Mining Data Streams

The Stream Model
Sliding Windows
Counting 1's
Exponentially Decaying Windows

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Data Management Vs. Stream Management

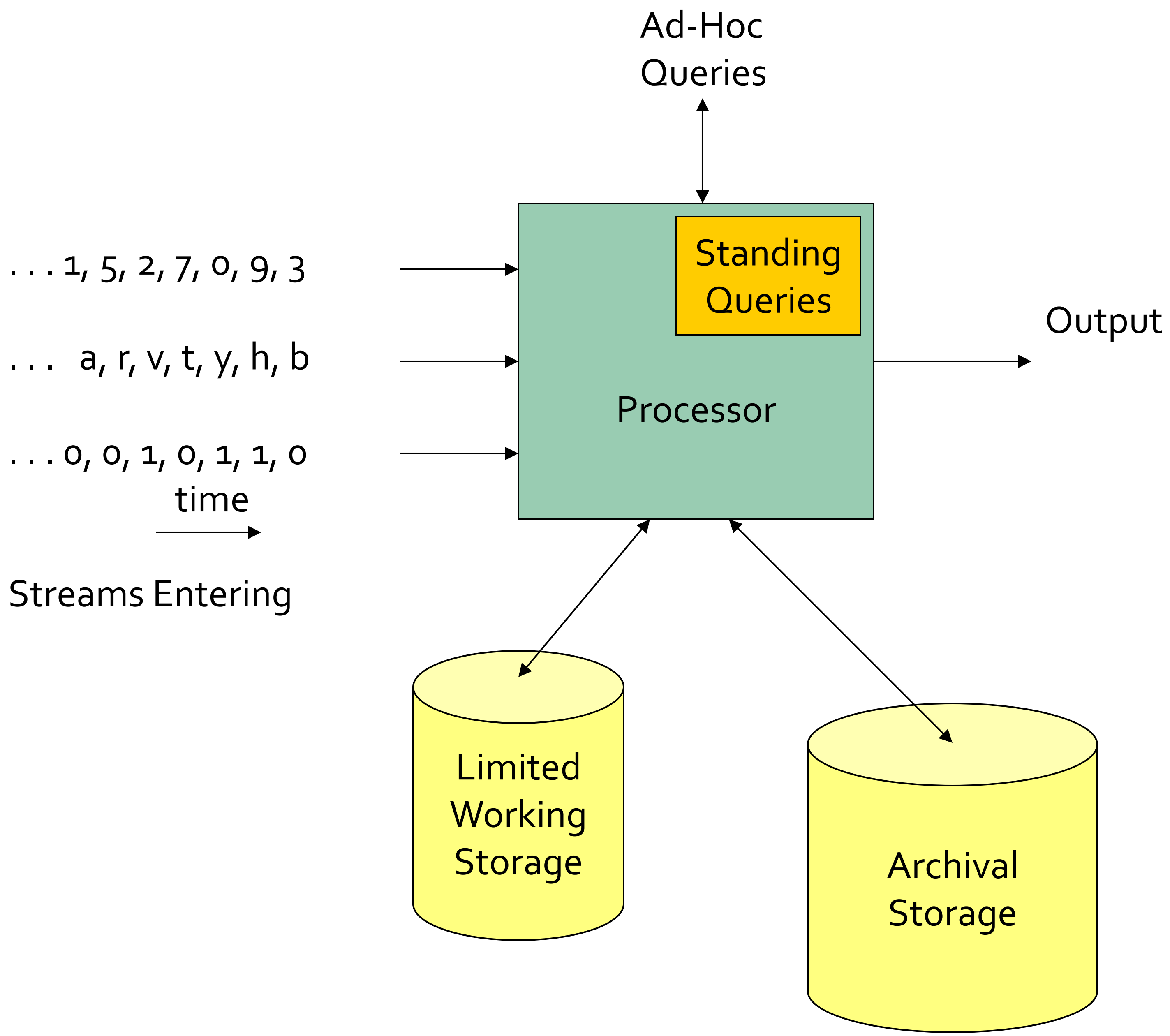
- In a DBMS, input is under the control of the owner.
  - SQL INSERT commands or bulk loaders.
- Stream management is important when the input rate is controlled externally.
  - Example: Google search queries.
  - Example: Amazon loves to get orders for products, but does not control when they come in.
  - Example: Satellites send down images, often petabytes/day.
    - More predictable, but too much to deal with efficiently.

The Stream Model

- Input tuples enter at a rapid rate, at one or more input ports.
- The system cannot store the entire stream accessibly.
- How do you make critical calculations about the stream using a limited amount of (primary or even secondary) memory?

Two Forms of Query

1. *Ad-hoc queries*: Normal queries asked one time about streams.
   - Example: What is the maximum value seen so far in stream *S*?
2. *Standing queries*: Queries that are, in principle, asked about the stream at all times.
   - Example: Report each new maximum value ever seen in stream *S*.

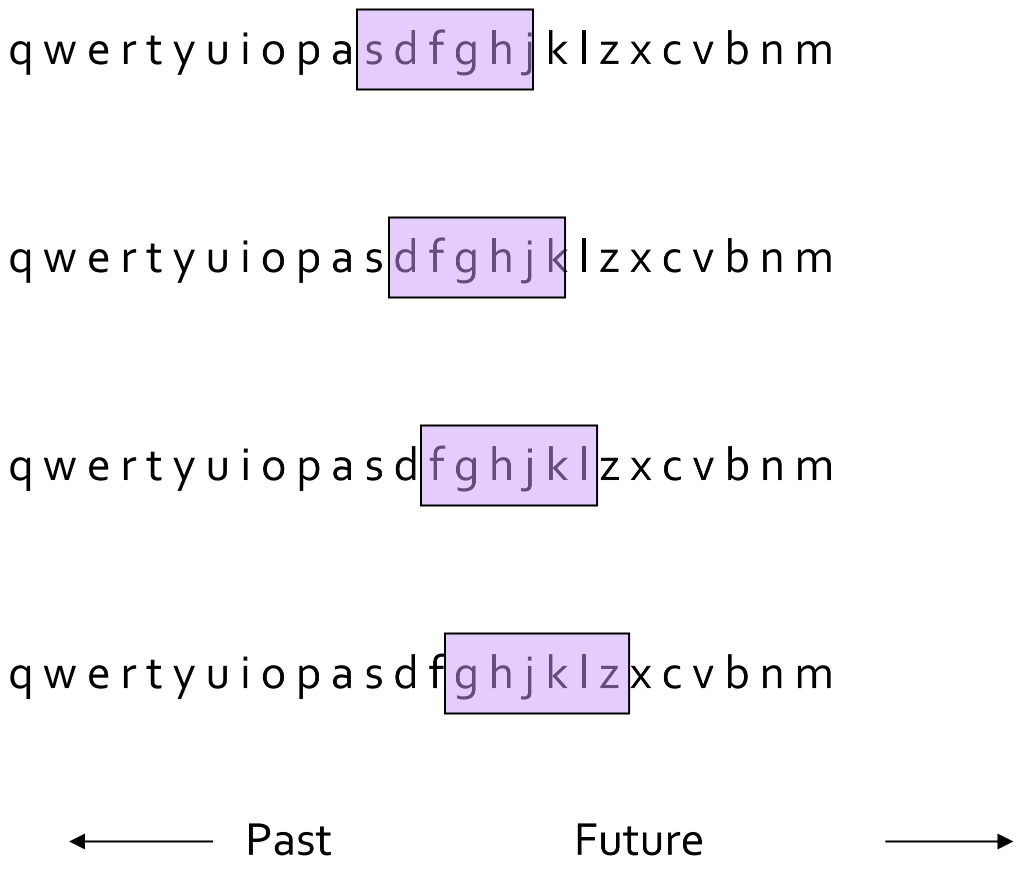

Ad-Hoc Queries
Standing Queries
Processor
Output
. . . 1, 5, 2, 7, 0, 9, 3
. . . a, r, v, t, y, h, b
. . . 0, 0, 1, 0, 1, 1, 0
time
Streams Entering
Limited Working Storage
Archival Storage

Applications

- Mining query streams.
  - Google wants to know what queries are more frequent today than yesterday.
- Mining click streams.
  - Yahoo! wants to know which of its pages are getting an unusual number of hits in the past hour.
    - Often caused by annoyed users clicking on a broken page.
- IP packets can be monitored at a switch.
  - Gather information for optimal routing.
  - Detect denial-of-service attacks.

Sliding Windows

- A useful model of stream processing is that queries are about a *window* of length *N* – the *N* most recent elements received.
- Interesting case: *N* is so large it cannot be stored in main memory.
  - Or, there are so many streams that windows for all do not fit in main memory.


q w e r t y u i o p a s d f g h j k l z x c v b n m
q w e r t y u i o p a s d f g h j k l z x c v b n m
q w e r t y u i o p a s d f g h j k l z x c v b n m
q w e r t y u i o p a s d f g h j k l z x c v b n m
Past
Future

Example: Averages

- Stream of integers, window of size *N*.
- Standing query: what is the average of the integers in the window?
- For the first *N* inputs, sum and count to get the average.
- Afterward, when a new input *i* arrives, change the average by adding (*i* - *j*)/*N*, where *j* is the oldest integer in the window before *i* arrived.
- Good: O(1) time per input.
- Bad: Requires the entire window in main memory.

Counting 1's

Approximating Counts
Exponentially Growing Blocks
DGIM Algorithm

Approximate Counting

- You can show that if you insist on an exact sum or count of the elements in a window, you cannot use less space than the window itself.
- But if you are willing to accept an approximation, you can use much less space.
- We’ll consider first the simple case of counting bits.

Counting Bits

- Problem: given a stream of 0's and 1's, be prepared to answer queries of the form "how many 1's in the most recent *k* bits?" where $k \leq N$.
- Obvious solution: store the most recent *N* bits.
- But answering the query will take O(*k*) time.
  - Very possibly too much time.
- And the space requirements can be too great.
  - Especially if there are many streams to be managed in main memory at once, or *N* is huge.

Example: Bit Counting

- Count recent hits on URL’s belonging to a site.
- Stream is a sequence of URL’s.
- Window size N = 1 billion.
- Think of the data as many streams – one for each URL.
- Bit on the stream for URL x is 0 unless the actual stream has x.

DGIM Method

- Name refers to the inventors:
  - Datar, Gionis, Indyk, and Motwani.
- Store only $O(\log^2 N)$ bits per stream.
  - N = window size.
- Gives approximate answer, never off by more than 50%.
  - Error factor can be reduced to any $\varepsilon > 0$, with more complicated algorithm and proportionally more stored bits, but the same $O(\log^2 N)$ space reqirement.

Timestamps

- Each bit in the stream has a *timestamp*, starting 0, 1, …
- Record timestamps modulo N (the window size), so we can represent any *relevant* timestamp in $O(\log_2 N)$ bits.

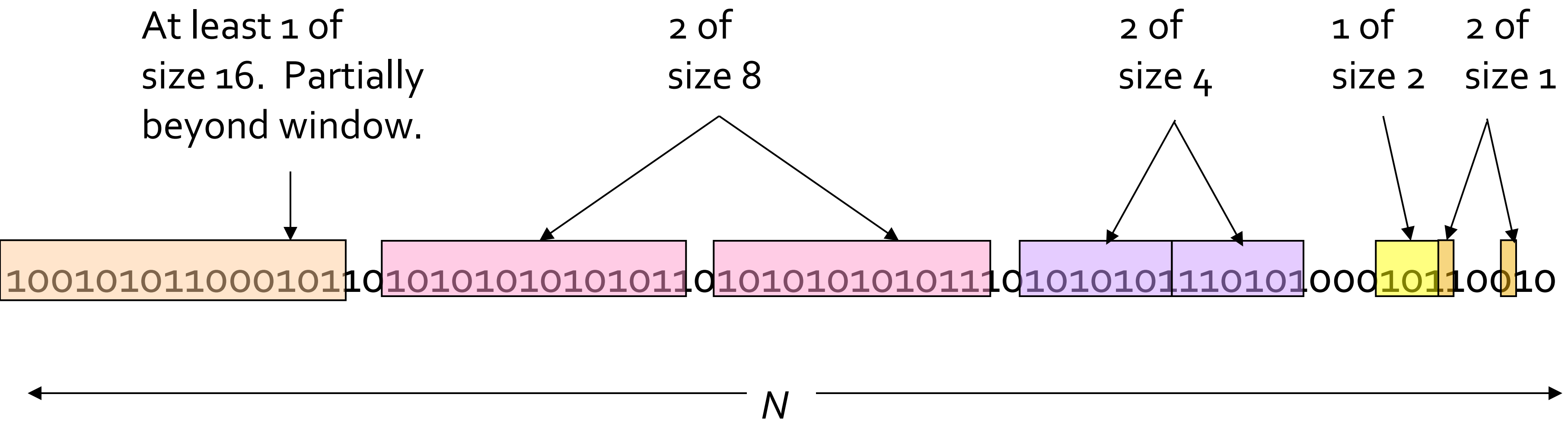
Buckets

- A *bucket* is a segment of the window ending with a 1; it is represented by a record consisting of:
  1. The timestamp of its end [O(log *N*) bits].
  2. The number of 1's between its beginning and end.
     - Number of 1's = *size* of the bucket.
- Constraint on bucket sizes: number of 1's must be a power of 2.
  - Thus, only O(log log *N*) bits are required for this count.

Representing a Stream by Buckets

- Either one or two buckets with the same power-of-2 number of 1's.
- Buckets do not overlap.
- Every 1 is in one bucket; 0's may or may not be in a bucket.
- Buckets are sorted by size.
  - Older buckets are not smaller than newer buckets.
- Buckets disappear when their end-time is > *N* time units in the past.

Example: Bucketized Stream



Updating the Set of Buckets

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to N time units before the current time.
- If the current bit is 0, no other changes are needed.

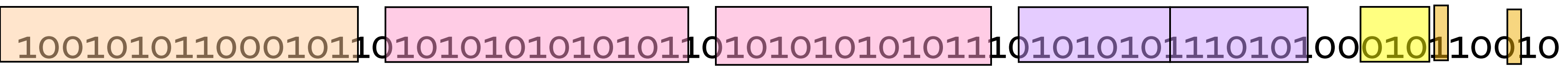
Updating Buckets: Input = 1

- If the current bit is 1:

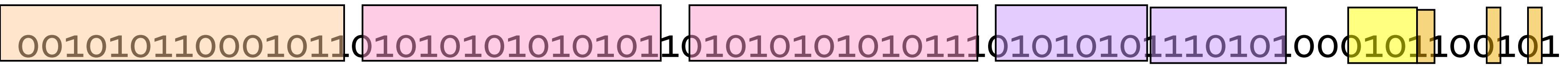
1. Create a new bucket of size 1, for just this bit.
   - End timestamp = current time.
2. If there are now three buckets of size 1, combine the oldest two into a bucket of size 2.
3. If there are now three buckets of size 2, combine the oldest two into a bucket of size 4.
4. And so on …

Example: Managing Buckets

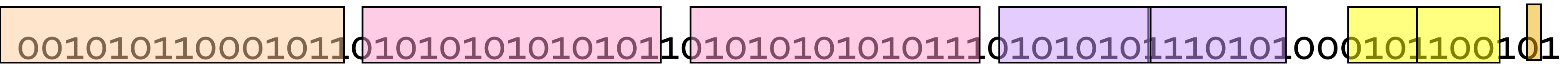
Initial



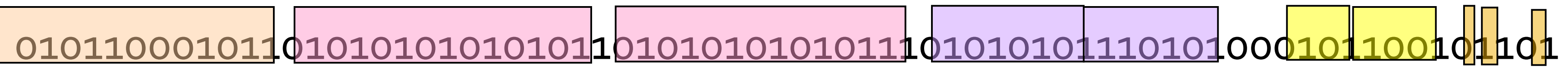
1 arrives; makes third block of size 1.



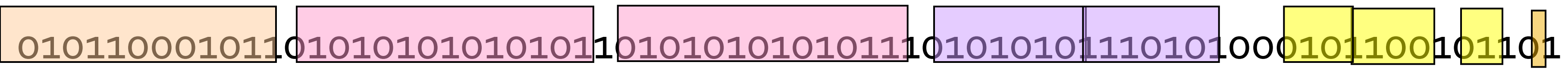
Combine the oldest two 1's into a 2.



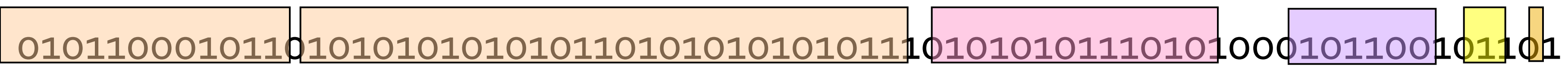
Later, 1, 0, 1 arrive. Now we have 3 1's again.



Combine the oldest two 1's into a 2.



The effect ripples all the way to a 16.



Querying

- To estimate the number of 1's in the most recent $k \leq N$ bits:
  1. Restrict your attention to only those buckets whose end time stamp is at most k bits in the past.
  2. Sum the sizes of all these buckets but the oldest.
  3. Add half the size of the oldest bucket.
- Remember: we don't know how many 1's of the last bucket are still within the window.

Error Bound

- Suppose the oldest bucket within range has size 2^i.
- Then by assuming 2^{i-1} of its 1's are still within the window, we make an error of at most 2^{i-1}.
- Since there is at least one bucket of each of the sizes less than 2^i, and at least 1 from the oldest bucket, the true sum is no less than 2^i.
- Thus, error at most 50%.
- Question for thought: Is the assumption 2^{i-1} 1's are still within the window the way to minimize maximum error?

Space Requirements

- We can represent one bucket in O(log N) bits.
  - It’s just a timestamp needing log N bits and a size, needing log log N bits.
- No bucket can be of size greater than N.
- There are at most two buckets of each size 1, 2, 4, 8,…
- That’s at most log N different sizes, and at most 2 of each size, so at most 2log N buckets.

Question for Thought

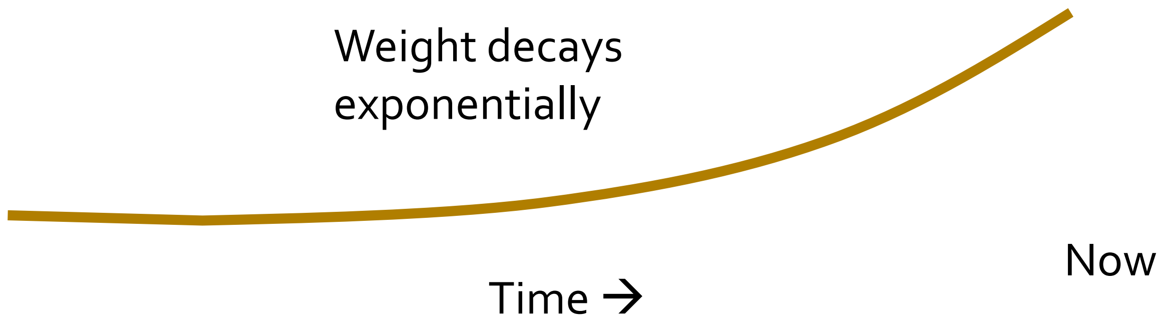
- Suppose the input stream is integers in the range 0 to M, and our goal is to estimate the sum of the last k elements in the window of size N.
- How would you take advantage of the DGIM idea to save space?

Exponentially Decaying Windows

Efficient Maintenance of E.D.W.'s
Application to Frequent Itemsets

Exponenially Decaying Windows

- Viewpoint: what is important in a stream is not just a finite window of most recent elements.
  - But all elements are not equally important; “old” elements less important than recent ones.
- Pick a constant c << 1 and let the “weight” of the i-th most recent element to arrive be proportional to $(1-c)^i$.



Numerical Streams

- Common case: elements are numerical, with a_i arriving at time i.
- The stream has a value at time t: $\Sigma_{i \leq t}\, a_i(1-c)^{t-i}$.
- Example: are we in a rainy period?
  - a_i = 1 if it rained on day i; 0 if not.
  - c = 0.1.
- If it rains every day, the value of the sum is $1+.9+(.9)^2+\ldots = 1/c = 10$.
- Value will be higher if the recent days have been rainy than if it rained long ago.

Maintaining the Stream Value

- Exponentially decaying windows make it easy to maintain this sum.
- When a new element x arrives:
  1. Multiply the previous value by 1-c.
     - Has the effect of devaluing every previous element by factor (1-c).
  2. Add x.

Maintaining Frequent Itemsets

- Imagine many streams, each Boolean, each representing the occurrence of one element.
- Example: sales of items.
  - One stream for each item.
    - Stream has a 1 when an instance of that item is sold.
- Special assumption: streams arrive synchronously, so at any time instance, we can view the set of items whose streams have 1 as a "basket."

Maintaining Frequent Itemsets

- Frequency of an item can be represented by the "value" of its stream in the decaying-window sense.
- Frequency of an itemset can be measured similarly by imagining there is a stream that has 1 if and only if all its items appear at a given time.
- But there are too many itemsets to maintain a value for every possible itemset.

A-Priori-Like Approach

- Take the support threshold s to be 1/2.
  - I.e., count a set only when the value of its stream is at least 1/2.
- Start by counting only the singleton items that are above threshold.
- Then, start counting a set when it occurs at time t, provided all of its immediate subsets were already being counted (before time t).
- Question for thought: Why not choose s = 1? Or s = 2?

Processing at Time t

1. Suppose set of items S are all the items sold at time t.
2. Multiply the value for each itemset being counted by (1-c).
3. Add 1 to the values for every set $T \subseteq S$, such that either:
   - T is a singleton, or
   - Every immediate subset of T was being counted at time t-1.
4. Drop any values < 1/2.