More Stream Mining

Bloom Filters
Sampling Streams
Counting Distinct Items
Computing Moments

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Example: Filtering Chunks

- Suppose we have a dataset stored in a distributed file system, spread over many chunks (e.g. blocks of 64MB).
- We want to find a particular value V, looking at as few chunks as possible.
- A Bloom filter on each chunk is a relatively short sequence of bits that provides an answer to the question “is there a value V anywhere in this chunk?”

Filtering Chunks – (2)

- If the Bloom filter says "no," then there definitely is no V there, so we do not have to retrieve this chunk.
- Unfortunately, there can be false positives – the Bloom filter says "yes," but there really is no instance of V there.
- We can make the probability of a false positive very low, but at the cost of a larger bit array.

How a Bloom Filter Works

- A *Bloom filter* is an array of bits, together with a number of hash functions.
- The argument of each hash function is a stream element, and it returns a position in the array.
- Initially, all bits are 0.
- When input x arrives, we set to 1 the bits h(x), for each hash function h.

Example: Bloom Filter

- Use N = 11 bits for our filter.
- Stream elements = integers.
- Use two hash functions:
  - $h_1(x)$ =
    - Take odd-numbered bits from the right in the binary representation of x.
    - Treat it as an integer i.
    - Result is i modulo 11.
  - $h_2(x)$ = same, but take even-numbered bits.

Example – Continued

Stream element	h_1	h_2	Filter contents
			00000000000
25 = 11001	5	2	00100100000
159 = 10011111	7	0	10100101000
585 = 1001001001	9	7	10100101010

Note: bit 7 was already 1.

Bloom Filter Lookup

- Suppose element y appears in the stream, and we want to know if we have seen y before.
- Compute h(y) for each hash function y.
- If all the resulting bit positions are 1, say we have seen y before.
  - We could be wrong.
    - Different inputs could have set each of these bits.
- If at least one of these positions is 0, say we have not seen y before.
  - We are certainly right.

Example: Lookup

- Suppose we have the same Bloom filter as before, and we have set the filter to 10100101010.
- Lookup element y = 118 = 1110110 (binary).
- $h_1(y)$ = 14 modulo 11 = 3.
- $h_2(y)$ = 5 modulo 11 = 5.
- Bit 5 is 1,
- But bit 3 is 0, so we are sure y is not in the set.

Performance of Bloom Filters

- Probability of a false positive depends on the density of 1's in the array and the number of hash functions.
  - = (fraction of 1's)$^{\text{\# of hash functions}}$.
- The number of 1's is approximately the number of elements inserted times the number of hash functions.
  - But collisions lower that number slightly.

Throwing Darts

- Turning random bits from 0 to 1 is like throwing *d* darts at *t* targets, at random.
- How many targets are hit by at least one dart?
- Probability a given target is hit by a given dart = 1/t.
- Probability none of d darts hit a given target is $(1-1/t)^d$.
- Rewrite as $(\underbrace{\boxed{1-1/t)}}_{\sim = 1/e}{}^{t(d/t)} \sim= e^{-d/t}$.

Example: Throwing Darts

- Suppose we use an array of 1 billion bits, 5 hash functions, and we insert 100 million elements.
- That is, $t = 10^9$, and $d = 5*10^8$.
- The fraction of 0's that remain will be $e^{-1/2} = 0.607$.
- Density of 1's = 0.393.
- Probability of a false positive = $(0.393)^5 = 0.00937$.

Sampling a Stream

What Doesn’t Work

Sampling Based on Hash Values

Key- and Value-Components of Data

The Use of Sampling

- Often, we can get a good approximate answer from a sample.
- Example: Frequent itemsets from a sample of baskets.
- If we take a 10% sample of baskets and lower the support threshold by a factor of 10, there is a good chance that an itemset is frequent in the sample iff it is frequent in the whole.

When Sampling Doesn't Work

- Suppose Google would like to examine its stream of search queries for the past month to find out what fraction of them were unique – asked only once.
- But to save time, we are only going to sample 10% of the stream.
- The fraction of unique queries in the sample != the fraction for the stream as a whole.
  - In fact, there is no way to adjust parameters to give the correct answer, as we could for frequent itemsets.

Example: Unique Search Queries

- The length of the sample is 10% of the length of the whole stream.
- Suppose a query is unique.
    - It has a 10% chance of being in the sample.
- Suppose a query occurs exactly twice in the stream.
    - It has an 18% chance of appearing exactly once in the sample.
- And so on … The fraction of unique queries in the stream is unpredictably large.

Sampling by Value

- My mistake: I sampled based on the position in the stream, rather than the value of the stream element.
- The right way: hash search queries to 10 buckets 0, 1,…, 9.
- Sample = all search queries that hash to bucket 0.
  - All or none of the instances of a query are selected.
  - Therefore the fraction of unique queries in the sample is the same as for the stream as a whole.

Controlling the Sample Size

- Problem: What if the total sample size must be limited?
- Solution: Hash to a large number of buckets.
- Adjust the set of buckets accepted for the sample, so your sample size stays within bounds.

Example: Fixed Sample Size

- Suppose we start our search-query sample at 10%, but we want to limit the size.
- Hash to (say) 100 buckets, 0, 1,…, 99.
  - Take for the sample those elements hashing to buckets 0 through 9.
- If the sample gets too big, get rid of bucket 9.
- Still too big, get rid of 8, and so on.

Sampling Key-Value Pairs

- This technique generalizes to any form of data that we can see as tuples (K, V), where K is the "key" and V is a "value."
- Distinction: We want our sample to be based on picking some set of keys only, not pairs.
  - In the search-query example, the data was "all key."
- Hash keys to some number of buckets.
- Sample consists of all key-value pairs with a key that goes into one of the selected buckets.

Example: Salary Ranges

- Data = tuples of the form (EmpID, Dept, Salary).
- Query: What is the average range of salaries within departments?
- If we sample tuples, we'll get a subset of the salaries for each department.
- We'll likely miss the highest and lowest salaries for a department.
- Thus, the estimate of the range will be biased to the low side.

Salary Ranges – (2)

- The correct way:
- Key = Dept.
- Value = (EmpID, Salary).
- Sample picks some departments, has salaries for all employees of that department, including its min and max salaries.
- Result will be an unbiased estimate of the average salary range.

Counting Distinct Elements

Flajolet-Martin Approximation Technique

Application to Counting Unions: Neighborhood Sizes

Counting Distinct Elements

- Problem: a data stream consists of elements chosen from a set of size n. Maintain a count of the number of distinct elements seen so far.
- Obvious approach: maintain the set of elements seen.

Some Applications

- How many different words are found among the Web pages being crawled at a site?
  - Unusually low or high numbers could indicate artificial pages (spam?).
- How many unique users visited Facebook this month?
- How many different pages link to each of the pages we have crawled.
  - Useful for estimating the PageRank of these pages.
    - Which in turn can tell a crawler which pages are most worth visiting.

Estimating Counts

- Real Problem: what if we do not have space to store the complete set?
  - Or we are trying to count lots of sets at the same time.
- Estimate the count in an unbiased way.
- Accept that the count may be in error, but limit the probability that the error is large.

Flajolet-Martin Approach

- Pick a hash function *h* that maps each of the *n* elements to at least $\log_2 n$ bits.
- For each stream element *a*, let *r*(*a*) be the number of trailing 0's in *h*(*a*).
  - Called the *tail length*.
  - Example: 000101 has tail length 0; 101000 has tail length 3.
- Record *R* = the maximum *r*(*a*) seen for any a in the stream.
- Estimate (based on this hash function) = 2^R.

Why It Works

- The probability that a given $h(a)$ ends in at least i 0's is 2^{-i}.
- If there are m different elements, the probability that $R \geq i$ is $1 - \boxed{\boxed{(1 - 2^{-i})}^{m}}$.

Prob. all h(a)'s end in fewer than i o's.

Prob. a given h(a) ends in fewer than i o's.

Why It Works – (2)

- Since 2^{-i} is small, $1 - (1-2^{-i})^m \approx 1 - e^{-m2^{-i}}$.
- If $2^i >> m$, $1 - e^{-m2^{-i}} \approx 1 - (1 - m2^{-i}) \approx m/2^i \approx 0$.
- If $2^i << m$, $1 - e^{-m2^{-i}} \approx 1$.
- Thus, 2^R will almost always be around m.

First 2 terms of the Taylor expansion of e^x

Same trick as "throwing darts." Multiply and divide m by 2^{-i}.

Why It Doesn't Work

- $E(2^R)$ is, in principle, infinite.
  - Probability of $\geq$ R 0's halves when R -> R+1, but value of 2^R doubles.
- Workaround involves using many hash functions and getting many samples.
- How are samples combined?
  - Average? What if one very large value?
  - Median? All values are a power of 2.

Solution

- Partition your samples into small groups.
  - $O(\log n)$, where n = size of universal set, suffices.
- Take the average within each group.
- Then take the median of the averages.

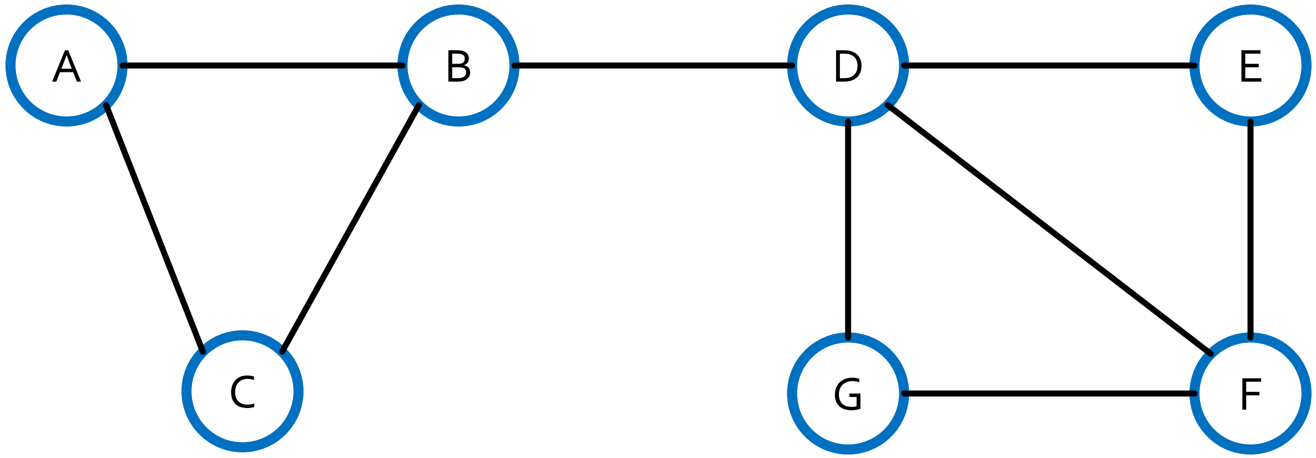
Application:Neighborhoods

Neighborhood of Distance d
Recursive Algorithm for Neighborhoods
Approximate Neighborhood Count

Neighbors and Neighborhoods

- If there is an edge between nodes u and v, then u is a *neighbor* of v and vice-versa.
- The *neighborhood* of node u at distance d is the set of all nodes v such that there is a path of length at most d from u to v.
    - Denoted n(u,d).
- Notice that if there are N nodes in a graph, then n(u,N-1) = n(u,N) = n(u,N+1) = … = all nodes reachable from u.

Example: Neighborhoods



$n(E,0) = \{E\}$

$n(E,1) = \{D,E,F\}$

$n(E,2) = \{B,D,E,F,G\}$

$n(E,3) = \{A,B,C,D,E,F,G\}$

Why Neighborhoods?

- The sizes of neighborhoods of small distance measure the “influence” a person has in a social network.
  - Note it is the size of the neighborhood, not the exact members of the neighborhood that is important here.

Algorithm for Finding Neighborhoods

- n(u,0) = {u} for every u.
- n(u,d) is the union of n(v, d-1) taken over every neighbor v of u.
- Not really feasible for large graphs, since the neighborhoods get large, and taking the union requires examining the neighborhood of each neighbor.
  - To eliminate duplicates.
- Note: Another approach where we take the union of neighbors of members of n(u, d-1) presents similar problems.

Approximate Algorithm for Neighborhood Sizes

- The idea behind Flajolet-Martin lets you estimate the number of distinct elements in the union of several sets.
- Pick several hash functions; let h be one.
- For each node u and distance d compute the maximum tail length among all nodes in n(u,d), using hash function h.

Approximate Algorithm – (2)

- Remember: if R is the maximum tail length in a set of values, then 2^R is an estimate of the number of distinct elements in the set.
- Since n(u,d) is u plus the union of all neighbors v of u of n(v,d-1), the maximum tail length of members of n(u,d) is the largest of
    1. The tail length of h(u), and
    2. The maximum tail length for all the members of n(v,d-1) for any neighbor v of u.

Approximate Algorithm – (3)

- Thus, we have a recurrence (on d) for the maximum tail length of any neighbor of any node u, using any given hash function h.
- Repeat for some chosen number of hash functions.
- Combine estimates to get an estimate of neighborhood sizes, as for the Flajolet-Martin algorithm.
- And voila! You have an efficient algorithm for estimating the size of each node’s neighborhood.

Moments

Surprise Numbers
AMS Algorithm

Generalization: Moments

- Suppose a stream has elements chosen from a set of n values.
- Let m_i be the number of times value i occurs.
- The kth *moment* of the stream is the sum of $(m_i)^k$ over all i.

Special Cases

- 0th moment = number of different elements in the stream.
  - The problem just considered.
- 1st moment = sum of counts of the numbers of elements = length of the stream.
  - Easy to compute.
- 2nd moment = *surprise number* = a measure of how uneven the distribution is.

Example: Surprise Number

- Stream of length 100; 11 values appear.
- Unsurprising: 10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9.
  Surprise # = $10^2 + 10*9^2 = 910$.
- Surprising: 90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1.
  Surprise # = $90^2 + 10*1^2 = 8{,}110$.

AMS Method

- Works for all moments; gives an unbiased estimate.
- We’ll talk about only the 2nd moment.
- Based on calculation of many random variables *X*.
  - Each requires a count in main memory, so number is limited.

One Random Variable

- Assume stream has length *n*.
- Pick a random time to start, so that any time is equally likely.
- Let the chosen time have element *a* in the stream.
- $X = n$ * ((twice the number of *a*'s in the stream starting at the chosen time) – 1).
  - Note: store *n* once, store count of *a*'s for each *X*.

Expected Value of X

- 2nd moment is $\Sigma_a(m_a)^2$.
- $E(X) = (1/n)\big(\Sigma_{\text{all times } t}\, n * $ (twice the number of times the stream element at time t appears from that time on) $- 1\big)$.
- $= \Sigma_a\, (1/n)(n)(1+3+5+\ldots+2m_a-1)$.
- $= \Sigma_a\, (m_a)^2$.

Group times by the value seen

Time when the last a is seen

Time when penultimate a is seen

Time when the first a is seen

Problem: Streams Never End

- We assumed there was a number *n*, the number of positions in the stream.
- But real streams go on forever, so *n* changes; it is the number of inputs seen so far.

Fixups

1. The variables *X* have *n* as a factor – keep *n* separately; just hold the count in *X*.
2. Suppose we can only store *k* counts. We cannot have one random variable X for each start-time, and must throw out some start-times as we read the stream.
    - Objective: each starting time *t* is selected with probability *k*/*n*.

Solution to (2)

- Choose the first *k* times for *k* variables.
- When the *n*th element arrives (*n* > *k*), choose it with probability *k*/*n*.
- If you choose it, throw one of the previously stored variables out, with equal probability.
- Probability of each of the first n-1 positions being chosen:

$$\underset{\substack{\uparrow \\ \text{n-th position} \\ \text{not chosen}}}{(n-k)/n} * \underset{\substack{\uparrow \\ \text{Previously} \\ \text{chosen}}}{k/(n-1)} + \underset{\substack{\uparrow \\ \text{n-th position} \\ \text{chosen}}}{k/n} * \underset{\substack{\uparrow \\ \text{Previously} \\ \text{chosen}}}{k/(n-1)} * \underset{\substack{\uparrow \\ \text{Survives}}}{(k-1)/k} = k/n$$

Final Remarks

- Thus, each variable has the second moment as its expected value.
- Use many (e.g., 100) such variables.
- Combine them as for Flajolet-Martin: average of groups of size O(log n), and then take the median of the averages.