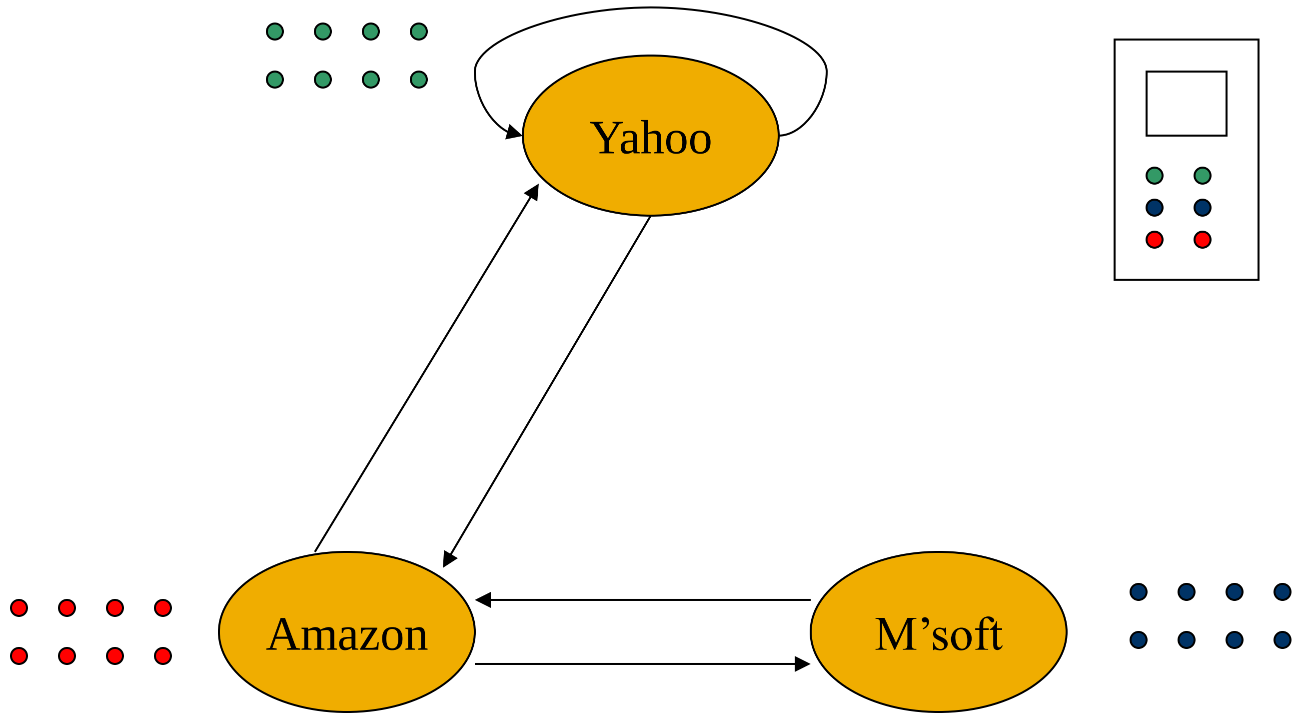
Example: Topic = Software

- Only Microsoft is in the teleport set.
- Assume 20% "tax."
  - I.e., probability of a teleport is 20%.

Only Microsoft in Teleport Set



Only Microsoft in Teleport Set



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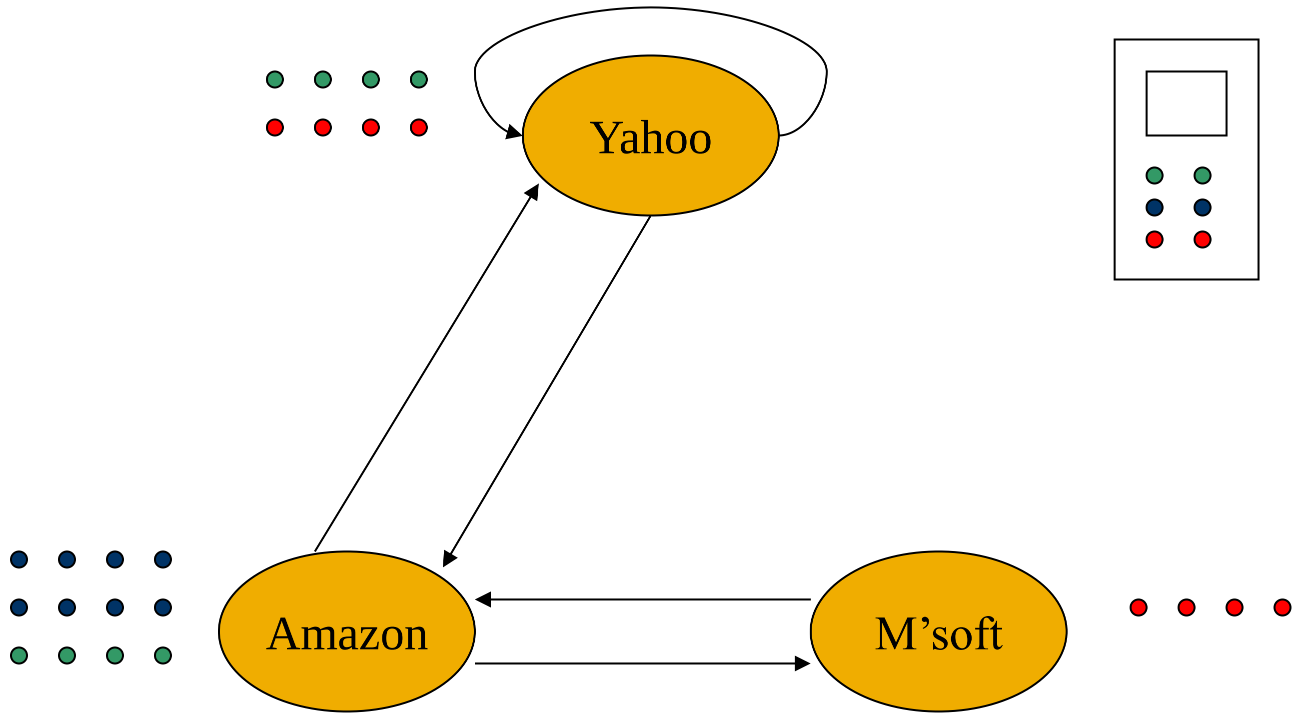

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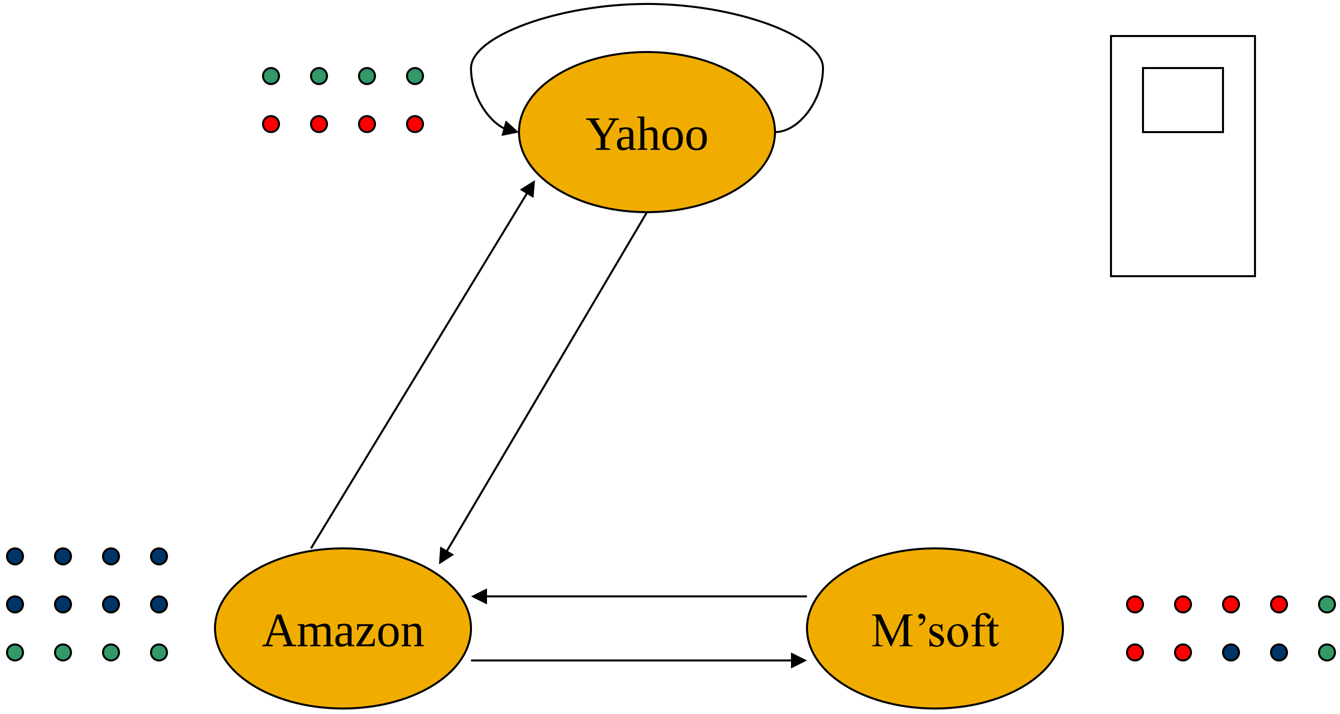

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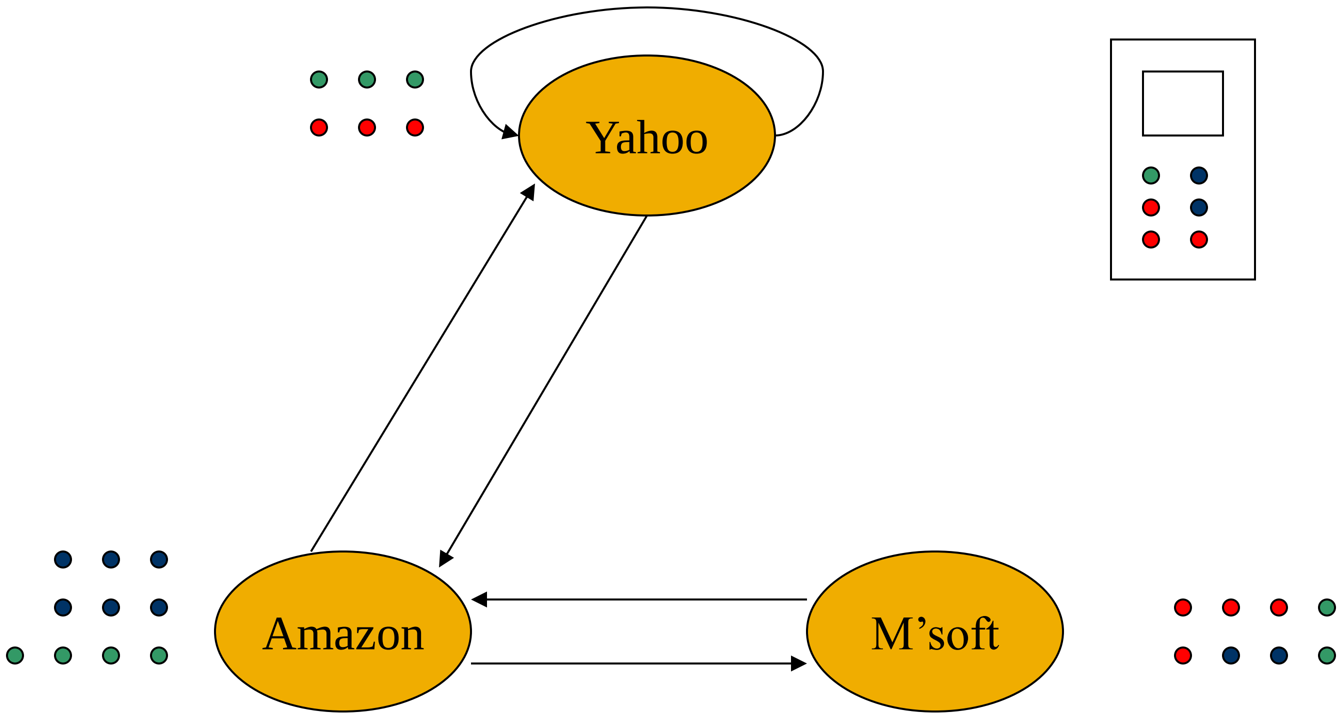

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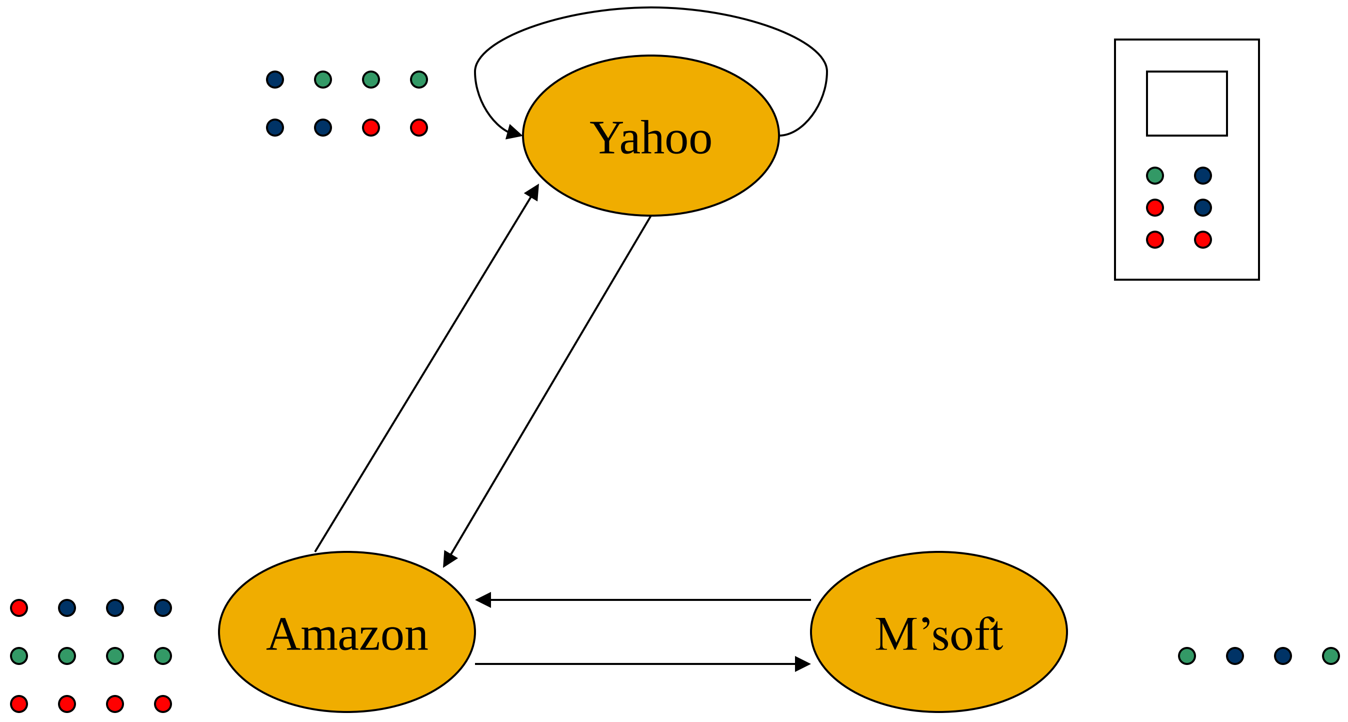

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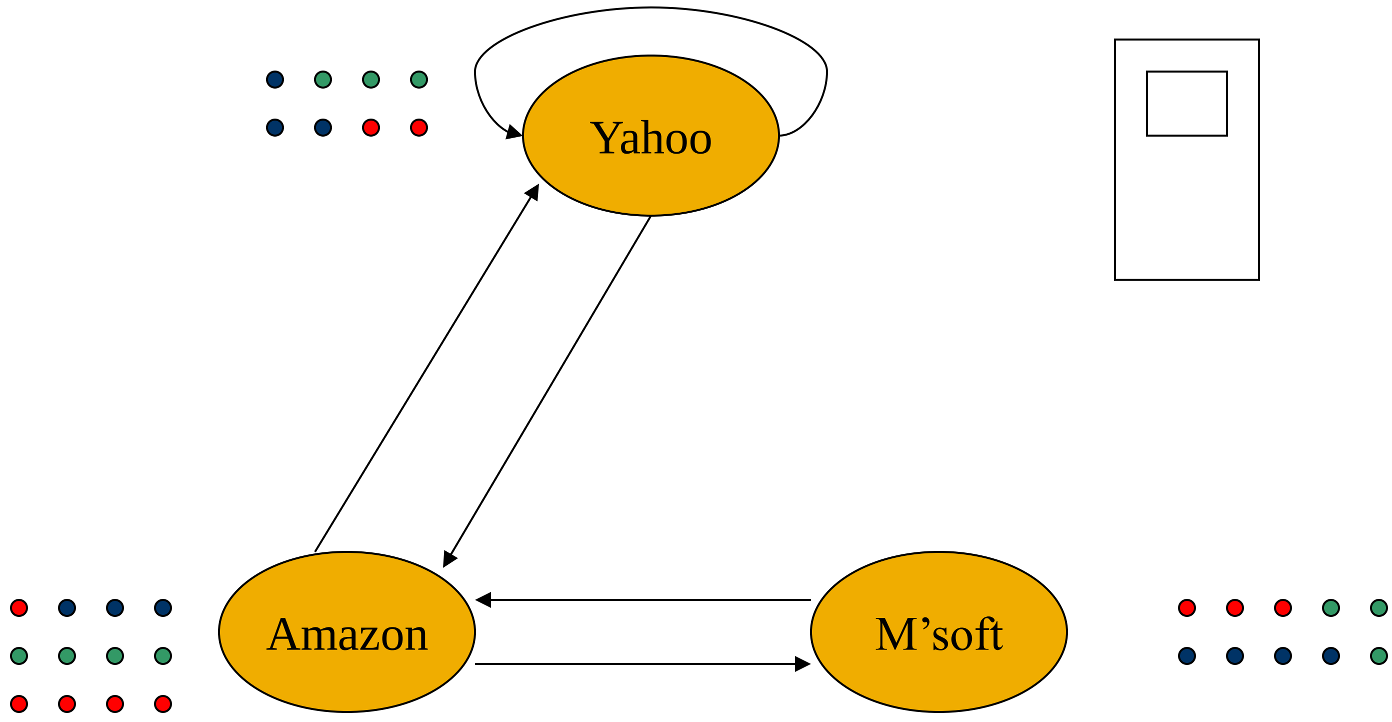


Picking the Teleport Set

1. One option is to choose the pages belonging to the topic in Open Directory.
2. Another option is to “learn,” from a training set (which could be Open Directory), the typical words in pages belonging to the topic; use pages heavy in those words as the teleport set.

Application: Link Spam

- Spam farmers create networks of millions of pages designed to focus PageRank on a few undeserving pages.
  - We'll discuss this technology shortly.
- To minimize their influence, use a teleport set consisting of trusted pages only.
  - Example: home pages of universities.

HITS

Hubs
Authorities
Solving the Implied Recursion

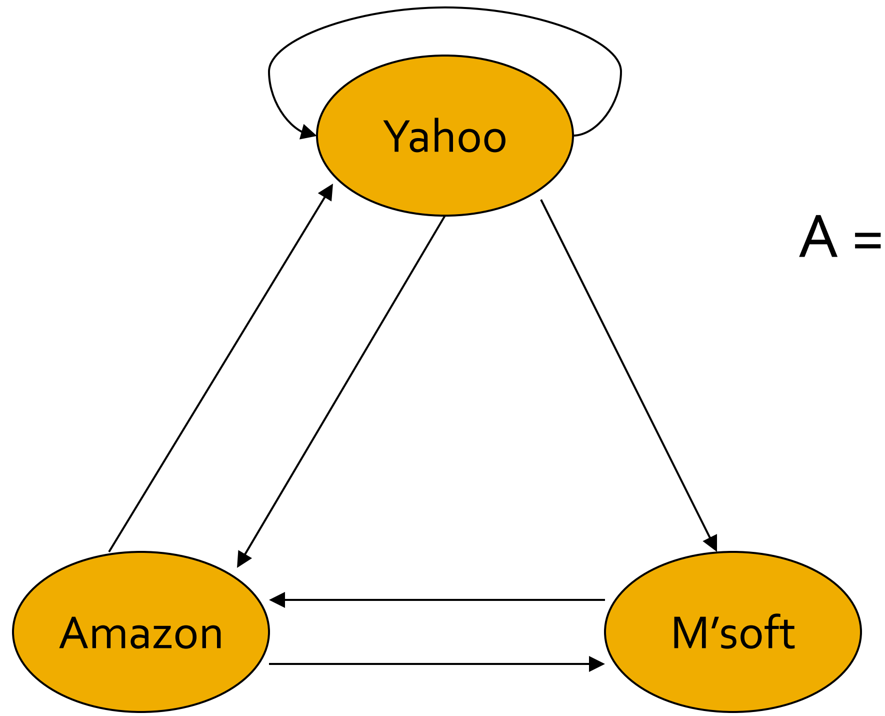
Hubs and Authorities ("HITS")

- Mutually recursive definition:
  - A *hub* links to many authorities;
  - An *authority* is linked to by many hubs.
- Authorities turn out to be places where information can be found.
  - Example: course home pages.
- Hubs tell where the authorities are.
  - Example: departmental course-listing page.

Transition Matrix A

- HITS uses a matrix $A[i, j] = 1$ if page i links to page j, 0 if not.
- A^T, the transpose of A, is similar to the PageRank matrix M, but A^T has 1's where M has fractions.
- Also, HITS uses column vectors **h** and **a** representing the degrees to which each page is a hub or authority, respectively.
- Computation of **h** and **a** is similar to the iterative way we compute PageRank.

Example: H&A Transition Matrix



$$A = \begin{array}{c|ccc|} & y & a & m \\ \hline y & 1 & 1 & 1 \\ a & 1 & 0 & 1 \\ m & 0 & 1 & 0 \\ \hline \end{array}$$

Using Matrix A for HITS

- Powers of A and A^T have elements whose values grow exponentially with the exponent, so we need scale factors λ and μ.
- Let **h** and **a** be column vectors measuring the "hubbiness" and authority of each page.
- Equations: $\mathbf{h} = \lambda A\mathbf{a}$; $\mathbf{a} = \mu A^T\mathbf{h}$.
  - Hubbiness = scaled sum of authorities of successor pages (out-links).
  - Authority = scaled sum of hubbiness of predecessor pages (in-links).

Consequences of Basic Equations

- From **h** = λA**a**; **a** = μA^T **h** we can derive:
  - **h** = $\lambda\mu AA^T$ **h**
  - **a** = $\lambda\mu A^TA$ **a**
- Compute **h** and **a** by iteration, assuming initially each page has one unit of hubbiness and one unit of authority.
- Technically, these equations let you solve for $\lambda\mu$ as well as **h** and **a**.
- In practice, you don't fix $\lambda\mu$, but rather scale the result at each iteration.
  - Example: scale to keep largest value at 1.

Scale Doesn't Matter

- Remember: it is only the direction of the vectors, or the relative hubbiness and authority of Web pages that matters.
- As for PageRank, the only reason to worry about scale is so you don't get overflows or underflows in the values as you iterate.

Example: Iterating H&A

$$\mathbf{a} = \lambda\mu A^T A\ \mathbf{a};\ \mathbf{h} = \lambda\mu A A^T\ \mathbf{h}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad AA^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad A^TA = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

a(yahoo)	=	1	5	24	114	. . .	$1+\sqrt{3}$
a(amazon)	=	1	4	18	84	. . .	2
a(m'soft)	=	1	5	24	114	. . .	$1+\sqrt{3}$

h(yahoo)	=	1	6	28	132	. . .	1.000
h(amazon)	=	1	4	20	96	. . .	0.735
h(microsoft)	=	1	2	8	36	. . .	0.268

A Better Way to Solve HITS

- Start with **h** = [1,1,…,1]; multiply by A^T to get first **a**; scale so largest component = 1; then multiply by A to get next **h**, and repeat until approximate convergence.
- You may be tempted to compute AA^T and A^TA first, then iterate multiplication by these matrices, as for PageRank.
- Question for thought: Why was the separate calculations of **h** and **a** actually less efficient than the method suggested above.