Small world phenomena (1)

CS 322: (Social and Information) Network Analysis
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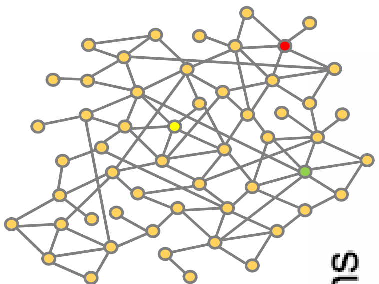
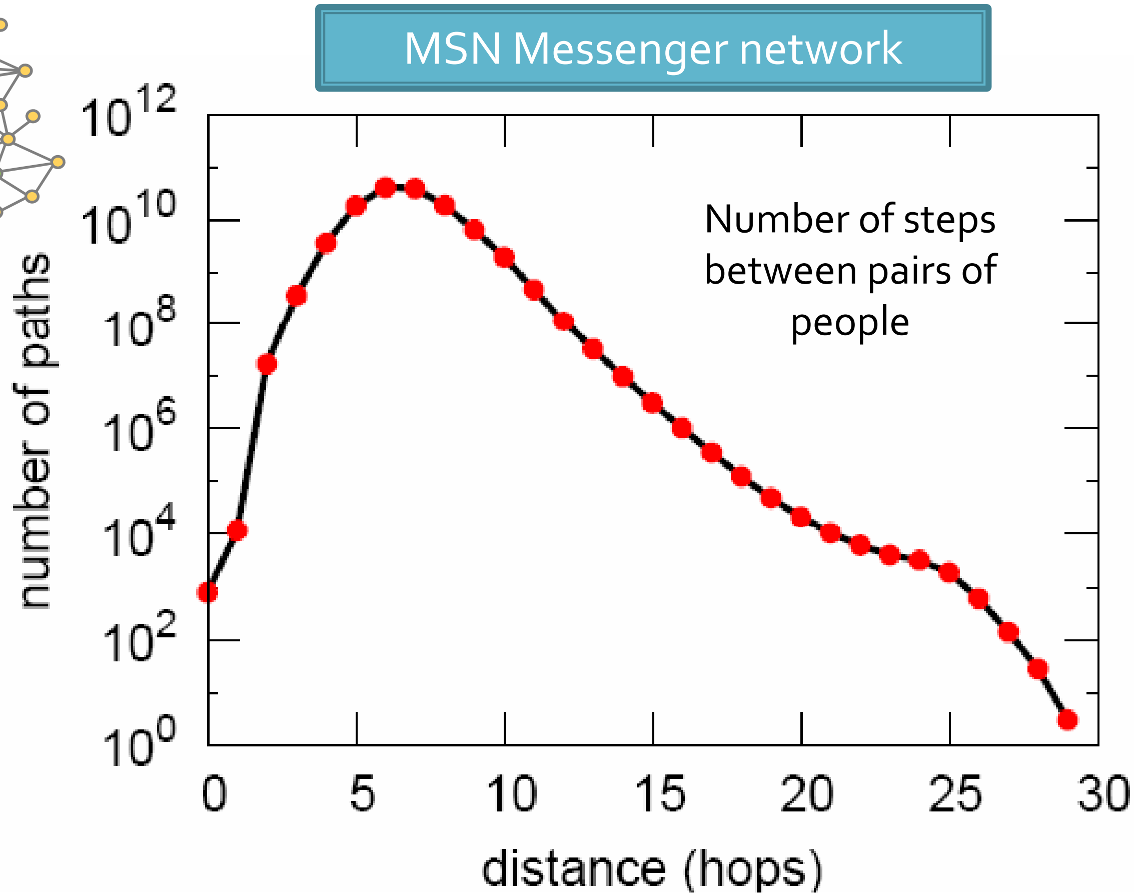
Stanford University

Network: Small world





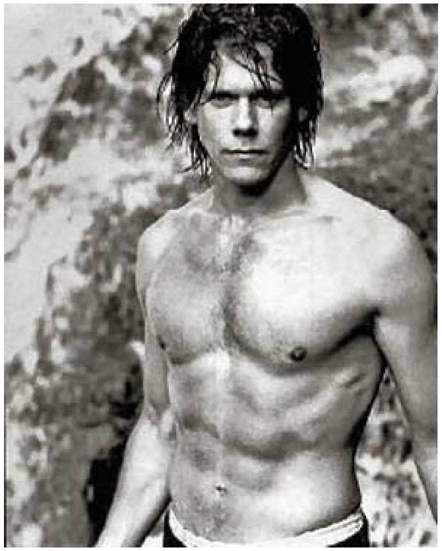
Hops	Nodes
0	1
1	10
2	78
3	3,96
4	8,648
5	3,299,252
6	28,395,849
7	79,059,497
8	52,995,778
9	10,321,008
10	1,955,007
11	518,410
12	149,945
13	44,616
14	13,740
15	4,476
16	1,542
17	536
18	167
19	71
20	29
21	16
22	10
23	3
24	2
25	3

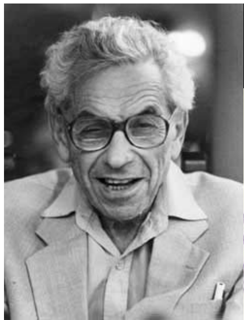
Avg. path length **6.6**

90% of the people can be reached in < 8 hops

Six Degrees of Kevin Bacon

- Bacon number:
  - Create a network of Hollywood actors
  - Connect two actors if they co-appeared in the movie
  - Bacon number: number of steps to Kevin Bacon
- As of Dec 2007, the highest (finite) Bacon number reported is 8
- Only approx. 12% of all actors cannot be linked to Bacon





Erdos numbers are small

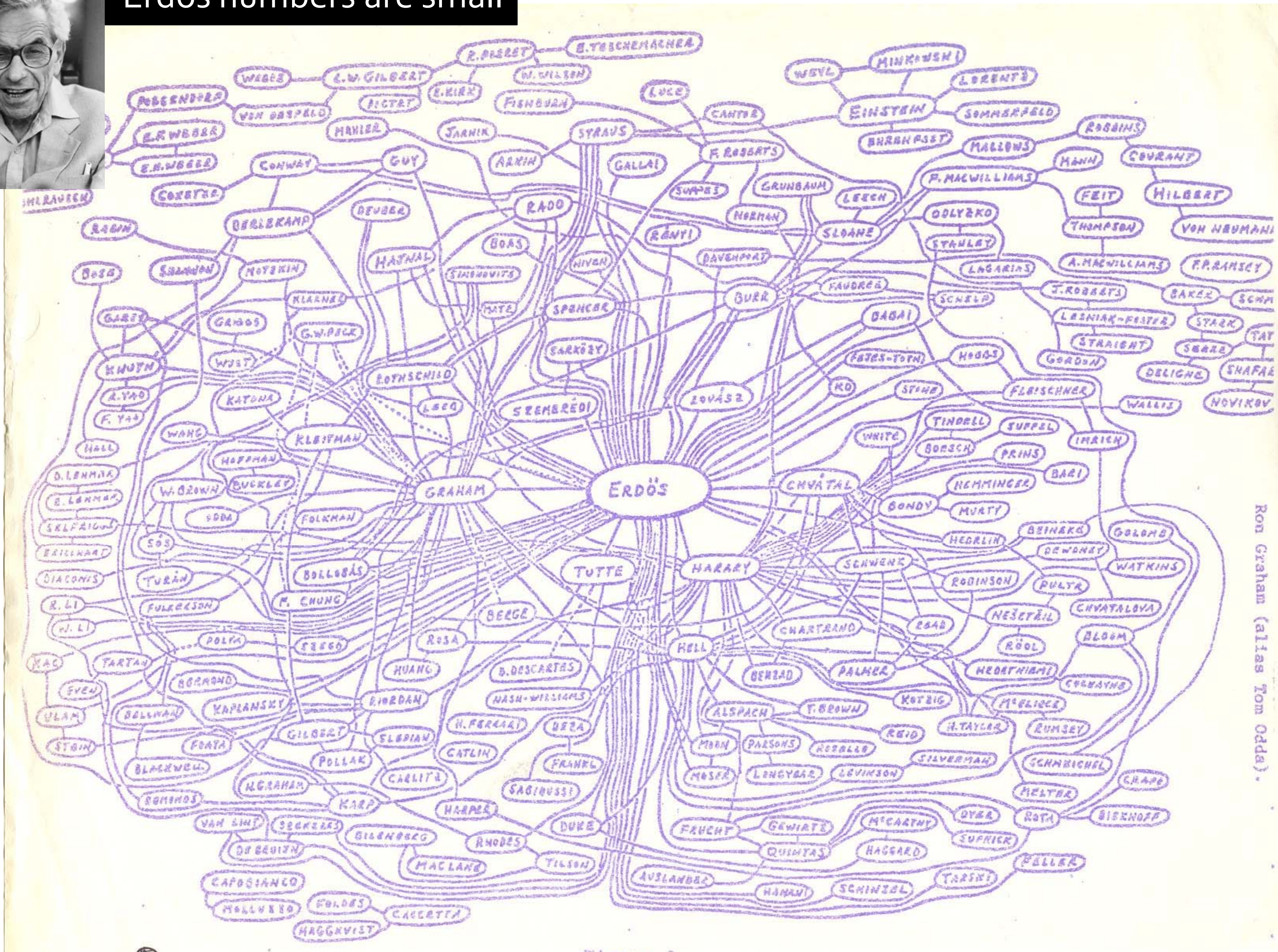


Figure 1

To appear in Topics in Graph Theory (F. Harary, ed.), New York Academy of Sciences (1979).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Shipping costs: Free
Standard Flat Rate Shipping Service
Meet the seller
Seller: gfporch (724)
Feedback: 100 % Positive
Member: since May-02-00 in United States
See detailed feedback
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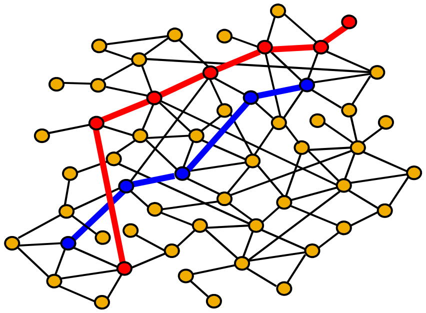
The Small-world experiment

- The Small-world experiment [Milgram '67]
  - Pick 300 people at random
  - Ask them to get a letter to a by passing it through friends to a stockbroker in Boston
- How many steps does it take?

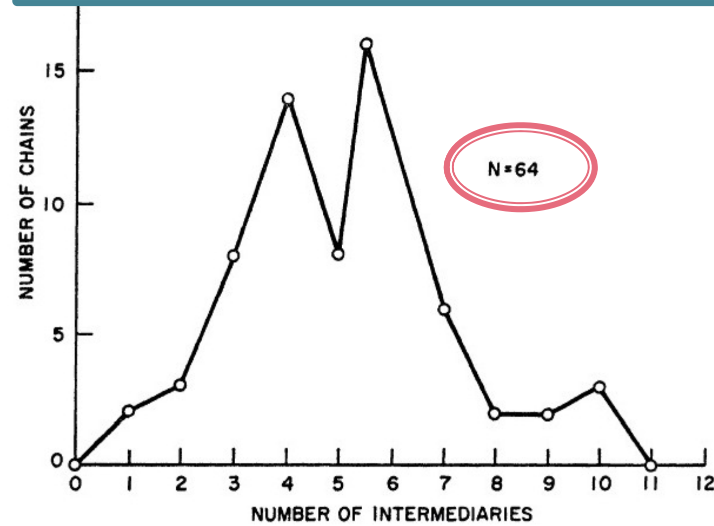
Stanley Milgram



The Small-world experiment

- 64 chains completed:
  - 6.2 on the average, thus "6 degrees of separation"

Milgram's small world experiment



- Further observations:
  - People what owned stock had shortest paths to the stockbroker than random people
  - People from the Boston area have even closer paths

6-degrees: Model?

- How can we understand the small world phenomena?
- What is a good model?

Simplest model?

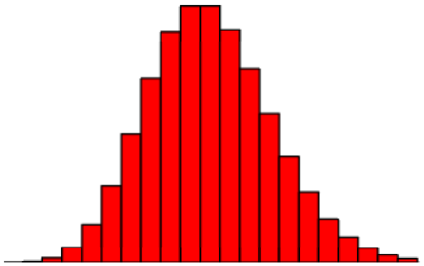
- Erdos-Renyi Random Graph model [Erdos-Renyi, '60]
  - aka.: Poisson/Bernoulli random graphs
- Two variants:
  - $G_{n,p}$: graph on n nodes and each edge (u,v) appears i.i.d. with prob. p. So a graph with m edges appears with prob. $p^m(1-p)^{M-m}$, where $M=n(n-1)/2$ is the max number of edges
  - $G_{n,m}$: graphs with n nodes, m uniformly at random picked edges

What kinds of networks does such process produce?

Properties of random graphs

- **Degree distribution** is Binomial (Poisson in the limit). Let p_k denote a fraction of nodes with degree k:

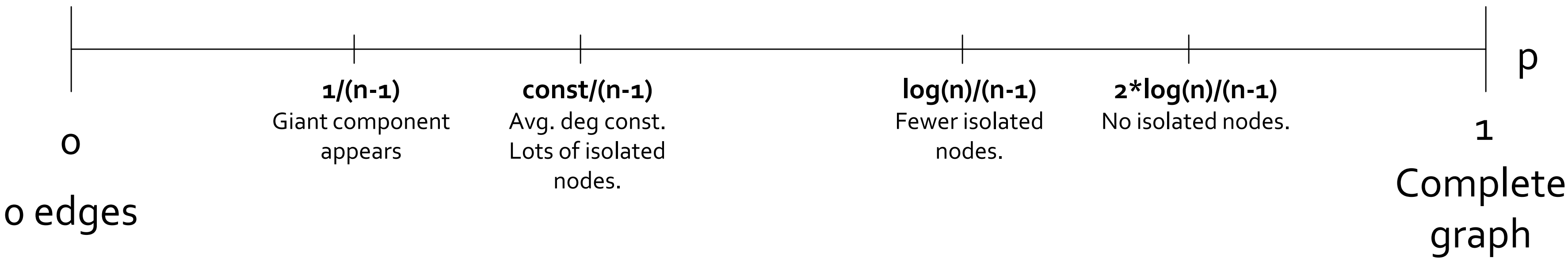
$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \approx \frac{z^k e^{-z}}{k!}$$



- What is expected degree of a node?
  - Prob. of node u linking to node v is p
  - u can link (flips a coin) for all of $(n-1)$ remaining nodes
  - Thus, the expected degree of a node is: $p(n-1)$

Properties of random graphs

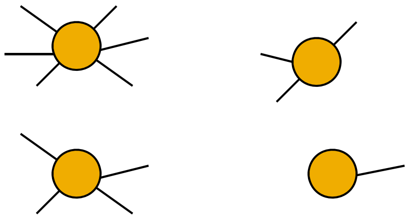
- **Graph structure as *p* changes:**

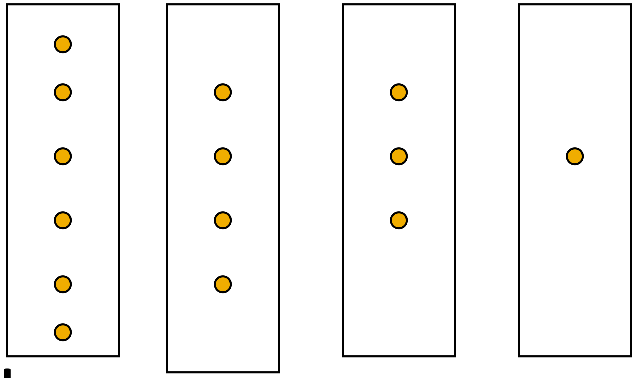


- **Emergence of a giant component:**
  avg. degree $k=2m/n$:
  - $k=1-\varepsilon$: all components are of size $\Omega(\log n)$
  - $k=1+\varepsilon$: 1 component of size $\Omega(n)$, others have size $\Omega(\log n)$

Configuration model

- Configuration model:





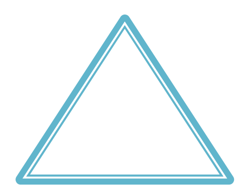
  and randomly connect the spokes
- Assume each node has *d* spokes (half-edges):
  - d=1: set of pairs
  - d=2: set of cycles
  - d=3: arbitrarily complicated graphs
- d-regular graphs

Properties of random graphs

- **Assume:**
  every node has degree exactly d
- **Then:**
  Diameter is $O((\log n) / \alpha)$:
  where G has expansion α
  if $\forall S \subseteq V$: $\#edges\ leaving\ S \geq \alpha \cdot \min(|S|, |\overline{S}|)$
  - Let S_j be a set of nodes within j steps of v.
    Then $|S_{j+1}| \geq |S_j| + \alpha|S_j|/d$.
    So in $O(\log n)$ steps $|S_j|$ grows to $\Theta(n)$.

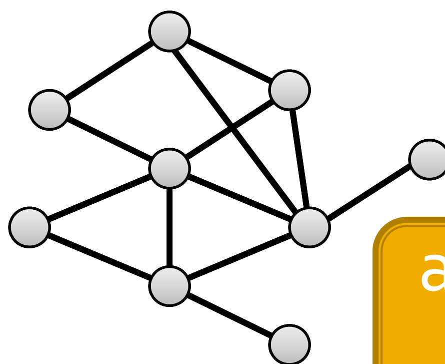
Small world: Reasoning 1

- Assume each human is connected to 100 other people:
- So:
  - In step 1 she can reach 100 people
  - In step 2 she can reach 100*100 = 10,000 people
  - In step 3 she can reach 100*100*100 = 100,000 people
  - In 5 steps she can reach 10 billion people
- What's wrong here?
  - Many edges are local ("short")



How "long" are the edges?

- We can actually directly observe atomic events of network evolution

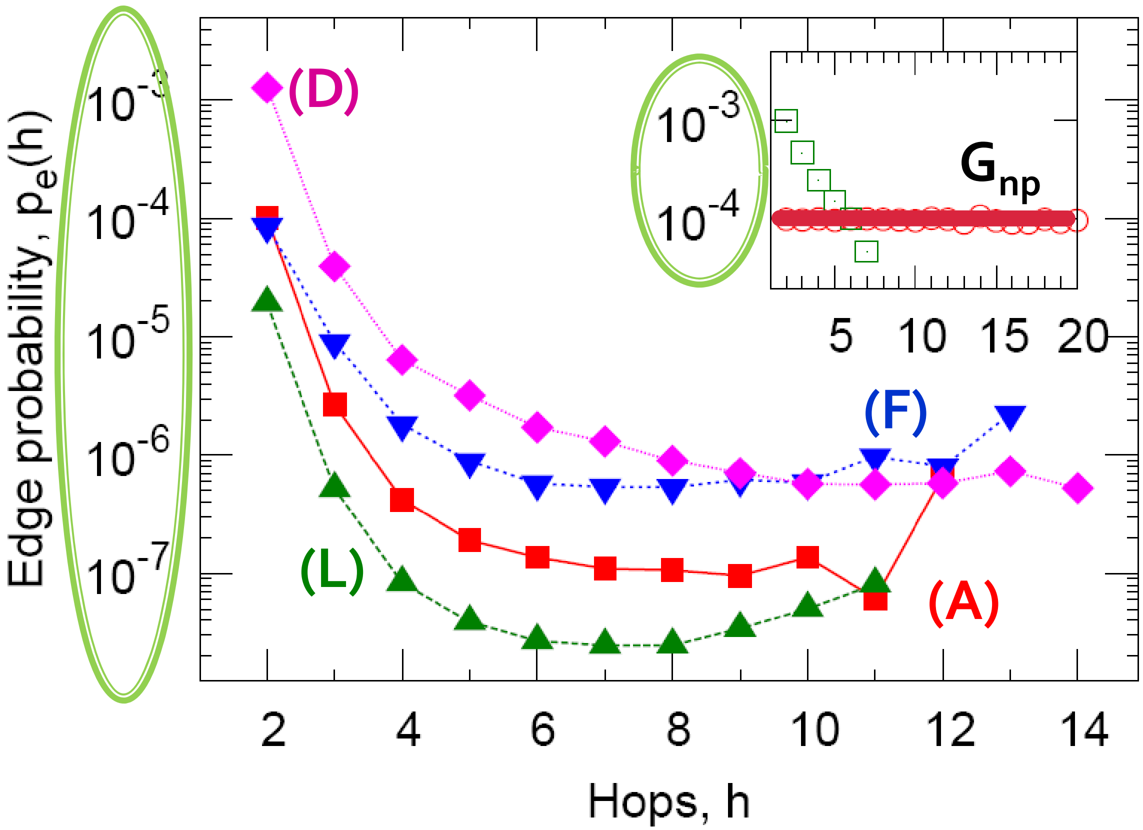


and so on for millions…

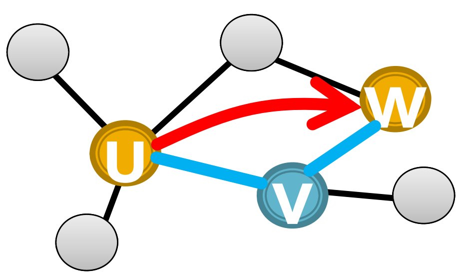
Network	T	N	E
FLICKR (03/2003–09/2005)	621	584,207	3,554,130
DELICIOUS (05/2006–02/2007)	292	203,234	430,707
ANSWERS (03/2007–06/2007)	121	598,314	1,834,217
LINKEDIN (05/2003–10/2006)	1294	7,550,955	30,682,028

Edges are local!

- Just before the edge *(u,v)* is placed how many hops is between *u* and *v*?



Fraction of triad closing edges	
Network	% Δ
F	66%
D	28%
A	23%
L	50%



Small-world model

- How to have local edges (lots of triangles) and small diameter?
- Small-world model [Watts-Strogatz 1998]:
  - Start with a low-dimensional regular lattice
  - Rewire:
    - Add/remove edges to create shortcuts to join remote parts of the lattice
    - For each edge with prob. *p* move the other end to a random vertex