New Topic: Infinite Data

- **High dim. data**
  - Locality sensitive hashing
  - Clustering
  - Dimensionality reduction

- **Graph data**
  - PageRank, SimRank
  - Community Detection
  - Spam Detection

- **Infinite data**
  - Filtering data streams
  - Queries on streams
  - Web advertising

- **Machine learning**
  - SVM
  - Decision Trees
  - Perceptron, kNN

- **Apps**
  - Recommender systems
  - Association Rules
  - Duplicate document detection
Data Streams

- In many data mining situations, we do not know the entire data set in advance.

- **Stream Management** is important when the input rate is controlled *externally*:
  - Google queries
  - Twitter or Facebook status updates

- We can think of the data as **infinite** and **non-stationary** (the distribution changes over time).
The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - We call elements of the stream **tuples**

- The system cannot store the entire stream accessibly

- **Q:** How do you make critical calculations about the stream using a limited amount of (secondary) memory?
Stochastic Gradient Descent (SGD) is an example of a stream algorithm. In Machine Learning we call this: Online Learning.

- Allows for modeling problems where we have a continuous stream of data.
- We want an algorithm to learn from it and slowly adapt to the changes in data.

Idea: Do slow updates to the model.

- SGD (SVM, Perceptron) makes small updates.
- So: First train the classifier on training data.
- Then: For every example from the stream, we slightly update the model (using small learning rate).
Streams Entering. Each is stream is composed of elements/tuples.
Types of queries one wants on a data stream: (we’ll do these today)

- Sampling data from a stream
  - Construct a random sample
- Queries over sliding windows
  - Number of items of type \( x \) in the last \( k \) elements of the stream
Problems on Data Streams

- Types of queries one wants on answer on a data stream: (we’ll do these on Thu)
  - Filtering a data stream
    - Select elements with property $x$ from the stream
  - Counting distinct elements
    - Number of distinct elements in the last $k$ elements of the stream
  - Estimating moments
    - Estimate avg./std. dev. of last $k$ elements
  - Finding frequent elements
Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday

- **Mining click streams**
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour

- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook
Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller

- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies

- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks
Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger
Sampling from a Data Stream

- Since we can not store the entire stream, one obvious approach is to store a sample.

- Two different problems:
  - (1) Sample a fixed proportion of elements in the stream (say 1 in 10)
  - (2) Maintain a random sample of fixed size over a potentially infinite stream
    - At any “time” $k$ we would like a random sample of $s$ elements
    - What is the property of the sample we want to maintain? For all time steps $k$, each of $k$ elements seen so far has equal prob. of being sampled.
Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as:** How often did a user run the same query in a single days
  - Have space to store \(1/10\)th of query stream
- **Naïve solution:**
  - Generate a random integer in \([0..9]\) for each query
  - Store the query if the integer is 0, otherwise discard
Simple question: What fraction of queries by an average search engine user are duplicates?

Suppose each user issues $x$ queries once and $d$ queries twice (total of $x+2d$ queries)

Correct answer: $d/(x+d)$

Proposed solution: We keep 10% of the queries

Sample will contain $x/10$ of the singleton queries and $2d/10$ of the duplicate queries at least once

But only $d/100$ pairs of duplicates

$d/100 = 1/10 \cdot 1/10 \cdot d$

Of $d$ “duplicates” $18d/100$ appear exactly once

$18d/100 = ((1/10 \cdot 9/10)+(9/10 \cdot 1/10)) \cdot d$

So the sample-based answer is

$$\frac{d}{100} \frac{x}{10} + \frac{d}{100} + \frac{18d}{100} = \frac{d}{10x+19d}$$
Solution:

- Pick \( \frac{1}{10} \)th of users and take all their searches in the sample
- Use a hash function that hashes the user name or user id uniformly into 10 buckets
Stream of tuples with keys:
- Key is some subset of each tuple’s components
  - e.g., tuple is (user, search, time); key is user
- Choice of key depends on application

To get a sample of $a/b$ fraction of the stream:
- Hash each tuple’s key uniformly into $b$ buckets
- Pick the tuple if its hash value is at most $a$

Hash table with $b$ buckets, pick the tuple if its hash value is at most $a$.

How to generate a 30% sample?
Hash into $b=10$ buckets, take the tuple if it hashes to one of the first 3 buckets
Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size
Problem 2: Fixed-size sample
Suppose we need to maintain a random sample $S$ of size exactly $s$ tuples
- E.g., main memory size constraint
Why? Don’t know length of stream in advance
Suppose at time $n$ we have seen $n$ items
- Each item is in the sample $S$ with equal prob. $s/n$

How to think about the problem: say $s = 2$
Stream: $\text{a x c y z j k o d e g…}$
At $n=5$, each of the first 5 tuples is included in the sample $S$ with equal prob.
At $n=7$, each of the first 7 tuples is included in the sample $S$ with equal prob.

Impractical solution would be to store all the $n$ tuples seen so far and out of them pick $s$ at random
Algorithm (a.k.a. Reservoir Sampling)

- Store all the first $s$ elements of the stream to $S$
- Suppose we have seen $n-1$ elements, and now the $n^{th}$ element arrives ($n > s$)
  - With probability $s/n$, keep the $n^{th}$ element, else discard it
  - If we picked the $n^{th}$ element, then it replaces one of the $s$ elements in the sample $S$, picked uniformly at random

Claim: This algorithm maintains a sample $S$ with the desired property:
- After $n$ elements, the sample contains each element seen so far with probability $s/n$
Proof: By Induction

- **We prove this by induction:**
  - Assume that after \( n \) elements, the sample contains each element seen so far with probability \( s/n \)
  - We need to show that after seeing element \( n+1 \) the sample maintains the property
    - Sample contains each element seen so far with probability \( s/(n+1) \)

- **Base case:**
  - After we see \( n=s \) elements the sample \( S \) has the desired property
    - Each out of \( n=s \) elements is in the sample with probability \( s/s = 1 \)
Proof: By Induction

- **Inductive hypothesis:** After \( n \) elements, the sample \( S \) contains each element seen so far with prob. \( s/n \)
- **Now element \( n+1 \) arrives**
- **Inductive step:** For elements already in \( S \), probability that the algorithm keeps it in \( S \) is:

\[
\left(1 - \frac{s}{n+1}\right) + \left(\frac{s}{n+1}\right)\left(\frac{s-1}{s}\right) = \frac{n}{n+1}
\]

- Element \( n+1 \) discarded
- Element \( n+1 \) not discarded
- Element in the sample not picked
- So, at time \( n \), tuples in \( S \) were there with prob. \( s/n \)
- Time \( n \rightarrow n+1 \), tuple stayed in \( S \) with prob. \( n/(n+1) \)
- So prob. tuple is in \( S \) at time \( n+1 \) = \( \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1} \)
Queries over a (long) Sliding Window
A useful model of stream processing is that queries are about a *window* of length $N$ – the $N$ most recent elements received

**Interesting case:** $N$ is so large that the data cannot be stored in memory, or even on disk

- Or, there are so many streams that windows for all cannot be stored

**Amazon example:**
- For every product $X$ we keep 0/1 stream of whether that product was sold in the $n$-th transaction
- We want answer queries, how many times have we sold $X$ in the last $k$ sales
Sliding Window: 1 Stream

- **Sliding window on a single stream:**

  \[
  \text{q w e r t y u i o p a s d f g h j k l z x c v b n m}
  \]

  \[
  \text{q w e r t y u i o p a s d f g h j k l z x c v b n m}
  \]

  \[
  \text{q w e r t y u i o p a s d f g h j k l z x c v b n m}
  \]

  \[
  \text{q w e r t y u i o p a s d f g h j k l z x c v b n m}
  \]

  Past \quad \text{Future}

  \[N = 6\]
Problem:
- Given a stream of 0s and 1s
- Be prepared to answer queries of the form How many 1s are in the last \( k \) bits? where \( k \leq N \)

Obvious solution:
- Store the most recent \( N \) bits
- When new bit comes in, discard the \( N+1^{st} \) bit

Suppose \( N=6 \)

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0

Past  Future
Counting Bits (2)

- You can not get an exact answer without storing the entire window

- Real Problem:
  What if we cannot afford to store $N$ bits?
  - E.g., we’re processing 1 billion streams and $N = 1$ billion

- But we are happy with an approximate answer
An attempt: Simple solution

- **Q**: How many 1s are in the last $N$ bits?
- A simple solution that does not really solve our problem: **Uniformity assumption**

Maintain 2 counters:
- $S$: number of 1s from the beginning of the stream
- $Z$: number of 0s from the beginning of the stream

How many 1s are in the last $N$ bits? $N \cdot \frac{S}{S+Z}$

But, what if stream is non-uniform?
- What if distribution changes over time?
DGIM Method

- DGIM solution that does **not** assume uniformity
- We store $O(\log^2 N)$ bits per stream
- Solution gives approximate answer, never off by more than 50%
  - Error factor can be reduced to any fraction $> 0$, with more complicated algorithm and proportionally more stored bits

[Datar, Gionis, Indyk, Motwani]
Idea: Exponential Windows

- Solution that doesn’t (quite) work:
  - Summarize \textit{exponentially increasing} regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region

We can reconstruct the count of the last $N$ bits, except we are not sure how many of the last 6 $1$s are included in the $N$
What’s Good?

- Stores only $O(\log^2 N)$ bits
  - $O(\log N)$ counts of $\log_2 N$ bits each

- Easy update as more bits enter

- Error in count no greater than the number of 1s in the “unknown” area
As long as the 1s are fairly evenly distributed, the error due to the unknown region is small—no more than 50%.

But it could be that all the 1s are in the unknown area at the end.

In that case, the error is unbounded!
**Fixup: DGIM method**

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of 1s:
  - Let the block *sizes* (number of 1s) increase exponentially

- When there are few 1s in the window, block sizes stay small, so errors are small
Each bit in the stream has a timestamp, starting 1, 2, ...

Record timestamps modulo \( N \) (the window size), so we can represent any relevant timestamp in \( O(\log_2 N) \) bits
A bucket in the DGIM method is a record consisting of:

- (A) The timestamp of its end \(O(\log N)\) bits
- (B) The number of 1s between its beginning and end \(O(\log \log N)\) bits

Constraint on buckets:

Number of 1s must be a power of 2

That explains the \(O(\log \log N)\) in (B) above
Either one or two buckets with the same power-of-2 number of 1s

- Buckets do not overlap in timestamps
- Buckets are sorted by size
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is $> N$ time units in the past
Example: Bucketized Stream

At least 1 of size 16. Partially beyond window.

2 of size 8

2 of size 4

1 of size 2

2 of size 1

Three properties of buckets that are maintained:
- Either one or two buckets with the same power-of-2 number of 1s
- Buckets do not overlap in timestamps
- Buckets are sorted by size
Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to $N$ time units before the current time.

- 2 cases: Current bit is 0 or 1

- If the current bit is 0:
  no other changes are needed
If the current bit is 1:

1. Create a new bucket of size 1, for just this bit
   - End timestamp = current time
2. If there are now three buckets of size 1, combine the oldest two into a bucket of size 2
3. If there are now three buckets of size 2, combine the oldest two into a bucket of size 4
4. And so on ...
Example: Updating Buckets

Current state of the stream:

```
10010101100010110101010101010110101010101011101010101110101000101100101
```

Bit of value 1 arrives

```
0010101100010110101010101010110101010101011101010101110101000101100101
```

Two orange buckets get merged into a yellow bucket

```
0010101100010110101010101010110101010101011101010101110101000101100101101
```

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

```
01011000101101010101010101110101010101110101000101100101101
```

Buckets get merged...

```
01011000101101010101010101110101010101110101000101100101101
```

State of the buckets after merging

```
01011000101101010101010101110101010101110101000101100101101
```
To estimate the number of 1s in the most recent $N$ bits:

1. Sum the sizes of all buckets but the last
   (note “size” means the number of 1s in the bucket)

2. Add half the size of the last bucket

Remember: We do not know how many 1s of the last bucket are still within the wanted window
Example: Bucketized Stream

At least 1 of size 16. Partially beyond window.

10010101110001011010101010101011010101010101110101010111010100010110010
Why is error 50%? Let’s prove it!

Suppose the last bucket has size $2^r$

Then by assuming $2^{r-1}$ (i.e., half) of its 1s are still within the window, we make an error of at most $2^{r-1}$

Since there is at least one bucket of each of the sizes less than $2^r$, the true sum is at least $1 + 2 + 4 + \ldots + 2^{r-1} = 2^r - 1$

Thus, error at most 50%
Instead of maintaining 1 or 2 of each size bucket, we allow either \( r-1 \) or \( r \) buckets \( (r > 2) \)

- Except for the largest size buckets; we can have any number between 1 and \( r \) of those

- Error is at most \( 1/r \)

- By picking \( r \) appropriately, we can tradeoff between number of bits we store and the error
Can we use the same trick to answer queries:

**How many 1’s in the last \( k \)?** where \( k < N \)?

- A: Find earliest bucket \( B \) that at overlaps with \( k \).
  Number of 1s is the *sum of sizes of more recent buckets* + \( \frac{1}{2} \) size of \( B \)

Can we handle the case where the stream is not bits, but integers, and we want the sum of the last \( k \) elements?
Stream of positive integers
We want the sum of the last \( k \) elements

- Amazon: Avg. price of last \( k \) sales

Solution:

1. If you know all integers have at most \( m \) bits
   - Treat \( m \) bits of each integer as a separate stream
   - Use DGIM to count 1s in each integer
   - The sum is \( \sum_{i=0}^{m-1} c_i 2^i \)

2. Use buckets to keep partial sums
   - Sum of elements in size \( b \) bucket is at most \( 2^b \)

<table>
<thead>
<tr>
<th>2 5 7 1 3 8 4 6 7 9 1 3 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 5 3 5 7 1 3 3 1 2 2 6</td>
</tr>
<tr>
<td>3 5 7 1 3 3 1 2 2 6 3</td>
</tr>
<tr>
<td>3 5 7 1 3 3 1 2 2 6 3</td>
</tr>
<tr>
<td>3 5 7 1 3 3 1 2 2 6 3</td>
</tr>
</tbody>
</table>

Idea: Sum in each bucket is at most \( 2^b \) (unless bucket has only 1 integer)

Bucket sizes:

16 8 4 2 1
Sampling a fixed proportion of a stream
- Sample size grows as the stream grows

Sampling a fixed-size sample
- Reservoir sampling

Counting the number of 1s in the last N elements
- Exponentially increasing windows
- Extensions:
  - Number of 1s in any last k (k < N) elements
  - Sums of integers in the last N elements