## Design Space of <br> Graph Neural Networks

CS224W: Machine Learning with Graphs Jure Leskovec, Stanford University http://cs224w.stanford.edu


## CS224W: Deep Learning in Graphs



## Key Questions for GNN Design

- GNN architectural design:
- How to find a good GNN design for a specific GNN task?
- Important but challenging:
- Domain experts want to use SOTA GNN on their specific tasks, however...
- There are tons of possible GNN architectures
- GCN, GraphSAGE, GAT, GIN, ...
- Issue: Best design in one task can perform badly for another task
- Redo hyperparameter grid search for each new task is NOT feasible
- Topic for today:
- Study for the GNN design space and task space
- GraphGym, a powerful platform for exploring different GNN designs and tasks


## Background: Terminology

- Design: a concrete model instantiation
- E.g., a 4-layer GraphSAGE
- Design dimensions characterize a design
- E.g., the number of layers $L \in\{2,4,6,8\}$
- Design choice is the actual selected value in the design dimension
- E.g., the number of layers L=2
- Design space consists of a Cartesian product of design dimensions
- Task: A specific task of interest
- E.g., node classification on Cora, graph classification on ENZYMES
- Task space consists of all the tasks we care about


## Recap: GNN Design Space

## Intra-layer Design:

GNN Layer = Transformation + Aggregation

- We propose a general instantiation under this perspective



## Recap: GNN Design Space

## Inter-layer Design

- We explore different ways of organizing GNN layers

Intra-layer Design: 4 dims


Learning Configuration: 4 dims

## Batch size

Learning rate Optimizer
Training epochs

Inter-layer Design: 4 dims


Pre-process layers:
Important when expressive node feature encoder is needed E.g., when nodes are images/text

## Skip connections:

Improve deep GNN's performance

Post-process layers: Important when reasoning or transformation over node embeddings are needed E.g., graph classification, knowledge graphs

## Recap: GNN Design Space



## Summary: GNN Design Space

Intra-layer Design: 4 dims

## Overall: A GNN design space

- Intra-layer design

| Batch Normalization | Dropout | Activation | Aggregation |
| :---: | :---: | :---: | :---: |
| True, False | False, 0.3, 0.6 | ReLU, PRELU, SwISH | MEAN, MAx, SUM |
| $\square$ | \|nter-layer deSign |  |  |
|  |  |  |  |


| Layer connectivity | Pre-process layers | Message passing layers | Post-precess layers |
| :---: | :---: | :---: | :---: | :---: |
| STACK, SKIP-SUM, SKIP-CAT | $1,2,3$ | $2,4,6,8$ | $1,2,3$ | $\mathbf{L}$

- Learning configuration

| Batch size | Learning rate | Optimizer | Training epochs |
| :---: | :---: | :---: | :---: |
| $16,32,64$ | $0.1,0.01,0.001$ | SGD, ADAM | $100,200,400$ |

## A General GNN Task Space

- Categorizing GNN tasks
- Common practice: node / edge / graph level task
- Reasonable but not precise
- Node prediction: predict clustering coefficient vs. predict a node's subject area in a citation networks - completely different task
- But creating a precise taxonomy of GNN tasks is very hard!
- Subjective; Novel GNN tasks can always emerge
- Our innovation: a quantitative task similarity metric
- Purpose: understand GNN tasks, transfer the best GNN models across tasks


## A General GNN Task Space

- Quantitative task similarity metric
- 1) Select "anchor" models ( $M_{1}, \ldots, M_{5}$ )
- 2) Characterize a task by ranking the performance of anchor models
- 3) Tasks with similar rankings are considered as similar

Task Similarity Metric

|  | Anchor Model <br> Performance ranking |  |  |  |  | Similarity <br> to Task $A$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Task $A$ | $M_{1}$ | $M_{2}$ | $M_{3}$ | $M_{4}$ | $M_{5}$ | 1.0 |
| Task $B$ | $M_{1}$ | $M_{3}$ | $M_{2}$ | $M_{4}$ | $M_{5}$ | 0.8 |
| Task $C$ | $M_{5}$ | $M_{1}$ | $M_{4}$ | $M_{3}$ | $M_{2}$ | -0.4 |

Task $A$ is similar to Task $B$
Task $A$ is not similar to Task $C$

- How do we select the anchor models?


## A General GNN Task Space

- Selecting the anchor models
- 1) Select a small dataset
- E.g., node classification on Cora
- 2) Randomly sample $N$ models from our design space, run on the dataset
- E.g., we sample 100 models
- 3) Sort these models based on their

Sorted by performance


- Goal: Cover a wide spectrum of models: A bad model in one task could be great for another task


## A General GNN Task Space

## We collect 32 tasks: node / graph classification

## Task name

node-AmazonComputers-N/A-N/A node-AmazonPhoto-N/A-N/A node-CiteSeer-N/A-N/A node-CoauthorCS-N/A-N/A node-CoauthorPhysics-N/A-N/A node-Cora-N/A-N/A node-scalefree-clustering-pagerank node-scalefree-const-clustering node-scalefree-const-pagerank node-scalefree-onehot-clustering node-scalefree-onehot-pagerank node-scalefree-pagerank-clustering node-smallworld-clustering-pagerank node-smallworld-const-clustering node-smallworld-const-pagerank node-smallworld-onehot-clustering node-smallworld-onehot-pagerank node-smallworld-pagerank-clustering
graph-PROTEINS-N/A-N/A graph-BZR-N/A-N/A
graph-COX2-N/A-N/A graph-DD-N/A-N/A graph-ENZYMES-N/A-N/A graph-IMDB-N/A-N/A graph-scalefree-clustering-path graph-scalefree-const-path graph-scalefree-onehot-path graph-scalefree-pagerank-path graph-smallworld-clustering-path graph-smallworld-const-path graph-smallworld-onehot-path graph-smallworld-pagerank-path graph-ogbg-molhiv-N/A-N/A
(We include link prediction results in the Appendix)


6 Real-world node classificationtasks

12 Synthetic node classification tasks
Predict node properties:

- Clustering coefficient
- PageRank

6 Real-world graph classification tasks

8 Synthetic graph classification tasks
Predict graph properties:

- Average path length


## Evaluating GNN Designs

- Evaluating a design dimension:
- "Is BatchNorm generally useful for GNNs?"
- The common practice:
- (1) Pick one model (e.g., a 5-layer 64-dim GCN)
- (2) Compare two models, with BN = True / False
- Our approach:
- Note that we have defined 315K (models) * 32 (tasks) $\approx \mathbf{1 0 M}$ model-task combinations
- (1) Sample from 10M possible model-task combinations
- (2) Rank the models with BN = True / False - How do we make it scalable \& convincing?


## Evaluating GNN Designs

- Evaluating a design dimension: Controlled random search
- a) Sample random model-task configurations, perturb BatchNorm = [True, False]
- Here we control the computational budget for all the models
(a) Controlled Random Search

| GNN Design Space |  |  |  |  | GNN Task Space |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BatchNorm | Activation | $\ldots$ | Message layers | Layer Connectivity | Task level | dataset |  |
| True | relu | $\ldots$ | 8 | skip_sum | node | CiteSeer |  |
| False | relu | $\ldots$ | 8 | skip_sum | node | CiteSeer |  |
| True | relu | $\ldots$ | 2 | skip_cat | graph | BZR |  |
| False | relu | $\ldots$ | 2 | skip_cat | graph | BZR |  |
|  |  |  |  |  |  |  |  |
| True | prelu | $\ldots$ | 4 | stack | graph | scale free |  |
| False | prelu | $\ldots$ | 4 | stack | graph | scale free |  |

## Evaluating GNN Designs

- b) Rank BatchNorm = [True, False] by their performance (lower ranking is better)
- c) Plot Average / Distribution of the ranking of BatchNorm = [True, False]
(b) Rank Design Choices by Performance

| GNN Design Space |
| :---: |
| BatchNorm |
| True |
| False |
| True |
| False |
|  |
| True |
| False |


| Experimental Results |  |
| :---: | :---: |
| Val. Accuracy | Design Choice Ranking |
| 0.75 | 1 |
| 0.54 | 2 |
| 0.88 | 1 (a tie) |
| 0.88 | 1 (a tie) |
|  |  |
| 0.89 | 1 |
| 0.36 | 2 |

(c) Ranking Analysis


- Summary: Convincingly evaluate any new design dimension, e.g., evaluate a new GNN layer we propose


## Results 1: A Guideline for GNN Design

- Certain design choices exhibit clear advantages
- Intra-layer designs:

Explanation:
GNNs are hard to optimize

## Explanation:

This is our new finding!


Explanation:
GNNs experience underfitting more often


Explanation:
Sum is the most expre aggregator

## Results 1: A Guideline for GNN Design

- Certain design choices exhibit clear advantages
- Inter-layer designs

Optimal number of layers is hard to decide Highly dependent on the task


PP'è-p̈r̈ōčēs̈š läÿèīs






Explanation: Skip connection enable hierarchical node representation

## Results 1: A Guideline for GNN Design

- Certain design choices exhibit clear advantages
- Learning configurations

Optimal batch size and learning rate is hard to decide Highly dependent on the task


Explanation:
Adam is more robust
More training epochs is better

## Results 2: Understanding GNN Tasks

- Best GNN designs in different tasks vary significantly
- Motivate that studying a task space is crucial


Dataset: BZR




Message passinğ läÿëris'
Dataset: scalefree

## Results 2: Understanding GNN Tasks

## - Build a GNN task space

Proposed task similarity (computed from 12 models) node-AmazonComputers-N/A-N/A-node-AmazonComputers-N/A-N/A-
node-AmazonPhoto-N/A-N/A-node-AmazonPhoto-N/A-N/A-
node-CiteSeer-N/A-N/A-
node-CoauthorCS-N/A-N/A-node-CiteSeer-N/A-N/A-
node-CoauthorCS-N/A-N/A-
node-CoauthorPhysics-N/A-N/A-node-CoauthorPhysics-N/A-N/A-node-Cora-N/A-N/A-graph-PROTEINS-N/A-N/A-
graph-BZR-N/A-N/A-graph-BZR-N/A-N/A-
graph-COX2-N/A-N/A-graph-COX2-N/A-N/A-
graph-DD-N/A-N/A-graph-ENZYMES-N/A-N/A-graph-IMDB-N/A-N/A-graph-scalefree-clustering-path-graph-scalefree-const-path-graph-scalefree-onehot-path-graph-scalefree-pagerank-path-graph-smallworld-const-path-graph-smallworld-onehot-path-graph-smallworld-pagerank-path-graph-smallworld-pagerank-path-
node-scalefree-clustering-pagerank-node-scalefree-const-clustering-node-scalefree-const-pagerank-
node-scalefree-onehot-clustering-node-scalefree-onehot-clustering-node-scalefree-pagerank-clustering-node-smallworld-clustering-pagerank-node-smallworld-const-clustering-node-smallworld-const-pagerank-node-smallworld-onehot-clustering-
node-smallworld-onehot-pagerank-node-smallworld-pagerank-clustering-

We compute pairwise similarities between all GNN tasks

Recall how we compute task similarity

|  | Anchor Model <br> Performance ranking |  |  |  |  | Similarity <br> to Task $A$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Task $A$ | $M_{1}$ | $M_{2}$ | $M_{3}$ | $M_{4}$ | $M_{5}$ | 1.0 |
| Task $B$ | $M_{1}$ | $M_{3}$ | $M_{2}$ | $M_{4}$ | $M_{5}$ | 0.8 |
| Task $C$ | $M_{5}$ | $M_{1}$ | $M_{4}$ | $M_{3}$ | $M_{2}$ | -0.4 |



Task similarity computation is cheap:
Using 12 anchor models is a good approximation!

## Results 2: Understanding GNN Tasks

- GNN task space is informative


Group 1:
Tasks rely on feature information Node/graph classification tasks, where input graphs have highdimensional features

- Cora graph has 1000+ dim node feature



## Results 2: Understanding GNN Tasks

- GNN task space is informative
 best architectures
- AmazonComputers
- AmazonPhoto

CiteSeer
CoauthorCS
CoauthorPhysics

BZR
COX2
DD

IMDB
scalefree
smallworld
Task-level

* Graph-level

| Best GNN Designs Found in Different Tasks |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Prelayers | MP layers | Postlayers | Connectivity | AGG |
| Task A | 2 | 8 | 2 | skip-sum | sum |
| Task B | 1 | 8 | 2 | skip-sum | sum |
| Task C | 2 | 6 | 2 | skip-cat | mean |

## Results 3: Transfer to Novel Tasks

- Case study: generalize best models to unseen OGB ogbg-molhiv task:
- ogbg-molhiv is unique from other tasks: 20x larger, imbalanced ( $1.4 \%$ positive) and requires out-of-distribution generalization
- Concrete steps for applying to a novel task:
- Step 1: Measure 12 anchor model performance on the new task
- Step 2: Compute similarity between the new task and existing tasks
- Step 3: Recommend the best designs from existing tasks with high similarity



## Results 3: Transfer to Novel Tasks

- Our task space can guide best model transfer to novel tasks!


## Findings:

We pick 2 tasks:
Task A: Similar to OGB
Task B: Not similar to OGB

Transfer the best model from Task A achieves SOTA on OGB
Transfer the best model from Task B performs badly on OGB

|  | Task $A:$ graph- <br> scalefree-const-path | Task $B:$ node- <br> CoauthorPhysics |
| :---: | :---: | :---: |
| Best design <br> in our design space | $(1,8,3$, skipcat, sum $)$ | $(1,4,2$, skipcat, max $)$ |
| Task Similarity <br> with ogbg-molhiv | $\mathbf{0 . 4 7}$ | -0.61 |
| Performance after <br> transferto ogbg-molhiv | $\mathbf{0 . 7 8 5}$ | 0.736 |

Previous SOTA: 0.771

## GNN Design Space: Summary

- Systematic investigation of:
- General guidelines for GNN design
- Understandings of GNN tasks
- Transferring best GNN designs across tasks
- GraphGym: Easy-to-use code platform for GNN
(a) GNN Design Space


Learning Configuration: $\mathbf{4}$ dims
Batch size
Learning rate
Optimizer Training epochs
3/16/2023
(b) GNN Task Space

(c) Controlled Random Search

| GNN Design Space |  |  |  |  | GNN Task Space |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BatchNorm | Act | $\ldots$ | MP layers | Connectivity | level | dataset |
| True | relu | $\ldots$ | 8 | skip_sum | node | CiteSeer |
| False | relu | $\ldots$ | 8 | skip_sum | node | CiteSeer |
| True | relu | $\ldots$ | 2 | skip_cat | graph | BZR |
| False | relu | $\ldots$ | 2 | skip_cat | graph | BZR |
| $\ldots$ |  |  |  |  |  |  |

(d) Rank Design Choices by Performance

| Experimental Results |  |
| :---: | :---: |
| Val. <br> Accuracy | Design Choice <br> Ranking |
| 0.75 | 1 |
| 0.54 | 2 |
| 0.88 | 1 (a tie) |
| 0.86 | 1 (a tie) |
| $\ldots$ |  |

(e) Ranking Analysis
 Jure Leskovec, Stanford CS224W: Ma chine Learning with Graphs

## Pre-Training Graph Neural Networks

CS224W: Machine Learning with Graphs Jure Leskovec, Stanford University http://cs224w.stanford.edu


## Graph ML in Scientific Domains

- Chemistry: Molecular graphs
- Molecular property prediction

Our running
example today
= toxic?

- Biology: Protein-protein association graphs
- Protein function prediction



## GNNs for Graph Classification

- GNNs obtain an embedding of an entire graph by following two steps
- Iteratively aggregate neighboring information to obtain node embeddings
- Pool node embeddings to obtain a graph embedding



## GNNs for Graph Classification

- Node embeddings capture local neighborhood structure
- The embedding of an entire graph is a global aggregation of such node embeddings

Capture local neighborhood structure
Molecule




Globallyaggregate local features


## Challenges of Applying ML

- Two fundamental challenges in applying ML to scientific domains

1. Scarcity of labeled data

- Obtaining labels requires expensive lab experiments
$\rightarrow$ ML models overfit to small training data

2. Out-of-distribution prediction

- Test examples tend to be very different from training examples
$\rightarrow$ ML models extrapolate poorly


## Challenges for Deep Learning (1)

- Deep learning models have a lot parameters to train (e.g., in the order of millions).
- \#(Labeled training data) << \#(Parameters)
- Deep learning models are extremely prone to overfitting on small labeled data.


## Challenges for Deep Learning (2)

- Deep learning models extrapolate poorly
- Models often make predictions based on spurious correlations in a dataset [Sagawa et al. ICML 2020]
" Ex) Image classification between "polar bear" and "brown bear"
- During training:


Adapted from
Wikipedia
" Most "polar bears" have the snow background
" Most "brown bears" have the grass background

- Model can learn to make prediction based on the image background, rather than the animal itself.
- At test time, what if we see "polar bear" on the grass?


## Injecting Domain Knowledge

- Goal: Improve model's out-of-distribution prediction performance even with limited data.
- Key idea: Inject domain knowledge into a model before training on scarcely-labeled tasks!
- The model already knows the domain knowledge before training on data
- So that the model can
- Generalize well without many task-specific labeled data
- Extract essential (non-spurious) pattern that allows better extrapolation.


## Effective Solution: Pre-Training

- We pre-train a model on relevant tasks, where data is abundant.
- After pre-training, the model parameters already contain domain knowledge.
- For downstream tasks (what we care about, typically with small \#labeled data)
- We start from the pre-trained parameters and finetuning them.

Pre-training
Raw
input

Output


Fine-tuning on downstream tasks


## Pre-Training is Successful

- Pre-training has been hugely successful in computer vision and natural language processing.
- Pre-training improves label-efficiency.
- Pre-training improves out-of-distribution performance [Hendrycks et al. ICML 2019]
- Pre-training is a powerful solution to the two ML challenges in scientific applications
- Scarce labels
- Out-of-distribution prediction


## Pre-training GNNs

- Let's consider pre-training GNNs!
- We design GNN pre-training strategies and systematically investigate

Q1. How effective is pre-training GNNs?
Q2. What is the effective pre-training strategy?

## How Effective is Pre-training GNNs?

## Let's think about molecular property prediction for drug discovery. <br> - Naïve strategy

Multi-task supervised pre-training on relevant labels.

Molecule

Diverse labels
from chemical database

## Experimental Setting

## - Molecule classification

- Task: Binary classification. ROC-AUC as metric

$$
f(, \mathcal{O})=\{0,1\}
$$

- Supervised pre-training data
- 1310 diverse binary bioassays annotated over ~450K molecules
- Downstream task (what we care about!)
- 8 molecular classification datasets (relatively-small, 1K100K molecules)
- Data split: Scaffold (test molecules are out-ofdistribution)


## How Effective is Pre-training GNNs?

- Naïve strategy:

Multi-task supervised pre-training on relevant labels.
$\rightarrow$ Limited performance improvement on downstream tasks. Often leads to negative transfer

Molecule classification performance


Downstream datasets

## What is the Effective Strategy?

- Key idea: Pre-train both node and graph embeddings.
$\rightarrow$ GNN can capture domain-specific knowledge of both local and global structure



## What is the Effective Strategy?

- Key idea: Pre-train both node and graph embeddings.

Embedding illustration



## Proposed Pre-Training Methods

Self-supervised
(No need for external labels)

| Attribute <br> prediction | Attribute <br> Masking | Supervised <br> Attribute <br> Prediction |
| :---: | :---: | :---: |
| Structure <br> prediction | Context <br> Prediction | Structural <br> Similarity <br> Prediction |

## Attribute Masking: Algorithm

- Mask node attributes
- Use GNNs to generate node embeddings.
- Use the embeddings to predict masked attributes.



## Attribute Masking: Intuition

## Intuition

- Through solving the masked attribute prediction task, a GNN is forced to learn domain knowledge, e.e.g., chemical rules.



## Context Prediction: Algorithm

- For each graph, sample one center node.
- Extract neighborhood and context graphs.
- Use GNNs to encode neighborhood and context graphs into vectors.
- Maximize/minimize the inner product between true/false (neighborhood, context) pairs.




## Context Prediction: Intuition

## - Intuition

Subgraphs that are surrounded by similar contexts are semantically similar.

- In natural language processing, this is called distributional hypothesis, and is exploited in the word2vec model [mikioloveta. N N1P52013].




## Supervised Attribute Prediction

- Multi-task supervised training on many relevant labels.

Iterative neighbor aggregation


## Overall Strategy

## 1. Node-level pre-training <br> 2. Graph-level pre-training 3. Fine-tuning on downstream tasks



## Results of Our Strategy

- Avoids negative transfer.
- Significantly improve the performance.

Non pretrained GNNs

Molecule classification performance


## Comparison of GNN models

- When different GNN models are pre-trained, the most expressive model (GIN) benefits the most from pre-training.
- Intuition: Expressive model can learn to capture more domain knowledge than less expressive models.

|  | Chemistry |  |  | Biology |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Non-pre-trained | Pre-trained | Gain | Non-pre-trained | Pre-trained | Gain |
| GIN | 67.0 | $\mathbf{7 4 . 2}$ | $\mathbf{+ 7 . 2}$ | $64.8 \pm 1.0$ | $\mathbf{7 4 . 2} \pm \mathbf{1 . 5}$ | $+\mathbf{+ 9 . 4}$ |
| GCN | $\mathbf{6 8 . 9}$ | 72.2 | +3.4 | $63.2 \pm 1.0$ | $70.9 \pm 1.7$ | +7.7 |
| GraphSAGE | 68.3 | 70.3 | +2.0 | $65.7 \pm 1.2$ | $68.5 \pm 1.5$ | +2.8 |
| GAT | 66.8 | 60.3 | -6.5 | $\mathbf{6 8 . 2} \pm \mathbf{1 . 1}$ | $67.8 \pm 3.6$ | -0.4 |

## Pre-Training GNNs: Summary

- GNNs have important applications in scientific applications, but they present challenges of
- Label scarcity
- Out-of-distribution prediction
- Pre-training is promising to tackle the challenges.
- However, naïve pre-training strategy gives suboptimal performance and even leads to negative transfer.
- Our strategy: Pre-train both node and graph embeddings $\rightarrow$ Leads to significant performance gain on downstream tasks.


## CS224W: Wrap-Up

CS224W: Machine Learning with Graphs Jure Leskovec, Stanford University http://cs224w.stanford.edu


## Modern ML Toolbox



Images


Text/Speech


Modern deep learning toolbox is designed for simple sequences \& grids

## This Course

## How can we develop neural networks that are much more broadly applicable?

## Graphs are the new frontier of deep learning

## Graphs and Relational Data



Image credit: ResearchGate
Code Graphs

Molecules

3D Shapes

## CS224W: Deep Learning in Graphs



## Models of Interest: Invariances



Perceptrons
Function regularity


CNNs
Translation


Group-CNNs
Translation+Rotation, Global groups


LSTMs
Time warping


DeepSets / Transformers
Permutation


GNNs
Permutation


Intrinsic CNNs
Isometry / Gauge choice

## The Bottom Line

- There is exciting relational structure in many many real-world problems
- Molecules/Proteins as strings vs. graphs
- Travel time duration over the map graph
- Identifying and harnessing this relational structure leads to better predictions
- AlphaFold
- Biomedicine
- Recommender systems


## You learned a lot!

- Theory:
- Models, architectures, approaches
- Practice:
- Collab notebooks
- Homeworks
- Creative research:
- Course project

The real-world use cases and applications

## What Next?

- Project write-ups:
- Tues March 21 Midnight (11:59PM) Pacific Time No late days!
- Courses:
- CS246: Mining Massive Datasets (Spring)
- Data Mining \& Machine Learning for big data
- (big==doesn’t fit in memory/single machine)
- Fast clever algorithms for real-world problems
- Distributed data processing frameworks: MapReduce, Spark


## In Closing...

- It has been a challenging year for everyone
- Back to campus, work from home, social distancing, fatigue, disease, well-being
- Virtual office hours, take home exam
- But we *all* did our best and did best given the challenging circumstances


## Thank you, team!!!

## Course Assistants



Mohammad Aljubran


Sharmila Nangi


Xuan Su


Serina Chang


Lun Yu (Tina) Li


Feiyang (Kathy) Yu


Anirudh Sriram


Aman Bansal


Paridhi Maheshwari


Arvind Sridhar


Zhuoyi Huang

## I am very proud of everyone!

- You Have Done a Lot!!!
- And (hopefully) learned a lot!!!
- Answered questions and proved many interesting results
- Implemented a number of methods
- And did excellently on the project!

> Thank You for the Hard Work!!!

