Advertising on the Web

CS246: Mining Massive Datasets
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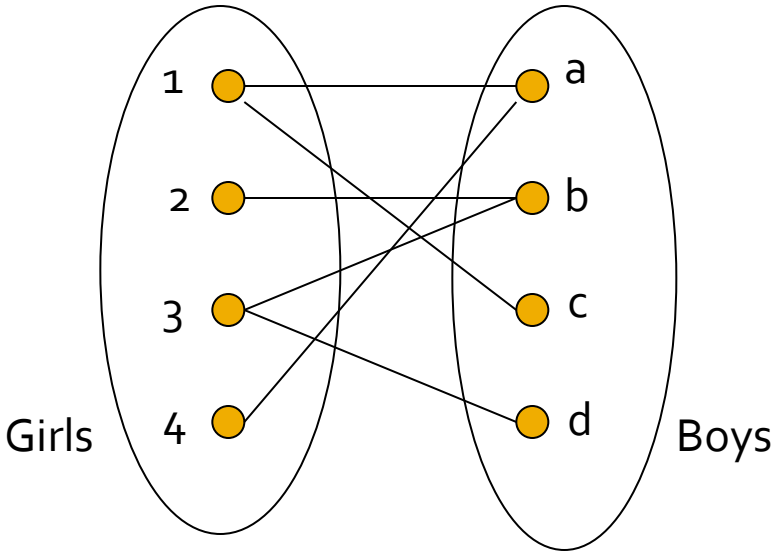
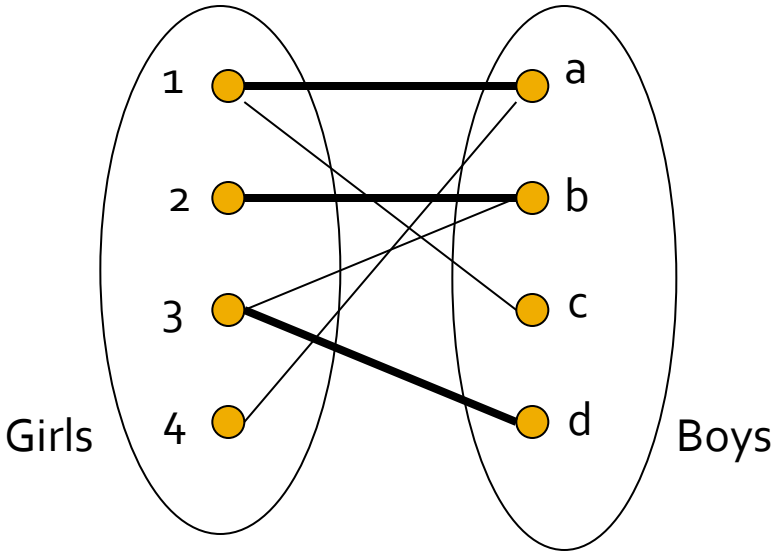
Online algorithms

- Classic model of algorithms
  - You get to see the entire input, then compute some function of it
  - In this context, "offline algorithm"
- Online algorithm
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
- Similar to data stream models

Example: Bipartite matching

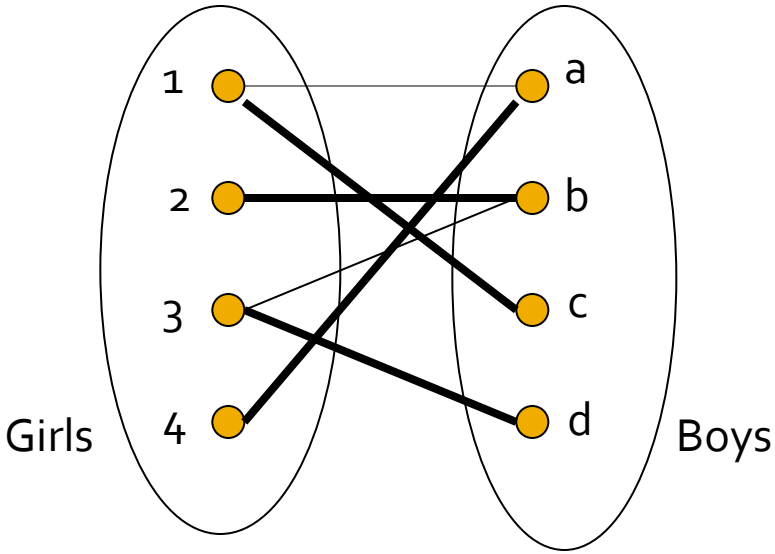


Example: Bipartite matching



M = {(1,a),(2,b),(3,d)} is a matching.
Cardinality of matching = |M| = 3

Example: Bipartite matching



M = {(1,c),(2,b),(3,d),(4,a)} is a perfect matching.

Matching Algorithm

- Problem: Find a maximum-cardinality matching for a given bipartite graph
  - A perfect one if it exists
- There is a polynomial-time offline algorithm (Hopcroft and Karp 1973)
- But what if we do not know the entire graph upfront?

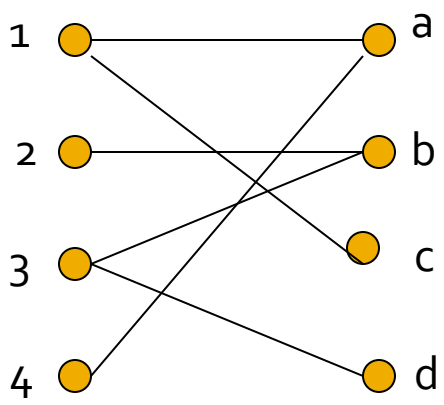
Online Graph Matching Problem

- Initially, we are given the set Boys
- In each round, one girl's choices are revealed
- At that time, we have to decide to either:
  - Pair the girl with a boy
  - Do not pair the girl with any boy

- Example of application:

  Assigning tasks to servers

Online Graph Matching: Example



(1,a)

(2,b)

(3,d)

Greedy Algorithm

- Greedy algorithm for online graph matching problem:
  - Pair the new girl with any eligible boy
    - If there is none, don't pair girl
  - How good is the algorithm?

Competitive Ratio

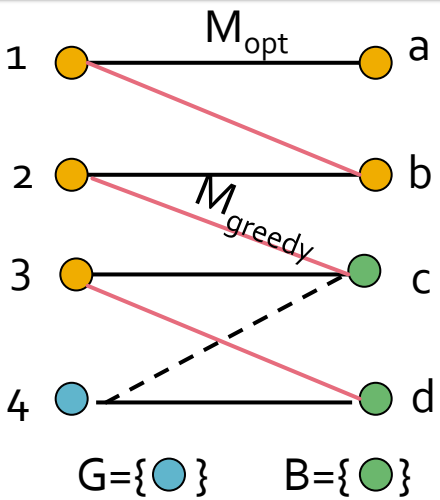
- For input I, suppose greedy produces matching M_{greedy} while an optimal matching is M_{opt}

Competitive ratio =

$$\min_{\text{all possible inputs } I} (|M_{greedy}|/|M_{opt}|)$$

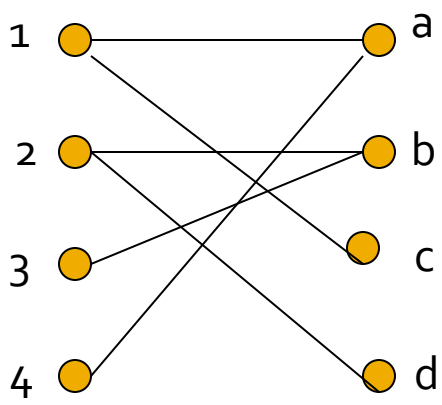
(what is greedy's worst performance over all possible inputs)

Analyzing the greedy algorithm



- Consider the set G of girls matched in M_{opt} but not in M_{greedy}
  - Then every boy B <u>adjacent</u> to girls in G is already matched in M_{greedy} : $|B| \leq |M_{greedy}|$
  - There are at least $|G|$ such boys ($|G| \leq |B|$) otherwise the optimal algorithm could, not have matched all the G girls. So: $|G| \leq |M_{greedy}|$
- By definition of G also: $|M_{opt}| \leq |M_{greedy}| + |G|$
- So $|M_{greedy}|/|M_{opt}| \geq 1/2$

Worst-case scenario



(1,a)

(2,b)

History of web advertising

- Banner ads (1995-2001)
  - Initial form of web advertising
  - Popular websites charged X$ for every 1000 "impressions" of ad
    - Called "CPM" rate
    - Modeled similar to TV, magazine ads
  - Untargeted to demographically targeted
  - Low clickthrough rates
    - low ROI for advertisers

Performance-based advertising

- Introduced by Overture around 2000
  - Advertisers "bid" on search keywords
  - When someone searches for that keyword, the highest bidder's ad is shown
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called "Adwords"

Ads vs. search results

Web Results **1** - **10** of about **2,230,000** for **geico**. (**0.04** secc

GEICO Car Insurance. Get an auto insurance quote and save today ...
GEICO auto insurance, online car insurance quote, motorcycle insurance quote, online insurance sales and service from a leading insurance company.
www.**geico**.com/ - 21k - Sep 22, 2005 - Cached - Similar pages
Auto Insurance - Buy Auto Insurance
Contact Us - Make a Payment
More results from www.geico.com »

Geico, Google Settle Trademark Dispute
The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.
www.clickz.com/news/article.php/3547356 - 44k - Cached - Similar pages

Google and **GEICO** settle AdWords dispute | The Register
Google and car insurance firm **GEICO** have settled a trade mark dispute over ... Car insurance firm **GEICO** sued both Google and Yahoo! subsidiary Overture in ...
www.theregister.co.uk/2005/09/09/google_**geico**_settlement/ - 21k - Cached - Similar pages

GEICO v. Google
... involving a lawsuit filed by Government Employees Insurance Company (**GEICO**). **GEICO** has filed suit against two major Internet search engine operators, ...
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Sponsored Links

Great Car Insurance Rates
Simplify Buying Insurance at Safeco
See Your Rate with an Instant Quote
www.Safeco.com

Free Insurance Quotes
Fill out one simple form to get multiple quotes from local agents.
www.HometownQuotes.com

5 Free Quotes. 1 Form.
Get 5 Free Quotes In Minutes!
You Have Nothing To Lose. It's Free
sayyessoftware.com/Insurance
Missouri

Web 2.0

- Performance-based advertising works!
  - Multi-billion-dollar industry
- Interesting problems:
  - What ads to show for a given query?
  - If I am an advertiser, which search terms should I bid on and how much should I bid?

Adwords Problem

- A stream of queries arrives at the search engine
  - q1, q2, …
- Several advertisers bid on each query
- When query q_i arrives, search engine must pick a subset of advertisers whose ads are shown

- Goal: maximize search engine's revenues

- Clearly we need an online algorithm!

Complications

- Each advertiser has a limited budget
  - Search engine guarantees that the advertiser will not be charged more than their daily budget
- Each ad has a different likelihood of being clicked
  - Advertiser 1 bids $2, click probability = 0.1
  - Advertiser 2 bids $1, click probability = 0.5
  - Clickthrough rate measured historically
  - Simple solution
    - Instead of raw bids, use the "expected revenue per click"

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Greedy algorithm

- The environment:
  - There is one ad shown for each query
  - All advertisers have the same budget
  - All adds are equally likely to be clicked
  - Value of each add is the same
- Simplest algorithm is greedy:
  - For a query pick any advertiser who has bid 1 for that query
- Competitive ratio of greedy is 1/2

Bad scenario for greedy

- Two advertisers A and B
  - A bids on query x, B bids on x and y
  - Both have budgets of $4
- Query stream: xxxxyyyy
  - Worst case greedy choice: BBBB____
  - Optimal: AAAABBBB
  - Competitive ratio = ½
- This is the worst case

BALANCE algorithm [MSVV]

- BALANCE by Mehta, Saberi, Vazirani, and Vazirani
  - For each query, pick the advertiser with the largest unspent budget
    - Break ties arbitrarily

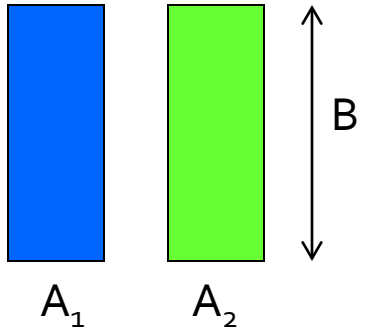
Example: BALANCE

- Two advertisers A and B
  - A bids on query x, B bids on x and y
  - Both have budgets of $4
- Query stream: xxxxyyyy
- BALANCE choice: ABABBB__
  - Optimal: AAAABBBB
- Competitive ratio = ¾

Analyzing BALANCE

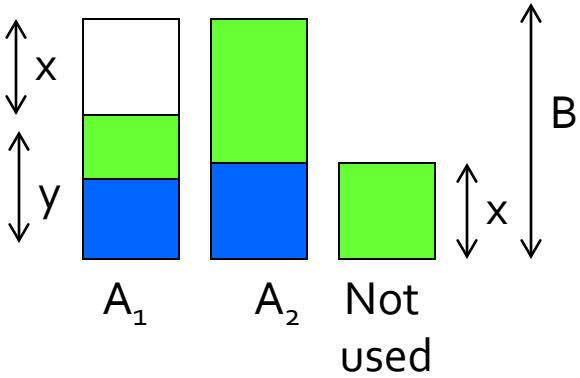
- Consider simple case:
  - Two advertisers, A_1 and A_2, each with budget B (assume $B \geq 1$)
- Assume optimal solution exhausts both advertisers' budgets
- BALANCE must exhaust at least one advertiser's budget
  - If not, we can allocate more queries
  - Assume BALANCE exhausts A_2's budget, but allocates x queries fewer than the optimal
  - BAL = 2B - x

Analyzing Balance



Queries allocated to A_1 in optimal solution

Queries allocated to A_2 in optimal solution



Opt revenue = 2B
Balance revenue = 2B-x = B+y

We have $y \geq x$
Balance revenue is minimum for x=y=B/2
Minimum Balance revenue = 3B/2
Competitive Ratio = 3/4

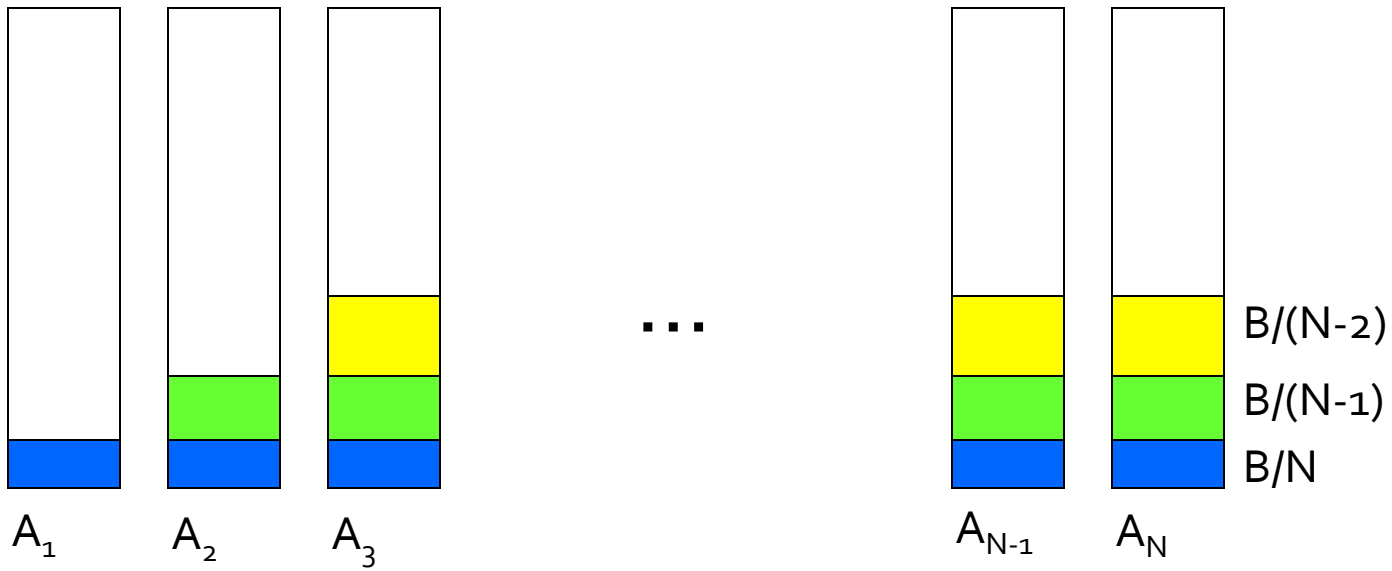
BALANCE: General Result

- In the general case, worst competitive ratio of BALANCE is $1-1/e$ = approx. 0.63
  - Interestingly, no online algorithm has a better competitive ratio!
- We do not through the details here, but let's see the worst case that gives this ratio

Worst case for BALANCE

- N advertisers: $A_1, A_2, \ldots A_N$
  - Each with budget $B > N$
- Queries:
  - $N \cdot B$ queries appear in N rounds of B queries each
- Bidding:
  - Round 1 queries: bidders $A_1, A_2, \ldots, A_N$
  - Round 2 queries: bidders $A_2, A_3, \ldots, A_N$
  - Round *i* queries: bidders $A_i, \ldots, A_N$
- Optimum allocation:
  Allocate round i queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE allocation



Balance assigns each of the queries in round 1 to N advertisers

After *k* rounds, sum of allocations to each of advertisers $A_k,\ldots,A_N$ is

$S_k = S_{k+1} = \ldots = S_N = \sum_{i=1\ldots k-1} B / (N-i+1)$

If we find the smallest k such that $S_k \geq B$, then after k rounds we cannot allocate any queries to any advertiser

BALANCE analysis

$B/1 \quad B/2 \quad B/3 \quad \ldots \quad B/(N-k+1) \quad \ldots \quad B/(N-1) \quad B/N$

S_1

S_2

$S_k = B$

$1/1 \quad 1/2 \quad 1/3 \quad \ldots \quad 1/(N-k+1) \quad \ldots \quad 1/(N-1) \quad 1/N$

S_1

S_2

$S_k = 1$

BALANCE analysis

- Fact:

  $H_n = \sum_{i=1..n} 1/i$ = approx. log(n) for large n

  - Result due to Euler

$$1/1 \quad 1/2 \quad 1/3 \quad \ldots \quad 1/(N-k+1) \quad \ldots \quad 1/(N-1) \quad 1/N$$

$\longleftrightarrow$ log(N)

$\longleftrightarrow$ log(N)-1 $\qquad$ $\longleftrightarrow$ $S_k = 1$

$S_k = 1$ implies $H_{N-k} = \log(N)-1 = \log(N/e)$

$N-k = N/e$

$k = N(1-1/e)$

BALANCE analysis

- So after the first N(1-1/e) rounds, we cannot allocate a query to any advertiser
- Revenue = B·N (1-1/e)
- Competitive ratio = 1-1/e

General version of problem

- Arbitrary bids, budgets
- Consider query q, advertiser i
  - Bid = x_i
  - Budget = b_i
- BALANCE can be terrible
  - Consider two advertisers A_1 and A_2
  - A_1: $x_1 = 1$, $b_1 = 110$
  - A_2: $x_2 = 10$, $b_2 = 100$

Generalized BALANCE

- Arbitrary bids; consider query q, bidder i
  - Bid = x_i
  - Budget = b_i
  - Amount spent so far = m_i
  - Fraction of budget left over $f_i = 1-m_i/b_i$
  - Define $\psi_i(q) = x_i(1-e^{-f_i})$
- Allocate query q to bidder i with largest value of $\psi_i(q)$
- Same competitive ratio (1-1/e)