

CS224W Project Milestone: Analysis and Prediction of Ride-Sharing and Public Transportation Traffic

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Abstract

We analyze the Uber Movement Dataset for San Francisco along with the General Transit Feed Specification (GTFS) for the San Francisco Municipal Railway (MUNI) to examine the spatial and temporal organization of transportation in cities and to further identify disparities between road and public transit networks. By identifying important nodes in each network by utilizing measures such as betweenness centrality, closeness centrality and degree, we attempt to examine the interactions and commonalities between these sets of key nodes. We conclude that the Uber Movement Dataset reflects demand for travel services while the MUNI's schedule reflects a more uniform distribution, and that the MUNI data reflects city travel while the Uber Movement Dataset also includes long distance travel.

1 Introduction

Understanding how urban landscapes shape travel patterns and vice versa has been an ongoing research topic in the fields of transportation and urban planning. With the advent of readily available GPS and mobile phone data, information regarding transportation data has become more accessible and useful than ever. Some of this data is owned by private companies such as Uber, a mobile-phone directed ride sharing company. Some other domains have open data on various modes of transportation – Transitland and Google have open public transportation data, while the New York City Open Data Project has provided researchers with taxi data which helps them investigate traffic

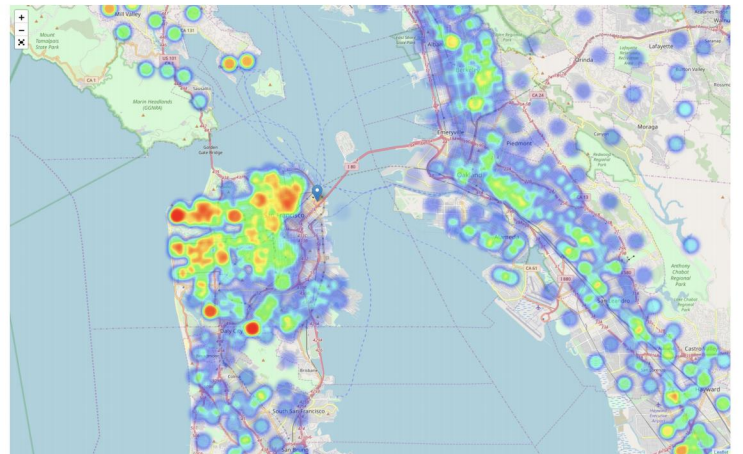


Figure 1: Time of travel from a single point near the Bay Bridge in northeast SF

flow.

Transportation data cannot solely be modeled off static information such as street patterns, because its behavior and demand are always fluctuating. Public transportation routes and stops shift in order to respond to travel needs, and traffic flow models rely on dynamic behavior and large spans of time. Uber Movement provides data associated with Uber trips taken in several major cities, including San Francisco. With the resource of open real-time traffic data from Uber Movement, we are able to assess travel patterns via rideshare and compare it to routes in a key public transit agency in San Francisco – the MUNI run by the San Francisco Municipal Transit Agency.

Defining and understanding the relationship between public transit and private vehicles is key

to seeing how different modes of transportation interact with one another. Ridesharing vehicles serve individuals who do not use their own personal vehicle – as a result, these two data sources are serving the same population with similar transportation needs – individuals without personal vehicles. Furthermore, because public transportation entails scheduled departure times to and from set locations, when comparing models of MUNI to the Uber Movement Dataset, we can utilize the passenger travel within the Uber Movement Dataset as an ideal. Uber allows for flexibility in timing, start location and end location, and therefore, when compared to the more rigidly scheduled public transportation systems, informs us the travel demands of passenger both temporally and spatially. In the city of San Francisco, we model both Uber Movement’s traffic data and GTFS data for the MUNI in order to compare transportation need and utilization.

We then analyze, in all of these networks, the key nodes that lead to heavy traffic in order to understand the disparities between car and public transportation. Understanding the key nodes and disparities among transportation allow urban planners to find ways to not only identify the nodes of the crucial to the transportation but also alleviate traffic in the city and optimize commute times for both private and public transportation. Also, understanding the key nodes with traffic elucidates the key nodes that have reduced traffic, providing insights to paths with shorter travel times. Additionally, after understanding the nodes in the network, we examine the edges of the Uber Movement Dataset and utilize them to determine the extent of public transportation coverage in San Francisco.

2 Related work

Throughout the process of researching this network project, we identified three key papers that assisted us in providing further context on the importance of certain nodes in a network, and key components in being able to visualize transportation networks in a graph setting, with nodes and vectors – both for roads / private transportation and public transportation.

2.1 Identifying Important Nodes

We used the paper “Identifying Important Nodes in Weighted Covert Networks using Generalized Centrality Measures” [3] in order to understand more about what key nodes were, and why they were relevant in a networking context. In order to understand key actors in a crime network, it was necessary to understand the relationships between nodes – or “actors” in this network to see who had the most influence. This idea of key nodes is highly relevant in transportation – in order to efficiently move freight, passengers and vehicles, it is important to see key bottlenecks or major nodes in which many pathways pass through.

In this paper, Memon incorporates a weighted network in their calculation of key nodes. Here, it is valuable to understand which nodes were most key or central in this network in the context of including both the number of edges, and the weight of those edges. Memon defined “node centrality” through three characteristics: degree, closeness, and betweenness. Each of these centrality measures were first explored in a non-weighted graph, and then further extended by combining both the number of edges linked to a node, and the weights itself. While this technical concept is applied to a different real-world network than transportation, the technique used to incorporate centrality and identify key nodes in a weighted graph is still important to flag here.

While the graph network here presents a viable method of determining what “key” nodes are the definition of “key nodes” was left more ambiguous here, leaving the reader to determine if this calculation is a viable method for their own real world graph.

The “key nodes” was left defined as simply “in the thick of the network”. However, different methods used to define nodes of relevance would not provide the same information, and might not be useful for other networking instances, like transportation. For our project, our challenge will be adapting this idea of node importance to transportation, where the travel time (weights of edges) shows the importance of various locations in the traffic network.

In [4] Traffic Flow Analysis Using Uber Movement Data, Pearson, Sagastuy and Samaniego incorporate various key characteristics in order to pinpoint important nodes. Each of these features reveal a different feature in real life regarding transportation.

By comparing the nodes that share these characteristics across the three graphs, we can begin to understand travel patterns between public transit and ride-sharing. These features are in-degree, out-degree, betweenness centrality, closeness centrality, PageRank, hubs and authorities, and community detection.

2.2 Road Networks and Key Nodes

The paper “Identification of Key Nodes in a Road Network Using the Fusion of Nodes with Degree Traffic Characteristics and LISH Model” [6] explored further concepts on the construction of a road network and key evaluation indices used to understand how transportation networks can be visualized. It acknowledges that road networks exhibit characteristics of a complex network and therefore, much can be derived from analyzing them in a graph based context. This research provides two useful contexts – the design of a spatial and traffic based network for road transportation, and the extended definition of key nodes.

The LISH model, before being combined with traffic characteristics considers the road topology only at first, including the structural and geographical features of a space network. However, in this paper, further additions are included to the LISH model by incorporating potential traffic characteristics.

The adapted version of the LISH model in this paper, while it does incorporate more elements of roads that can contribute to traffic, still does not completely visualize the actual flow of people. Road grade and road section length do capture hypothetical throughput of vehicles on a road – however, it does not reflect the movement and travel demand of real people heading from Point A to Point B. Examining the LISH model incorporating edge weight will help us understand traffic flow and, consequently, node importance.

In [1], Understanding urban traffic-flow characteristics: a rethinking of betweenness centrality, Gao, Wang, Gao and Liu also emphasize the importance of understanding why temporal and spatial factors both play a large part in being able to visualize key nodes in a road network. They stress that although roads are outlined spatially on a map, it is the relationship between human behavior over time and these roads that ultimately determine which nodes are “key” in transportation.

Furthermore, in exploring [2], Revealing travel patterns and city structure with taxi trip data, we examine how the city structure beneath the complex travel-flow system shows the inherent connection patterns within the city, on the basis of massive taxi trip data of Shanghai. Here, Liu, Gong, Gong and Liu overlaid traffic analysis data (obtained through taxi trip data) with the spatial layout of a city to understand how the two interacted with one another. Their further explorations on these sub-network structures and how they interacted with one another demonstrated the relationship between urban and suburban centers and how they influence local traffic. By incorporating the land use of centers from the travel pattern perspective, they were able to investigate sub-regions within the city.

3 Algorithms

3.1 Degree

All of our graphs are directed, therefore, we measured both in degree and out degree.

3.2 PageRank

In order to gain more insight into the most important zones of the Uber Movement Dataset, we ran PageRank to understand the importance of certain nodes based on how many edges are connected to that node from neighbors.

3.3 Clustering Coefficient

The clustering coefficient measures how closely nodes cluster together. The clustering coefficient of node i with degree k_i and e_i number of edges between the neighbors of node i is calculated with

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$

3.4 Betweenness Centrality

Betweenness centrality measures the probability that a random shortest path passes through a given node or edge. With σ_{yz} equal to the number of shortest paths going from y to z and $\sigma_{yz}(x)$ equal to the number of such paths that also pass through x , we

Algorithm 1: PageRank Algorithm

Input: Graph $G = (V, E)$, parameter β **Output:** PageRanke vector r $t = 1$ $\forall j : r_j^{(0)} = 1/N$ **do** **for all nodes j do** **if j in-degree = 0 then** $r_j^{(t)} = 0$ **else** $r_j^{(t)} = \sum_{i \rightarrow j} \beta r_i^{(t-1)} / d_i$ **for all nodes j do** $S = \sum_i r_i^{(t)}$ $r_j^{(t)} = r_j^{(t)} + (1 - S)/N$ $t = t + 1$ **while** $\sum_j |r_j^{(t)} - r_j^{(t-1)}| > \epsilon;$ **return** r

use the equation

$$c_{bet}(x) = \sum_{y,z \neq x, \sigma_{yz} \neq 0} \frac{\sigma_{yz}(x)}{\sigma_{yz}}$$

3.5 Closeness Centrality

Closeness centrality examines which nodes are more central by examining which have the smaller distances, assuming that the more central nodes have smaller distances. With $d(y, x)$ equal to the length of the shortest path from y to x , we use the equation

$$c_{clos}(x) = \frac{1}{\sum_y d(y, x)}$$

3.6 Harmonic Centrality

Harmonic centrality is a measure closely related to closeness centrality, in that they both measure which nodes act as bridges within the network. Harmonic centrality, however harmonic centrality can be applied to graphs that aren't strongly connected:

$$c_{har}(x) = \sum_{d(y,x) < \infty, y \neq x} \frac{1}{d(y, x)}$$

3.7 HITS Centrality

HITS Centrality assigns each node in a graph a hub score and an authority score. A hub score typically

indicates a node's quality as an expert, and an authority score typically indicates quality as a content provider. In our context, however, authorities can be taken as locations where traffic commonly flows through. We use the following equations

$$c_{aut}(x) = \sum_{y \rightarrow x} c_{hub}(y)$$

$$c_{hub}(x) = \sum_{x \rightarrow y} c_{aut}(y)$$

4 Data

We gathered data from the San Francisco Municipal Transportation (MUNI) through publicly available GTFS data (General Transit Feed Specification) and Uber Movement for the city of San Francisco. For the GTFS, we utilized the standard files, stops.txt and stop_times.txt to represent nodes and time intervals. stops.txt contains familiar names along with longitude and latitude coordinates for each stop within the MUNI system, while stop_times.txt contains entire routes, labeled with arrival times, departure times, unique stop ids, and route ids, and stop number (within a route).

In order to utilize this GTFS data to create a network, we developed a python script to process these two files, assigning each unique stop to a node, and weighted edges between each of those nodes with the average time difference between the different stops. Therefore, an edge between two given nodes represents the average scheduled time of travel between two stops. For the MUNI networks, this is the scheduled interval of time on a transit route. The SF MUNI graph was created in the manner of \mathbb{L} -space graphs, as \mathbb{L} -space graphs have been shown to provide unique insights regarding transportation and provide the most cohesive representation of transportation routes.[5].

The Uber Movement data for the San Francisco area provided the source ID, destination ID and geometric mean travel time between the two ID locations. The sourceIDs and destinationIDs correspond to arbitrarily drawn census tracts of San Francisco. An edge between two given nodes (source ID to destination ID) here represents the average

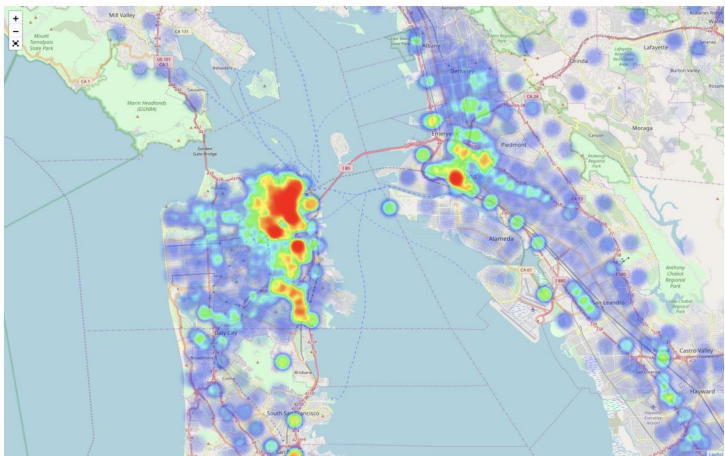
travel time between spatially adjacent census tracts.

In order to effectively look at travel demands throughout the day, we divided the data for MUNI and Uber Movement into specific time sections. These time intervals were as follows (in military or 24 hour time)

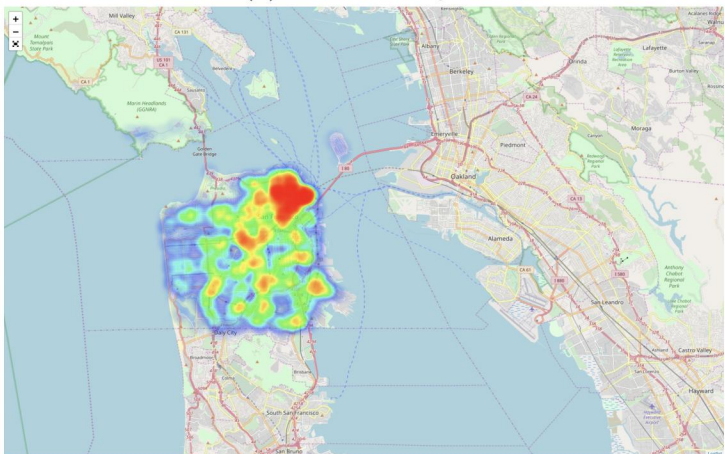
- **Early:** 0 - 5
- **To Work:** 6 - 11
- **Midday:** 12 - 17
- **From Work:** 18 - 23

5 Results

5.1 Degree



(a) Uber In Degree



(b) MUNI In Degree

In degree for Uber represents the locations that passengers want to travel to, as the more edges to directed toward the node shows that more people

from different areas wanted to travel to this location. In the city of San Francisco, the highest demand occurs on the eastern side of the city, extending from the southeast part to the northeast part. This roughly corresponds with the commercial and prime office space regions in San Francisco. Since San Francisco is a key urban area, this data makes sense logically – less private vehicle ownership means more individuals are taking alternative transportation to get to commercial areas or their place of employment. Besides SFO, (the international San Francisco Bay Area Airport), which has the highest in-degree of 1398, an intersection near the Bay Bridge in northeast SF had the largest in degree of 1198.

We contrast this with the MUNI system. Instead of directly representing ridership demand and request as Uber Movement does, the MUNI bus system data shows the major areas supported by the official San Francisco bus system. When normalized, the MUNI system has a seemingly more uniform distribution across the city than that of the Uber dataset, with nodes with higher in-degrees on the northeast corner of the San Francisco city limits. This corresponds roughly to the high-travel demand in Market Street and Embarcadero, locations that are travel-heavy for tourists and residents alike. The node with the highest in-degree of 5 is at an intersection near Market Street in Northeast San Francisco.

The characteristics of the in-degree between these two datasets represent both travel demand and relative coverage. The Uber Movement in-degree data showcases more concentrated, high traffic areas such as downtown San Francisco, downtown Oakland, and SFO Airport. However, the Uber data show a higher degree near the southeastern part of the city, a part which is not covered as strongly by MUNI as the northeastern part.

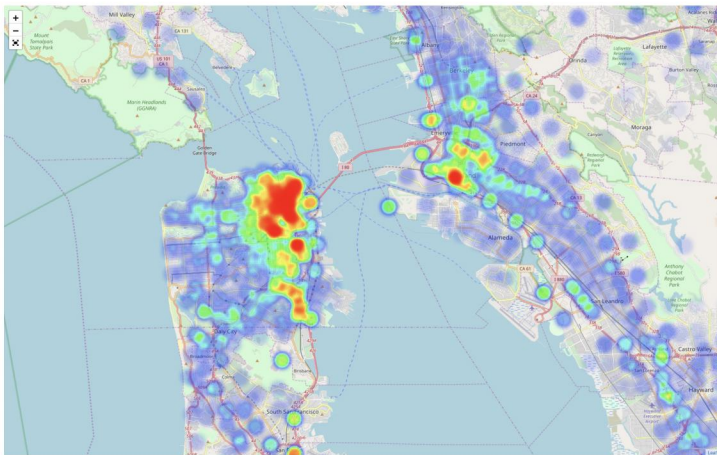
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We analyzed the in and out degrees of both the Uber and MUNI datasets and created gifs for them in the link above. We analyzed them for the four time periods: early (0-5), to work (6-11), midday (12-18), and

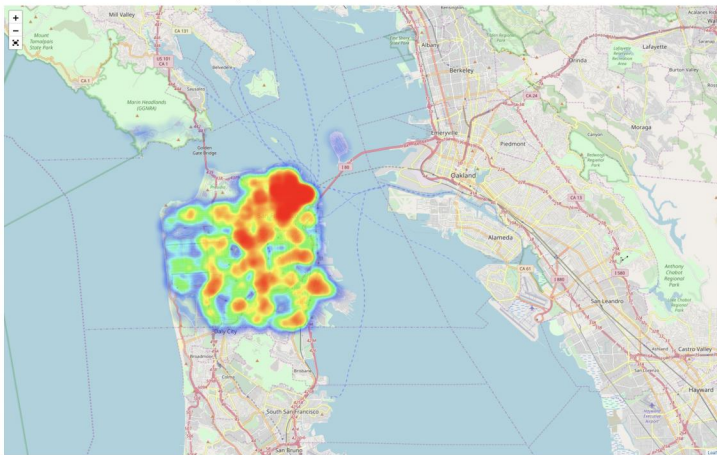
from work (18-23). For the Uber dataset, the areas with the highest in/out degrees tend to stay consistent for all time periods. However, the areas with lower degrees at early time period tend to slightly increase in degree during the other time periods. However, in the MUNI dataset, we get huge spikes in the degree during the daytime hours and huge falls during nighttime hours. Thus, we can see that there is still a demand for transportation at nighttime hours; however, the bus system does not provide services during that time, and transportation is limited to those who use Uber.

out degree of 4. This was an unpopular location for Uber passengers, and it further shows the economic disparity between Uber and MUNI riders, as lower economic areas suffer from higher juvenile incarceration rates.

5.2 PageRank

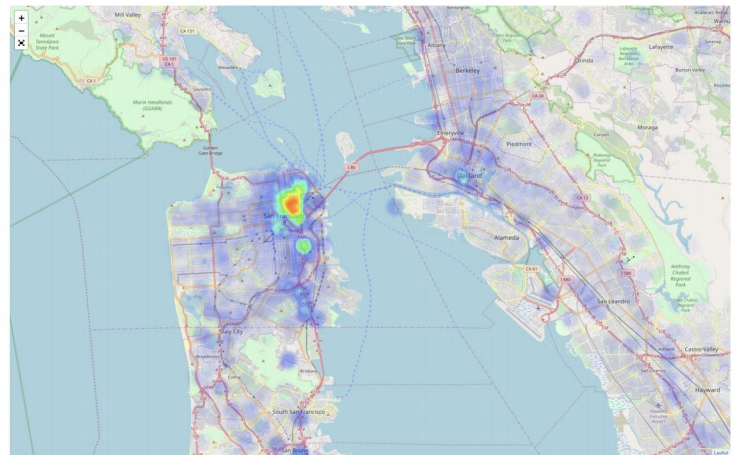


(a) Uber Out Degree

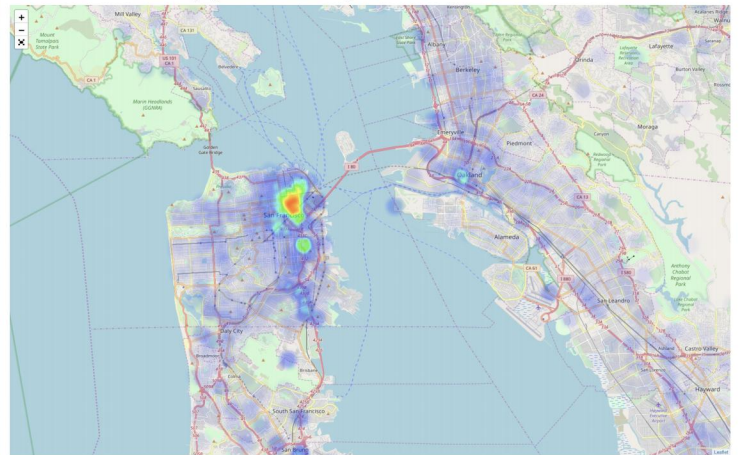


(b) MUNI Out Degree

Out degree informs where the demand for transportation lies. For instance, the location with the highest out degree had the most Ubers called to that location. The same intersection near the Bay Bridge with the highest in degree also had the highest out degree of 1150 (besides SFO). This shows the demand is highest in the northeast part of SF. For the MUNI system, that location also had a high out degree; however, the San Francisco Superior Court Juvenile Justice Center in central SF had the highest



(a) Uber PageRank $\alpha = 0.95$

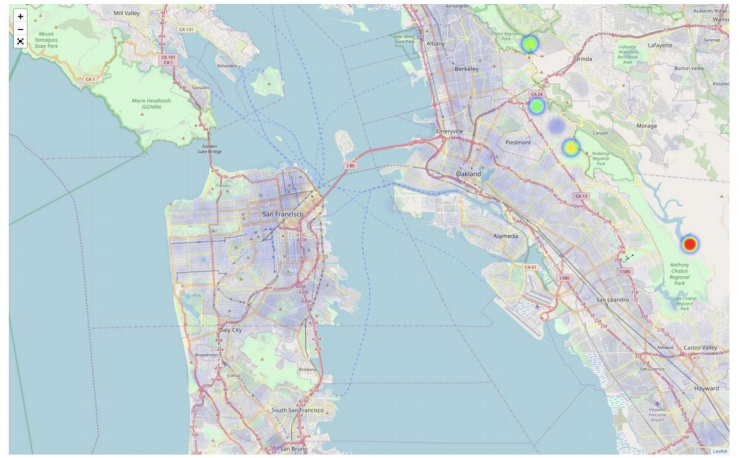


(b) Uber PageRank $\alpha = 0.85$

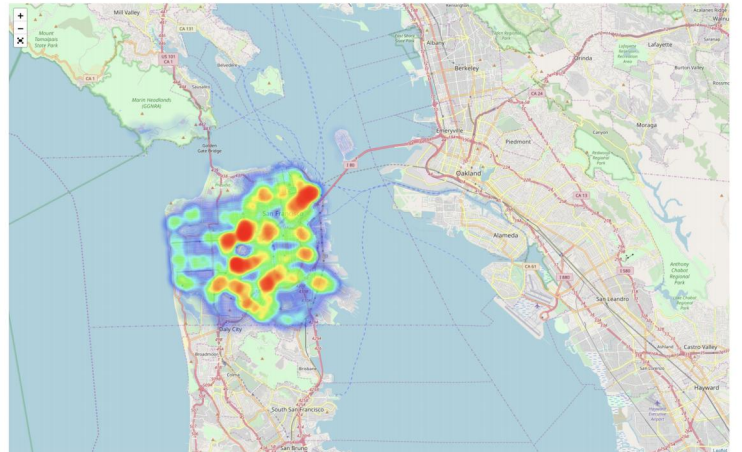
PageRank identifies the important nodes recursively examining the nodes that are "cited" by important nodes. We ran PageRank only on the Uber dataset, as it failed to converge on the MUNI dataset. The top two node with the highest PageRank of 0.006127 for both $\alpha = 0.85, 0.95$ lies in a small town past Sacramento. This node is a spider trap, as it has many in edges but no out edges. SFO lies in third place, with a score of 0.003476. Based on this information, it is inferred that passengers from SFO have rode to this town near Sacramento, giving rise to the largest PageRank score for this town.

5.3 Clustering Coefficient

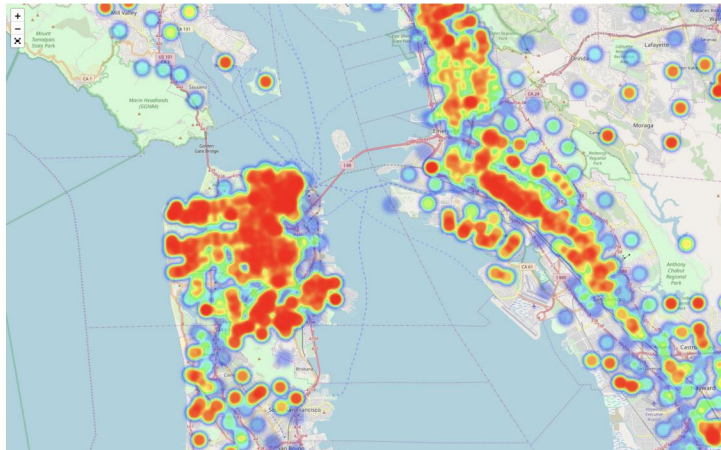
Clustering coefficient measures the proportion of connections between neighboring nodes of a particular node. For the MUNI dataset, we can see that there are a few small clusters with a high clustering coefficient, while the larger part of the graph is bare. This makes sense because bus lines are very linear and don't tend to have many connections between closely neighboring stops. On the other hand, for the Uber Movement Dataset, we see that a much larger area has a high clustering coefficient, reflecting the flexible nature of Uber. Since passengers have the freedom to start and end a ride anywhere they want, more clusters have the potential of forming. Furthermore, the manner in which the Uber Movements Dataset is structured, measuring the average travel time between spatially adjacent census tracts, also promotes the appearance of clusters.



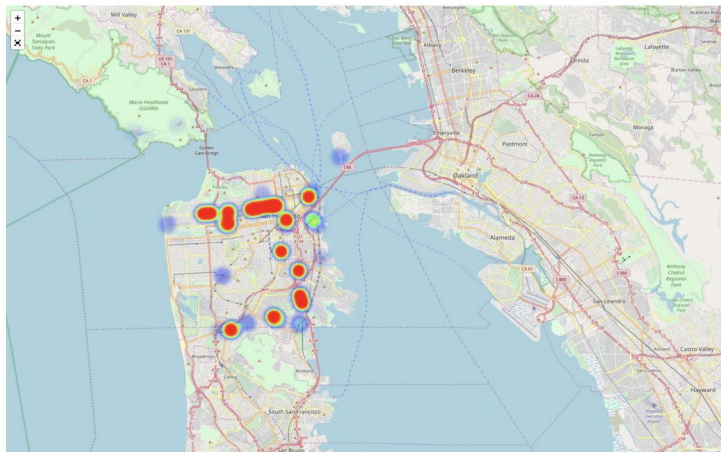
(a) Uber Betweenness Centrality



(b) MUNI Betweenness Centrality



(a) Uber Clustering Coefficient



(b) MUNI Clustering Coefficient

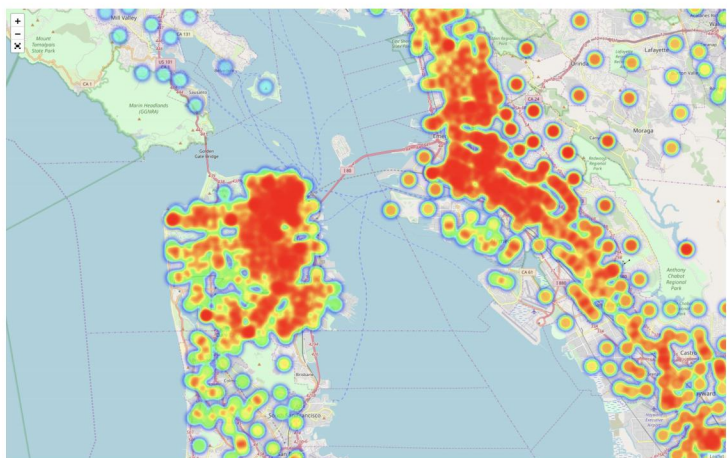
5.4 Betweenness Centrality

Betweenness centrality measures the probability that a random shortest path passes through a given node. For the Uber Dataset, the betweenness centrality did not prove useful for our analysis, as the nodes with the highest values were all located outside of SF in the East Bay. While this information is outside the scope of our recommendations in order to improve the SF MUNI, we can infer some conclusions based on the behavior of Uber riders and drivers. Because Uber can be used for long distance travel across the Bay Area (unlike MUNI, which is used for the city limits of San Francisco), many routes travel up and down the Bay Area, from San Jose to San Francisco, Berkeley, Sacramento and beyond. The reason why the nodes with high betweenness centrality are located in the East Bay is because due to more development in Silicon Valley (which comprises of the South Bay and the peninsula), the East Bay has relatively less traffic and therefore less travel time. therefore, since edges

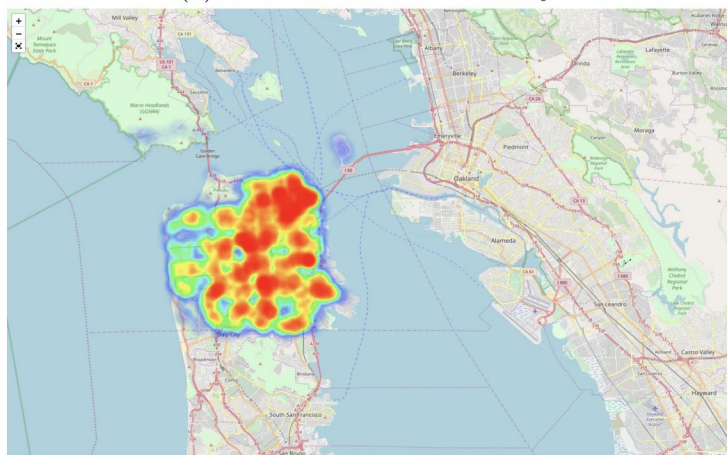
are weighted based on time, many nodes in the East Bay have a high betweenness centrality due to the low traffic.

In contrast, the MUNI system’s betweenness centralities of all the nodes roughly corresponds to the even distribution of the MUNI map itself. This is because rather than corresponding to ridership demand and traffic response, public transit schedule times and routes are fixed. The MUNI dataset node with the highest betweenness centrality was the Laguna Hospital in central SF. This implies this node intersects with many different MUNI routes, suggesting that the Laguna Hospital is both a significant stopping point in San Francisco and intersects spatially among various traveling communities.

5.5 Closeness Centrality and Harmonic Centrality



(a) Uber Closeness Centrality



(b) MUNI Closeness Centrality

We determined the closeness centrality of both the Uber Movement data and the MUNI data. In a connected graph, closeness centrality (or closeness) of a node is a measure of centrality in a network, calculated as the reciprocal of the sum of the length of the shortest paths between the node and all other nodes in the graph. Therefore, the more central a node is, the closer it is to all other nodes. In the context of public transportation, closeness centrality therefore implies that a stop (or node) has more routes linked to that particular stop. The heat map for both harmonic and closeness centrality approximately represents the distribution of routes in the SF MUNI system. Areas with more routes have a greater number of nodes that are "central" to the system – higher closeness centrality values.

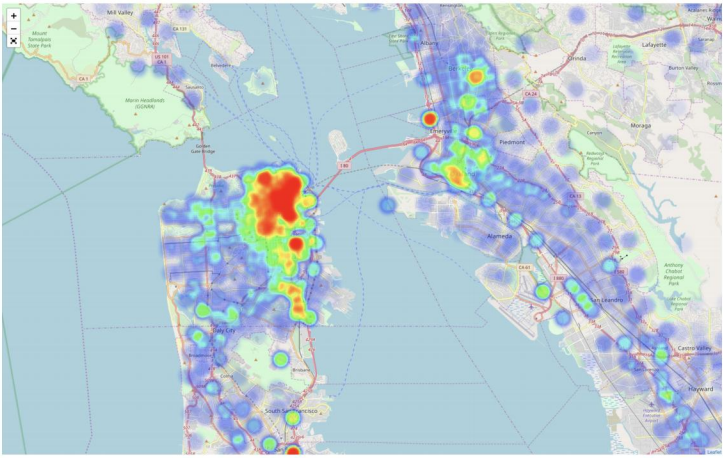
The closeness centrality value for the Uber Movement data is also similar in this regard – locations/nodes have a greater centrality measure in ways that correspond to the places in which drivers drive through often. Nodes with high closeness centrality values in the Uber Movement dataset center therefore center around relatively low traffic areas.

5.6 HITS Centrality

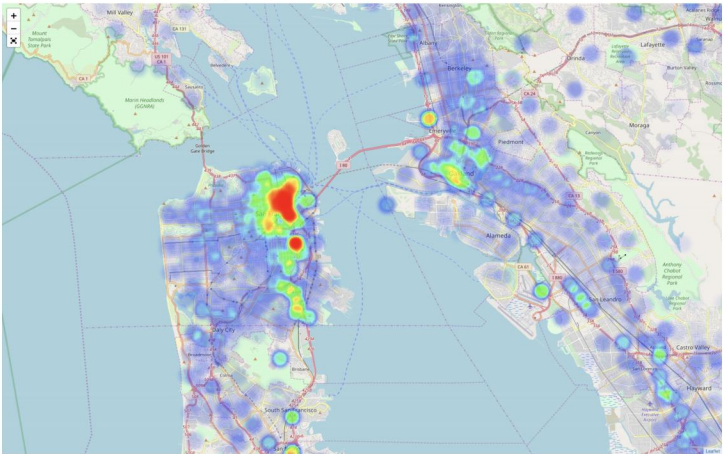
One interesting aspect we noticed was that instead of clearly detecting strong hubs pointing to strong authorities, we see a cluster in downtown San Francisco, or the Northeast. Usually, we would expect to see strongest hubs and authorities outside of the city as well, but what we see here is that the entire hubs and authorities interaction is occurring within this small cluster of San Francisco.

6 Analysis

Overall, we can see that the SF MUNI and Uber Datasets have a significant area of overlap. The following graph overlays the MUNI’s paths over a heatmap of outdegrees, showing Uber Demand from specific locations. Uber demand can be related to outdegree of a location, since outdegree represents the number of locations that have been visited starting from that location. As demonstrated in the graph, Downtown San Francisco has the highest demand for Ubers, which makes sense because it is a region where tourists and workers flock. An interest-



(a) Uber HITS Hub Score



(b) Uber HITS Authority Score

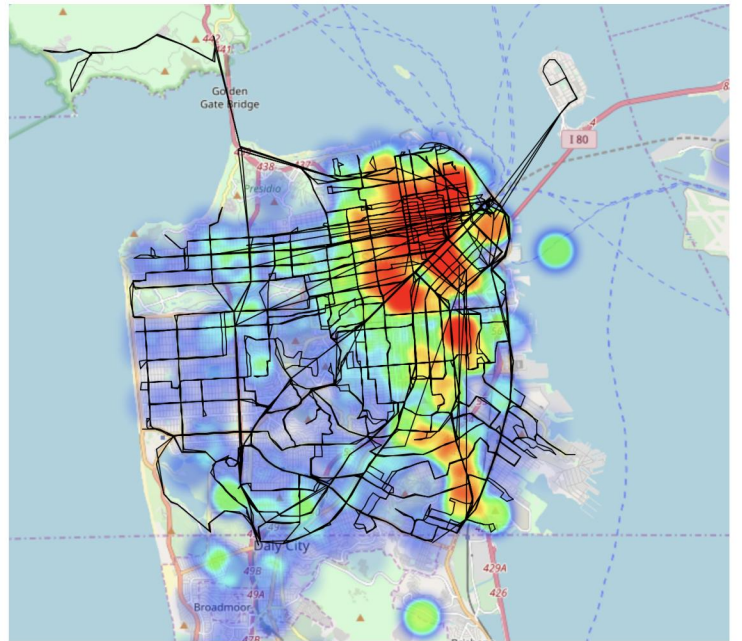


Figure 9: MUNI Paths and Uber Demand

ing thing to note is that both MUNI bus stop concentration and Uber demand increase in this downtown area, reflecting the increased need for transportation at a central point in the city. Although from the plotting of their bus lines alone, the SF MUNI appears capable of transporting passengers all over this region of the Bay, Uber is still in high demand. The reasons for this may have to do with social biases, discussed in the following section.

6.1 Biases

We want to identify the biases that have occurred in the data that are not easily identifiable simply by the information that is provided by these graphs. The primary example of this is the demographics of each user base. For instance, Uber tends to be used by those who have the ability to afford individual rides to a direct destination. This means that Uber data skews more towards a more affluent community that have different transportation demands than those with less financial resources. In

contrast, MUNI routes are set by city planners, and these routes are more financially accessible. Even though MUNI does not allow its user base to directly influence the data the way Uber riders would, MUNI does represent a wider financial demographic.

This is important to acknowledge because shaping MUNI – or other public transit lines – routes around Uber data could skew public transportation in favor of a more privileged demographic. The purpose of this report is to determine differences between ridership demand (reflected in Uber Movement data) and public transit routes, and from there, effectively make recommendations on redesigning these routes. However, if the difference between these networks is due to economic disparity, then these MUNI public transit routes should not be modified to address the needs of a primarily wealthier demographic.

7 Conclusions

Ultimately, directly comparing the SF MUNI system and the Uber Movement dataset did not necessarily lead to direct recommendations, but did provide some valuable insights on accessibility and variety of alternative transportation methods, and why both public transit and rideshare methods exist.

One of these insights was the key difference

between long distance travel and city-specific travel. The SF MUNI system is constrained to the city limits of San Francisco, while Uber Movement provides data on the entirety of the San Francisco Bay Area, and Uber as a private company, has operated with free authority in the overall Bay Area. This enables Uber to provide more long-range transportation needs, while SF MUNI covers a wider swath of the city of San Francisco itself. We see this in a number of our analyses, in which high-profile locations in Uber Movement are highlighted, such as downtown San Francisco, downtown Oakland, and SFO International Airport, to name a few. In contrast, while the SF MUNI system does address high-traffic and commercial areas in San Francisco, such as Market Street, the city overall provides more extensive, uniform transportation coverage because it cannot respond in real-time to immediate transportation needs.

This brings us to another key insight – the difference and merit in the uniform distribution for a public-facing service such as SF MUNI, versus a private company responding to ridership demand in the context of Uber. A prime example of this is, as previously discussed, how the SF MUNI is operated on a relatively speaking, uniform set of routes across the city of San Francisco, while Uber Movement’s dataset responds to high ridership demand. While our initial proposal suggested that by responding to ridership demand (in comparing the Uber Movement data), SF MUNI could improve its services, through analyzing this information ourselves, we see that SF MUNI provides a more accessible service for a variety of passenger demographics, through its extensive coverage of San Francisco.

Finally, the time lapse indicating the in and out degree of both SF MUNI GTFS data and Uber Movement data indicated significant differences in service and demand throughout a typical day. In general, while Uber Movement indicated that there was a key center of the city (downtown San Francisco), and ridership demand near the center gradually expanded throughout the day, the MUNI service did not reflect that. Instead, due to potential myriad factors yet to be explored, MUNI service declined in the "From Work" interval (18:00 - 23:00), at a point where Uber Movement data indicated that demand for travel services was rising.

8 Further Work

In further work, we would be interested in examining the Uber dataset on an hourly basis, instead of the time buckets. This examination would give us the hourly behavior of passenger and provide more insight to the disparities between public transport and Uber.

Another aspect we would be interested in examining is travel time prediction. With use of node2vec embeddings of the network and distance information from the streets (ie the shortest Manhattan distance between two nodes), we could attempt to predict the travel times between edges. These node2vec embeddings with a DFS approach would build clusters and allow us to see "distance" between the nodes. A BFS approach would allow us to examine the local structure of the given node.

9 Contributions

Krishna: Preliminary and final data processing, coded and ran all algorithms, ran tests, wrote algorithm descriptions, generated an algorithm that didn’t make it into the paper, preliminary and general analysis

Christine: Problem formulation and literature review, citations, decided which algorithms to use, wrote transit-related rationale and conclusions on network data/outcomes

Trevor: Plotted overlay and heat maps, provided data analytics, made conclusions from data, identified biases in data, created animations

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