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Mining Data Streams

CS246: Mining Massive Datasets
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New Topic: Infinite Data

High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	PageRank, SimRank	Filtering data streams	Decision Trees	Recommender systems
Clustering	Community Detection	Queries on streams	SVM	Association Rules
Dimensionality reduction	Spam Detection	Web advertising	Parallel SGD	Duplicate document detection

So far

- So far we have worked datasets or data bases where all data is available
- In contrast, in **data streams**, data arrives one element at a time often at a **rapid rate** that:
  - If it is not **processed immediately** it is lost forever.
  - It is not feasible to **store** it all

Data Streams

- **In many data mining situations, we do not know the entire data set in advance**
- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter posts or Facebook status updates
  - e-Commerce purchase data.
  - Credit card transactions
- Think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)
  - This is the fun part and why interesting algorithms are needed

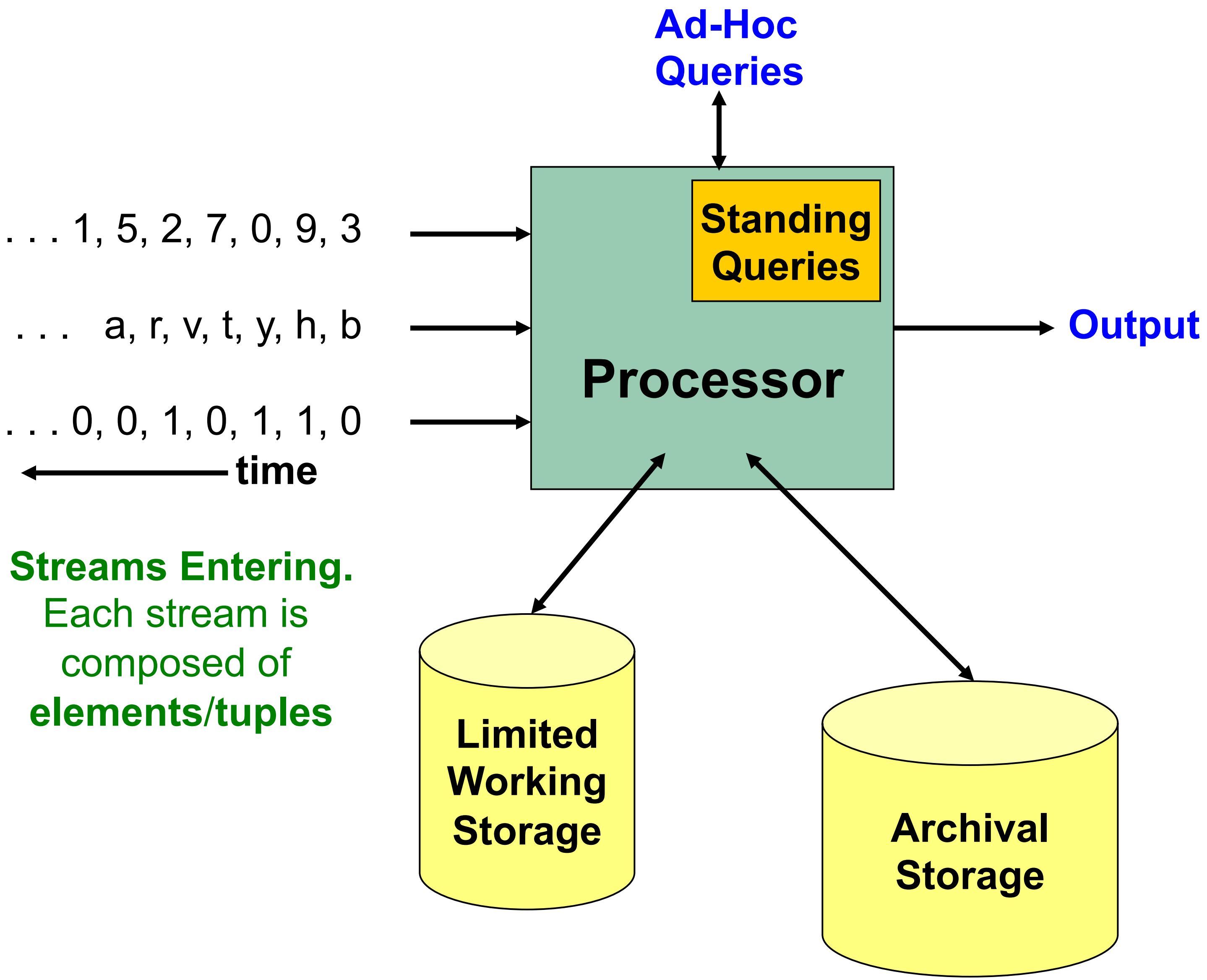
The Stream Model

- Input **elements** enter at a rapid rate,
  at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

Side note: SGD is a Streaming Alg.

- **Stochastic Gradient Descent (SGD) is an example of a streaming algorithm**
- **In Machine Learning we call this: Online Learning**
  - Allows for modeling problems where we have a continuous stream of data
  - We want an algorithm to learn from it and slowly adapt to the changes in data
- **Idea: Do small updates to the model**
  - **SGD** makes small updates
  - **So:** First train the classifier on training data
  - **Then:** For every example from the stream, we slightly update the model (using small learning rate)

General Stream Processing Model



Streams Entering. Each stream is composed of **elements/tuples**

Problems on Data Streams

- **Types of queries one wants to answer on a data stream:**
  - **Sampling data from a stream**
    - Construct a random sample
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **finding most frequent elements**

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday
- **Mining click streams**
  - Wikipedia wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - Look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

- Why is this important?
  - Since we cannot store the entire stream, a representative **sample** can act like the stream
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements of the stream 1..*k*
      - **What is the property of the sample we want to maintain?**
        For all time steps ***k***, each of the ***k*** elements seen so far must have **equal probability** of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling a fixed proportion**
  - E.g. sample 10% of the stream
  - As stream gets bigger, sample gets bigger

- **Naïve solution:**
  - Generate a random integer in **[0...9]** for each query
  - Store the query if the integer is **0**, otherwise discard

- **Any problem with this approach?**
  - We have to be very careful what query we answer using this sample

Problem with Naïve Approach

- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Question:** What fraction of unique queries by an average user are duplicates?
    - Suppose each user issues x queries once and d queries twice (total of $x+2d$ query instances) then the correct answer to the query is $d/(x+d)$
  - **Proposed solution:** **We keep 10% of the queries**
    - Sample will contain ***(x+2d)*/10** elements of the stream
    - Sample will contain ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - There are ***(10x+19d)/100*** unique elements in the sample
      - **(*x+2d)*/10 - *d*/100 = *(10x+19d)/100***
  - **So the sample-based answer is** $\frac{\frac{d}{100}}{\frac{10x}{100}+\frac{19d}{100}} = \frac{d}{10x+19d}$

Problem with Naïve Approach

- **Scenario:** Search engine query stream
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  - **So the sample-based answer is** $\frac{\frac{d}{100}}{\frac{10x}{100}+\frac{19d}{100}} = \frac{d}{10x+19d}$

Sample underestimates

Solution: Sample Users

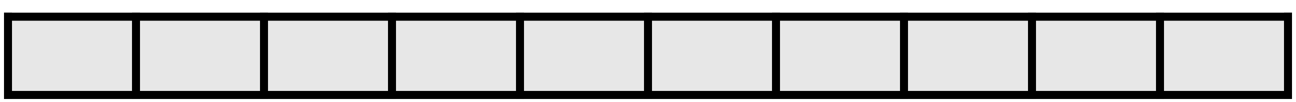
Solution:

- Don’t sample queries, sample users instead
- Pick **1/10th** of **users** and take all their search queries in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple’s components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application
- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple’s key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

The sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose by time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the ***n***th element arrives ($n > s$)
    - With probability ***s/n***, keep the ***n***th element, else discard it
    - If we picked the ***n***th element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

- **We prove this by induction:**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing element ***n+1*** the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Inductive step:**
  - **New element *n+1* arrives, it goes to S with prob s/(n+1)**
  - **For all other elements currently in S:**
    - They were in ***S*** with prob. **s/n**
    - The probability that they remain in S:

$$\left(1-\frac{s}{n+1}\right)+\left(\frac{s}{n+1}\right)\left(\frac{s-1}{s}\right)=\frac{n}{n+1}$$

Element **n+1** discarded

Element **n+1** not discarded

Element in the sample not picked

    - tuples stayed in ***S*** with prob. **n/(n+1)**
- So ***P(tuple is in S at time n+1)*** $= \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

Filtering Data Streams

Filtering Data Streams

- **Each element of data stream is a tuple**
- Given a list of keys **S (which is our filter)**
- **Determine which tuples of stream have key in *S***

- **Obvious solution: Hash table**
  - But suppose we **do not have enough memory** to store all of ***S*** in a hash table
    - E.g., we might be processing millions of filters on the same stream

Applications

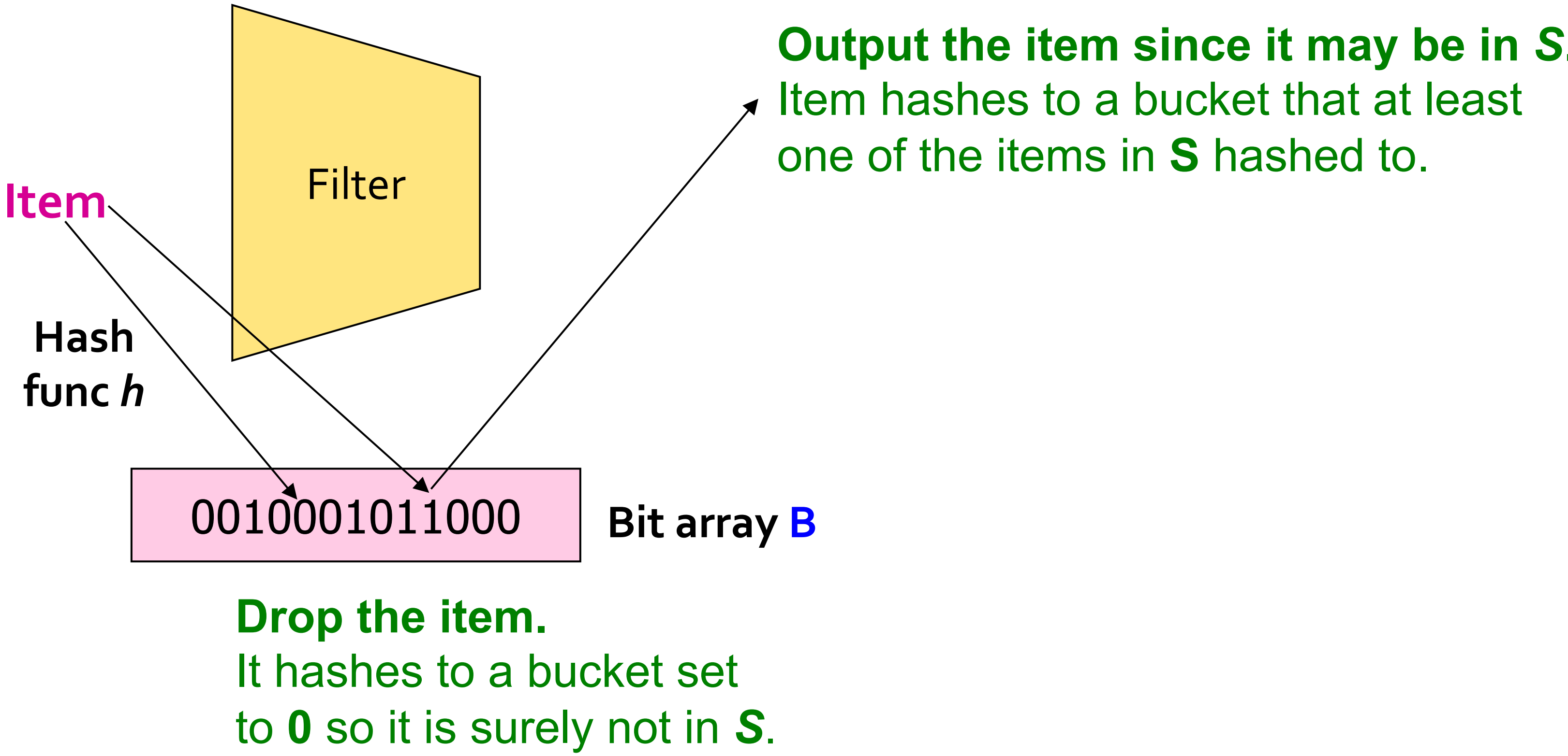
- **Example: Email spam filtering**
  - 1 million users, each user has 1000 “good” email addresses (trusted addresses)
  - If an email comes from one of these, it is **NOT** spam
- **Publish-subscribe systems**
  - You are collecting lots of messages (news articles)
  - People express interest in certain sets of keywords
  - Determine whether each message matches a user’s interest
- **Content filtering**
  - You want to make sure the user does not see the same ad/recommendation multiple times

First Cut Solution (1)

Given a set of keys *S* that we want to filter

- Create a **bit array *B*** of ***n*** bits, initially all ***0*s**
- Choose a **hash function *h*** with range **[*0,n*)**
- Hash each member of $s \in S$ to one of n buckets, and set that bit to **1**, i.e., $B[h(s)]=1$
- Hash each element ***a*** of the stream and output only those that hash to bit that was set to **1**
  - **Output *a* if B[h(a)] == 1**

First Cut Solution (2)



- **Creates false positives**
  - Items that are hashed to a 1 bucket may or may not be in S
- **but no false negatives**
  - Items that are hashed to 0 bucket are surely not in S

First Cut Solution (3)

- **|S| = 1 billion email addresses**
  |B|= 1GB = 8 billion bits
- If the email address is in ***S***, then it surely hashes to a bucket that has the bit set to **1**, so it always gets through (***no false negatives***)
- Approximately **1/8** of the bits are set to **1**, so about **1/8th** of the addresses not in ***S*** get through to the output (***false positives***)
  - Actually, less than **1/8th**, because more than one address might hash to the same bit

Analysis: Throwing Darts (1)

- **Let's do a more accurate analysis of number of false positives, we know that:**
  - **Fraction of 1s in array B = prob. of false positive**
- **Darts & Targets:** If we throw ***m*** darts into ***n*** equally likely targets, **what is the probability that a target gets at least one dart?**
- **In our case:**
  - **Targets** = bits/buckets
  - **Darts** = hash values of items

Analysis: Throwing Darts (2)

- We have ***m*** darts, ***n*** targets
- **What is the probability that a target gets at least one dart?**

Equals **1/e** as **n** $\rightarrow\infty$

Equivalent

$$\boxed{1 - \boxed{(1 - 1/n)}^{n(m/n)}}$$

$$\mathbf{1 - e^{-m/n}}$$

Probability some target **X** not hit by a dart

Probability at least one dart hits target **X**

Approximation is especially accurate when n is large

Analysis: Throwing Darts (3)

- **Fraction of 1s in the array B = probability of false positive = $1 - e^{-m/n}$**
- **Example:** 10^9 darts, $8 \cdot 10^9$ targets
  - Fraction of **1s** in **B** = $1 - e^{-1/8} = 0.1175$
    - Compare with our earlier estimate: **1/8 = 0.125**
- **To reduce false positive rate of bloom filter we use multiple hash functions**

Bloom Filter

- Consider: **|S| = *m keys*, |B| = *n bits***
- Use k independent hash functions $h_1, \ldots, h_k$
- **Initialization:**
  - Set **B** to all **0s**
  - Hash each element $s \in S$ using each hash function h_i, set **B[$h_i(s)$] = 1** (for each $i = 1,.., k$)

(**note:** we have a single array B!)

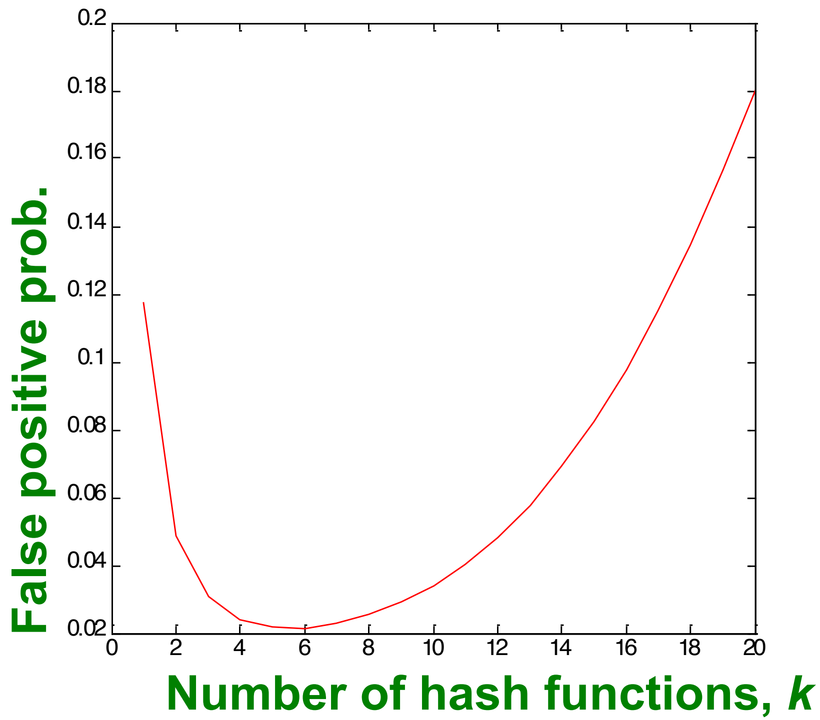
- **Run-time:**
  - When a stream element with key x arrives
    - If **B[$h_i(x)$] = 1** for all i **= 1,..., k** then declare that x is in S
      - That is, x hashes to a bucket set to **1** for every hash function $h_i(x)$
    - Otherwise discard the element x

Bloom Filter – Analysis

- **What fraction of the bit vector B are 1s?**
  - Throwing $k \cdot m$ darts at n targets
  - So fraction of **1**s is $(1 - e^{-km/n})$
- But we have k independent hash functions and we only let the element x through **if all** k hash element x to a bucket of value **1**
- So, **false positive probability** = $(1 - e^{-km/n})^k$

Bloom Filter – Analysis (2)

- ***m* = 1 billion, *n* = 8 billion**
  - **k = 1**: $(1 - e^{-1/8}) =$ **0.1175**
  - **k = 2**: $(1 - e^{-1/4})^2 =$ **0.0489**



- **What happens as we keep increasing *k*?**
- Optimal value of ***k***: $\frac{n}{m} \ln 2$
  - **In our case:** Optimal **k = 8 ln(2) = 5.54 ≈ 6**
    - **Error at k = 6**: $(1 - e^{-3/4})^6 =$ **0.0216**

Optimal *k*: *k* which gives the lowest false positive probability

Bloom Filter: Wrap-up

- **Bloom filters guarantee no false negatives, and use limited memory**
  - Great for pre-processing before more expensive checks
- **Suitable for hardware implementation**
  - Hash function computations can be parallelized
- Is it better to have **1** big **B** or ***k*** small **B**s?
  - **It is the same:** $(1 - e^{-km/n})^k$ vs. $(1 - e^{-m/(n/k)})^k$
  - But keeping **1 big B** is simpler

Counting Distinct Elements

Counting Distinct Elements

- **Problem:**
  - Data stream consists of a universe of elements chosen from a set of size ***N***
  - Maintain a count of the number of distinct elements seen so far

- **Obvious approach:**
  Maintain a dictionary of elements seen so far
  - keep a hash table of all the distinct elements seen so far
  - What if number of distinct elements are huge?
  - What if there are many streams that need to be processed at once?

Applications

- **How many unique users a website has seen in each given month?**
  - Universal set = set of logins for that month
  - Stream element = each time someone logs in
- **How many different words are found at a site which is among the Web pages being crawled?**
  - Unusually low or high numbers could indicate artificial pages (spam?)
- **How many distinct products have we sold in the last week?**

Using Small Storage

- **Real problem: What if we do not have space to maintain the set of elements seen so far in every stream?**
  - **We have limited working storage**
- **We use a variety of hashing and randomization to get approximately what we want**
- **Estimate the count in an unbiased way**
- **Accept that the count may have a little error, but limit the probability that the error is large**

Flajolet-Martin Approach

- Estimates number of distinct elements by hashing elements to a bit-string that is sufficiently long
  - The length of the bit-string is large enough that it produces more result that size of universal set.
- **Idea**: the more different elements we see in the stream, the more different hash values we shall see.
  - Number of trailing **0**s in these hash values estimates number of distinct elements.

Flajolet-Martin Approach

- Pick a hash function h that maps each of the N elements to at least $\log_2 N$ bits
- For each stream element a, let $r(a)$ be the number of trailing **0s** in $h(a)$
  - **r(a)** = position of first 1 counting from the right
    - E.g., say $h(a) = 12$, then *12* is *1100* in binary, so $r(a) = 2$
- Record ***R* = the maximum *r*(*a*) seen**
  - $R = \max_a r(a)$, over all the items a seen so far
- **Estimated number of distinct elements = 2^R**

Why It Works: Intuition

- **Very rough and heuristic intuition why Flajolet-Martin works:**
  - ***h(a)*** hashes ***a*** with **equal prob.** to any of ***N*** values
  - All elements have **equal prob**. to have a tail of ***r*** zeros
  - That is ***2^{-r}*** fraction of all ***a***s have a tail of ***r*** zeros
    - About 50% of ***a***s hash to *****0**
    - About 25% of ***a***s hash to ****00**
    - So, if we saw the longest tail of ***r=2*** (i.e., item hash ending ***100**) then we have probably seen **about *4*** distinct items so far
  - **So, it takes to hash about *2^r* items before we see one with zero-suffix of length *r***

Why It Works: More formally

- Now we show why Flajolet-Martin works
- Let m be the number of distinct elements seen so far in the stream
- We show that probability of finding a tail of *r* zeros:
  - Goes to 1 if $m \gg 2^r$
  - Goes to 0 if $m \ll 2^r$
- Thus, 2^R will almost always be around *m!*

Why It Works: More formally

- **What is the probability that a given *h*(*a*) ends in at least *r* zeros? It is 2^{-r}**
  - **h(a)** hashes elements uniformly at random
  - Probability that a random number ends in at least ***r*** zeros is $\mathbf{2^{-r}}$
- Then, the probability of **NOT** seeing a tail of length ***r*** among ***m*** elements:

$$\boxed{(\boxed{1-2^{-r}})^{m}}$$

Prob. all end in fewer than ***r*** zeros.

Prob. that given **h(a)** ends in fewer than ***r*** zeros

Why It Works: More formally

- **Note:** $(1-2^{-r})^m = (1-2^{-r})^{2^r(m2^{-r})} \approx e^{-m2^{-r}}$
- **Prob. of NOT finding a tail of length *r* is:**
  - If ***m << 2^r***, then prob. tends to **1**
    - $(1-2^{-r})^m \approx e^{-m2^{-r}} = 1$ as **$m/2^r \to 0$**
    - So, the probability of finding a tail of length ***r*** tends to **0**
  - If ***m >> 2^r***, then prob. tends to **0**
    - $(1-2^{-r})^m \approx e^{-m2^{-r}} = 0$ as **$m/2^r \to \infty$**
    - So, the probability of finding a tail of length ***r*** tends to **1**
- **Thus, 2^R will almost always be around *m*!**

Why It Doesn't Work

- **E[2^R] is actually infinite**
  - Probability halves when ***R*** → ***R+1***, but value doubles
- **Workaround involves using many hash functions h_i and getting many samples of R_i**
- **How are samples R_i combined?**
  - **Average?** What if one very large value 2^{R_i}?
  - **Median?** All estimates are a power of **2**
  - **Solution:**
    - Partition your samples into small groups
    - Take the median of groups
    - Then take the average of the medians

Counting frequent items/itemsets

Counting Itemsets

- **New Problem: Given a stream of itemsets, which itemsets appear more frequently?**
- **Application:**
  - What are most frequent products bought together?
  - What are some "hot" gift items bought together?
- **Solution: Exponentially decaying windows**
  - We first use it to count singular items
    - Popular movies, most bought products, etc.
  - Then we extend it to counting itemsets

Exponentially Decaying Windows

- **Exponentially decaying windows: A heuristic for selecting likely frequent items (itemsets)**
  - **What are "currently" most popular movies?**
    - Instead of computing the raw count in last ***N*** elements
    - Compute a **smooth aggregation** over the whole stream
- **Smooth aggregation**: If stream is $a_1, a_2, \ldots$ then the smooth aggregation at time $t = \sum_{t=1}^{T} a_t (1-c)^{T-t}$
    - **c** is a constant, presumably tiny, like 10^{-6} or 10^{-9}
    - a_t is a non-negative integer in general
- **When new a_{t+1} arrives:**
  Multiply current sum by **(1-c)** and add a_{t+1}

A binary stream per item

- Think of the stream of itemsets as one binary stream per item
  - For every item, form a binary stream
    - **1** = item present; **0** = not present

Stream of items:

brtbhbgbbgzcbabbcbdbdbnbrbpbqbbsbtbababebcbbbvbwbxbwbbbcbdbcgfbabbzdba

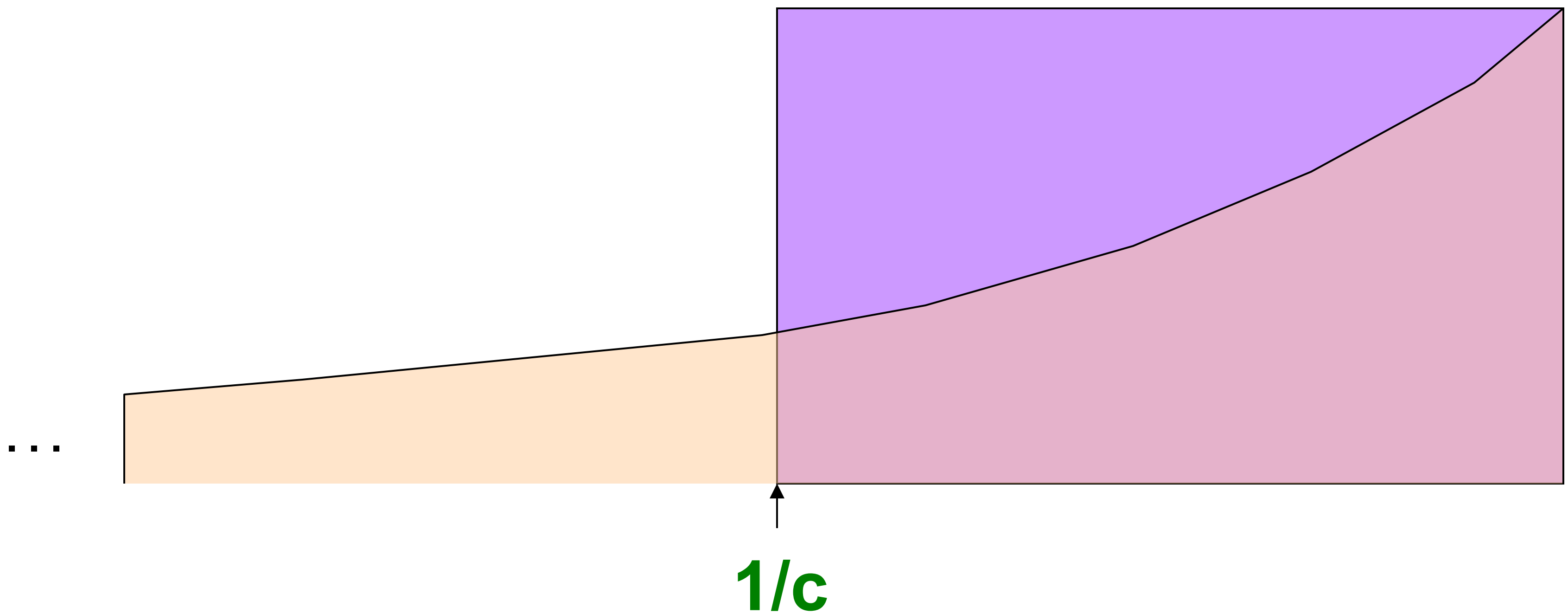
Binary stream for “b”

1001010110001011010101010101011010101010101110101010111010100010110010

Counting Items

- If each a_t is an "item" we can compute the **characteristic function** of each item x as an **Exponentially Decaying Window**:
  - That is: $\sum_{t=1}^{T} \delta_t \cdot (1-c)^{T-t}$
    where $\delta_t = 1$ if $a_t = x$, and 0 otherwise
  - **In other words:** Imagine that for each item x we have a binary stream (1 if x appears, 0 if x does not appear)
  - Then, when a new item a_t arrives:
    - Multiply the summation by $(1-c)$
    - Add **+1** to the summation if item = x
- **Call this sum the "weight" of item x**

Counting Items: Decaying Windows



- **Important property:** Sum over all weights
  $\sum_t 1 \cdot (1-c)^t = 1/[1-(1-c)] = 1/c$

$$\sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}$$

Counting Individual Items

- **What are "currently" most popular movies?**
- **Suppose we want to find movies of weight > ½**
  - **Important property:** Sum over all weights $\sum_t \delta_t \cdot (1-c)^t$ is 1/[1 – (1 – c)] = $\mathbf{1/c}$
    - **That means that no item can have weight greater than 1/c**
    - The item will have weight $\mathbf{1/c}$ if its stream is [1,1,1,1,1…]. Note we have a separate binary stream for each item. So, at a given time only one item will have a δ_t=1, and other items will get a 0.
- **Thus:**
  - There cannot be more than $\mathbf{2/c}$ movies with weight of ½ or more
    - Why? Assume weight of item is ½. How many items *n* can we have so that the sum is <1/c; **Answer:** ½n<1/c → $n < 2/c$
- So, $\mathbf{2/c}$ is a limit on the number of movies being counted at any time

Extension to Itemsets

- **Extension: Count (some) itemsets**
  - **What are currently "hot" itemsets?**
    - **Problem:** Too many itemsets to keep counts of all of them in memory
- **When a basket B comes in:**
  - Multiply all counts by $(1 - c)$
  - For uncounted items in B, create new count
  - Add 1 to count of any item in B and to any **itemset** contained in B that is already being counted
  - **Drop counts < ½**
  - Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset $S \subseteq B$ if every proper subset of ***S*** had a count prior to arrival of basket ***B.***
  - **Intuitively:** If all subsets of **S** are being counted this means they are "**frequent/hot**" and thus ***S*** has a potential to be "**hot**"
- **Example:**
  - Start counting ***S*={i, j}** iff both **i** and **j** were counted prior to seeing ***B***
  - Start counting ***S*={i, j, k}** iff **{i, j}**, **{i, k}**, and **{j, k}** were all counted prior to seeing ***B***

How many counts do we need?

- Counts for single items **< (2/c)·(avg. number of items in a basket)**
- **Counts for larger itemsets = ??**
- **But we are conservative about starting counts of large sets**
  - If we counted every set we saw, one basket of **20** items would initiate **1M** counts

Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Check existence of a set of keys in the stream**
  - Bloom filter
- **Counting distinct elements in a stream**
  - Flajolet-Martin algorithm
- **Counting frequent elements in a stream**
  - Exponentially decaying window