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Stanford CS224W: Geometric Deep Learning

CS224W: Machine Learning with Graphs
Jure Leskovec and Charilaos Kanatsoulis, Stanford
University
<http://cs224w.stanford.edu>



Announcements

- **Exam** opens this Today – BEST OF LUCK
 - 11/21 5pm to 11/23 5am (36 hour window)
 - 2 hours long (can't stop + start) - on gradescope
 - Up to Lecture 13
 - Recitation: recording on Ed
 - More info on Website + Ed
 - Question from last time: yes, we require writing code in the exam!
 - Use private Ed post for clarification questions.
- **Colab 4 + 5** due after the break (12/3 and 12/5)

Announcements

- **We need your Medium Account Usernames**
 - We will add all of you as writers to our CS224w publications.
 - Question from last time: We need a medium account for each team (not student)
 - **Please fill out the google form on Ed with your medium account username by Wednesday EOD.**
 - <https://forms.gle/UtJ3x9dGpNTGCQz1A>
 - This is a requirement!

Outline

- Geometric Graphs
- Geometric Graph NNs
 - Invariant GNNs
 - Equivariant GNNs
- Geometric Generative Models (time-permitted)
 - Diffusion Models

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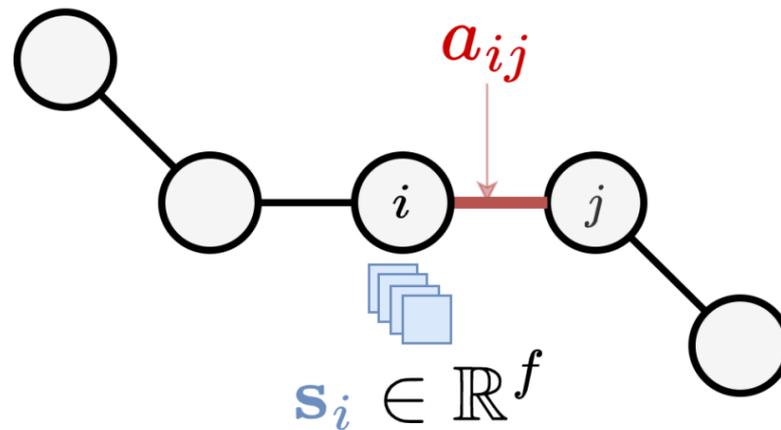
Stanford CS224W: Geometric Graphs

CS224W: Machine Learning with Graphs
Jure Leskovec, Stanford University
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Graphs

A graph $G = (A, S)$ is a set V of n nodes connected by **edges**. Each node has **scalar attributes**, e.g. atom type for molecules.



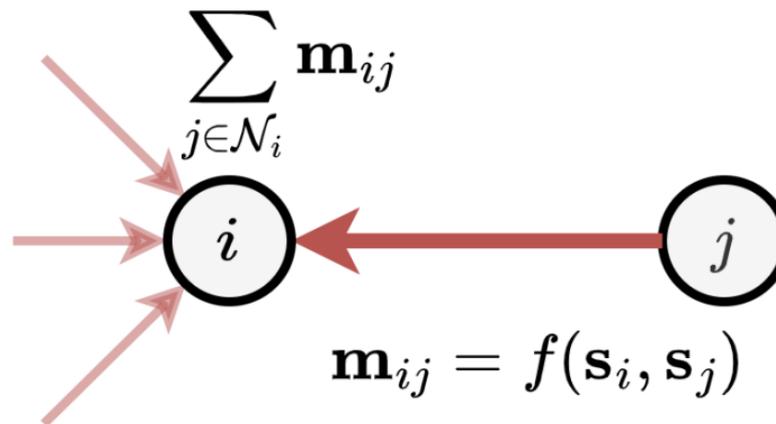
- A : an $n \times n$ adjacency matrix.
- $S \in \mathbb{R}^{n \times f}$: scalar features.

Hamilton, Will, Zhitao Ying, and Jure Leskovec. "Inductive representation learning on large graphs." *Advances in neural information processing systems* 30 (2017).
Joshi, Chaitanya K., et al. "On the expressive power of geometric graph neural networks."

Message Passing Neural Nets

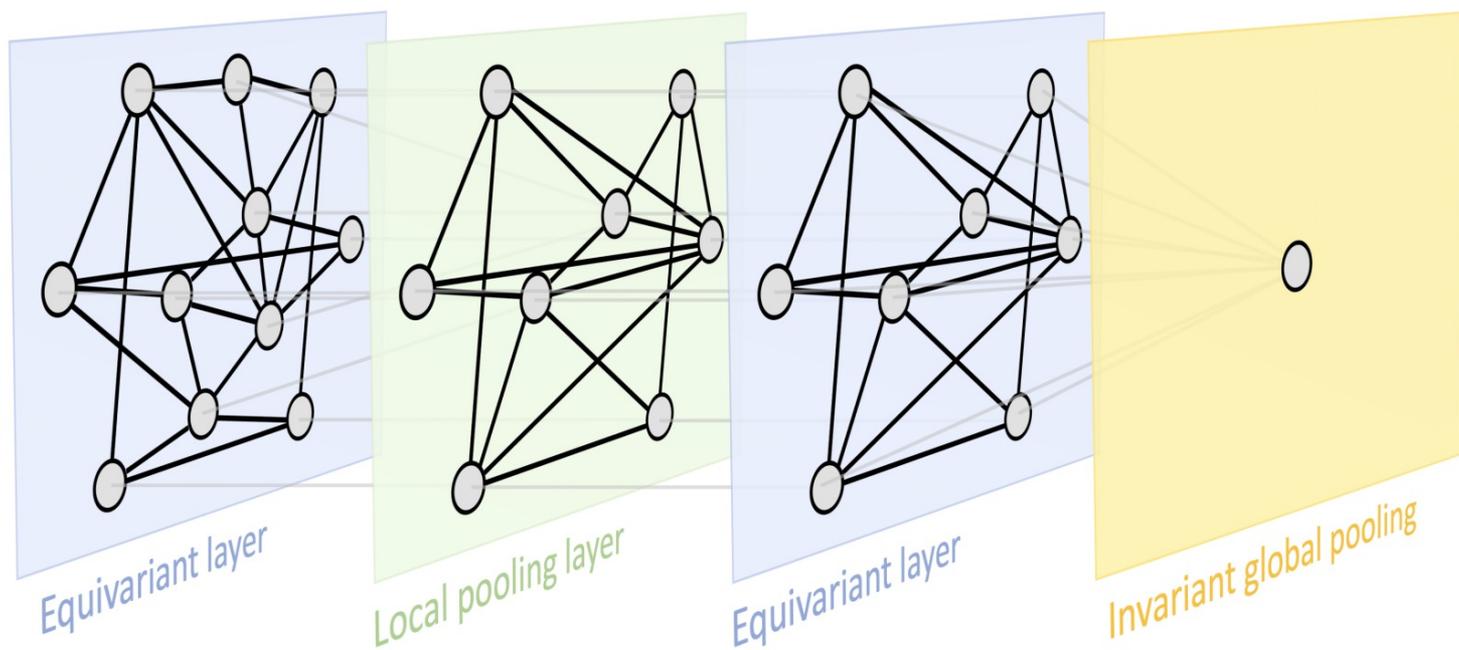
- Node features are updated from layer t to $t+1$ via learnable **permutation invariant** neighborhood **aggregate AGG** and **update UPD**:

$$\mathbf{m}_i^{(t)} = \text{AGG} \left(\left\{ \left(\mathbf{s}_i^{(t)}, \mathbf{s}_j^{(t)} \right) \mid j \in \mathcal{N}_i \right\} \right)$$
$$\mathbf{s}_i^{(t+1)} = \text{UPD} \left(\mathbf{s}_i^{(t)}, \mathbf{m}_i^{(t)} \right)$$



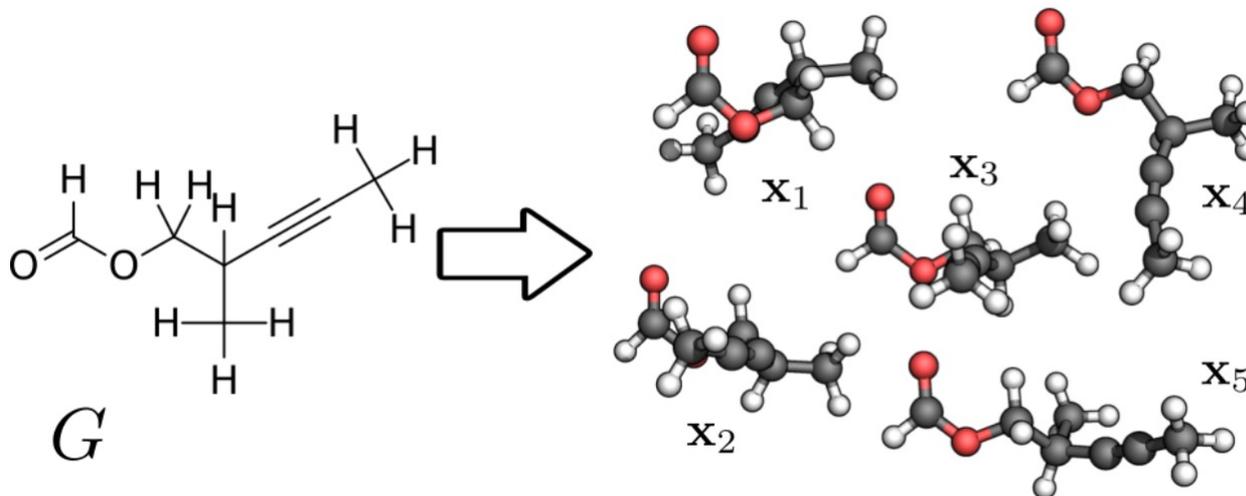
Graph-level tasks with GNNs

- GNNs make **pooling** over node features, which are then used to make a **graph-level** prediction.



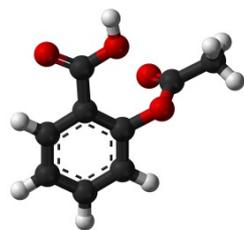
Molecular Graphs

- **Molecules** can be represented as a graph G with node features S_i and edge features a_{ij} .
 - Node features: atom type, atom charges...
 - Edge features: bond type, bond length...
 - However, sometimes, we also know the 3D positions x_i , which is actually more **informative**

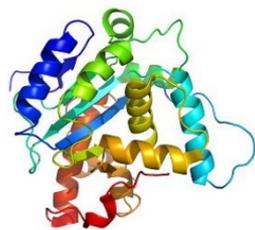


Simm, Gregor NC, and José Miguel Hernández-Lobato. "A generative model for molecular distance geometry." *ICML 2020*

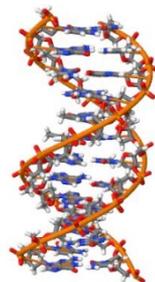
More Geometric Graphs



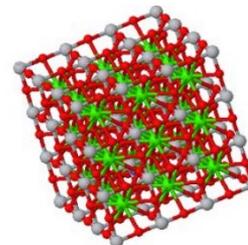
Small
Molecules



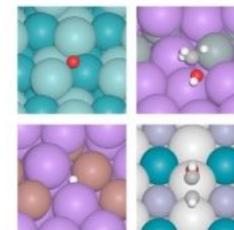
Proteins



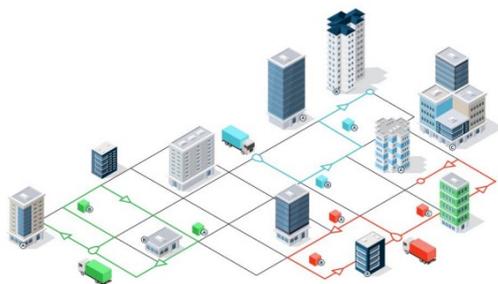
DNA/RNA



Inorganic
Crystals



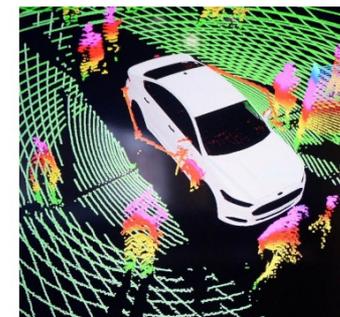
Catalysis
Systems



Transportation &
Logistics



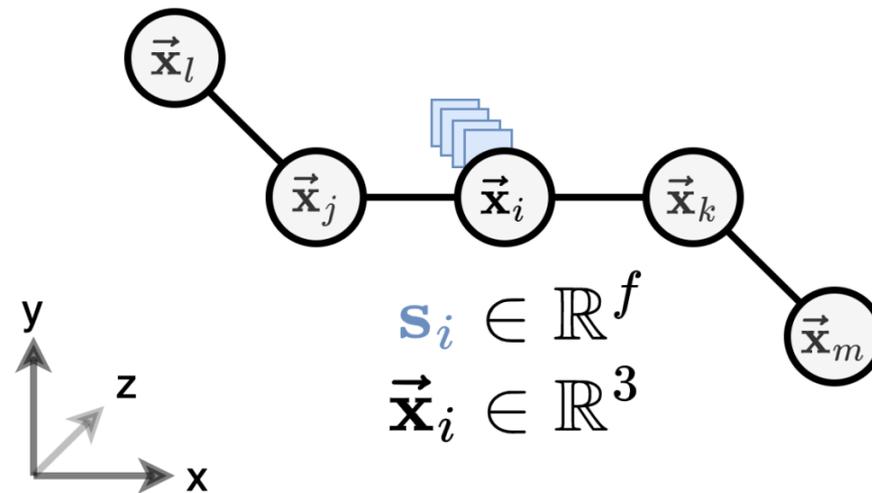
Robotic
Navigation



3D Computer
Vision

Geometric Graphs

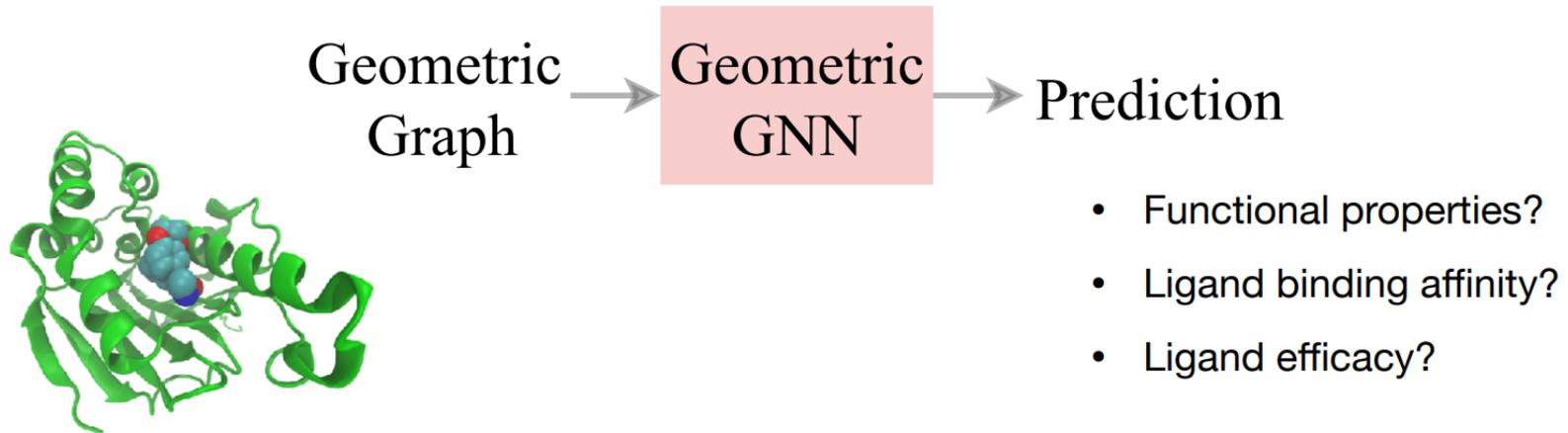
- A geometric graph $G = (A, S, X)$ is a graph where each node is embedded in d -dimensional **Euclidean space**:



- A : an $n \times n$ adjacency matrix.
- $S \in \mathbb{R}^{n \times f}$: **scalar** features.
- $X \in \mathbb{R}^{n \times d}$: **tensor** features, e.g., coordinates.

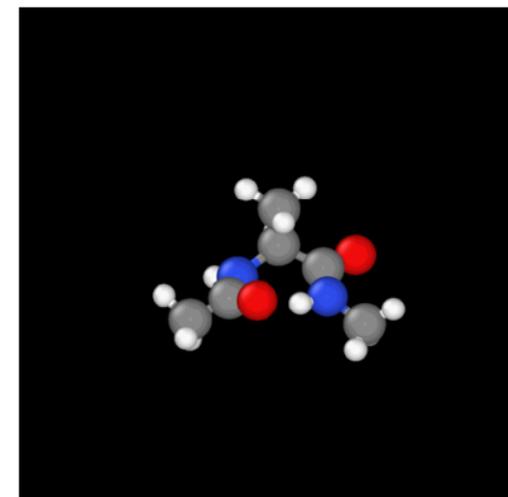
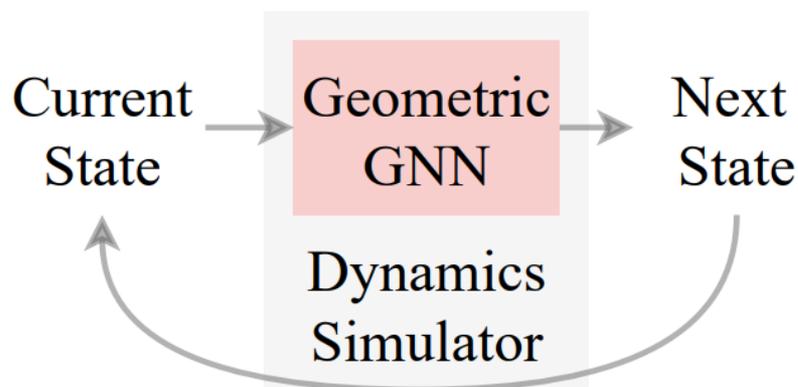
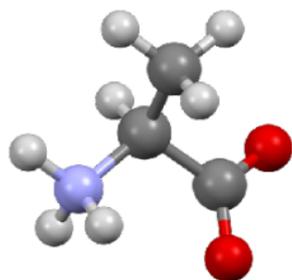
Broad Impact on Sciences

- Supervised Learning: Prediction
 - Properties prediction



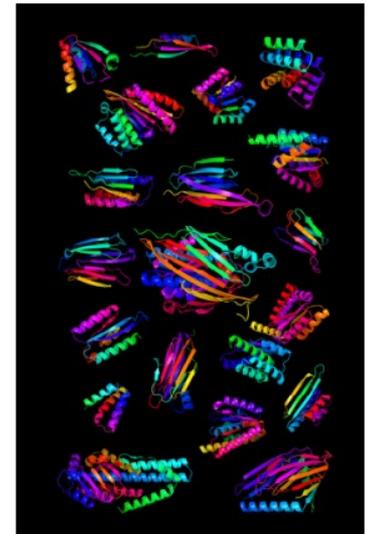
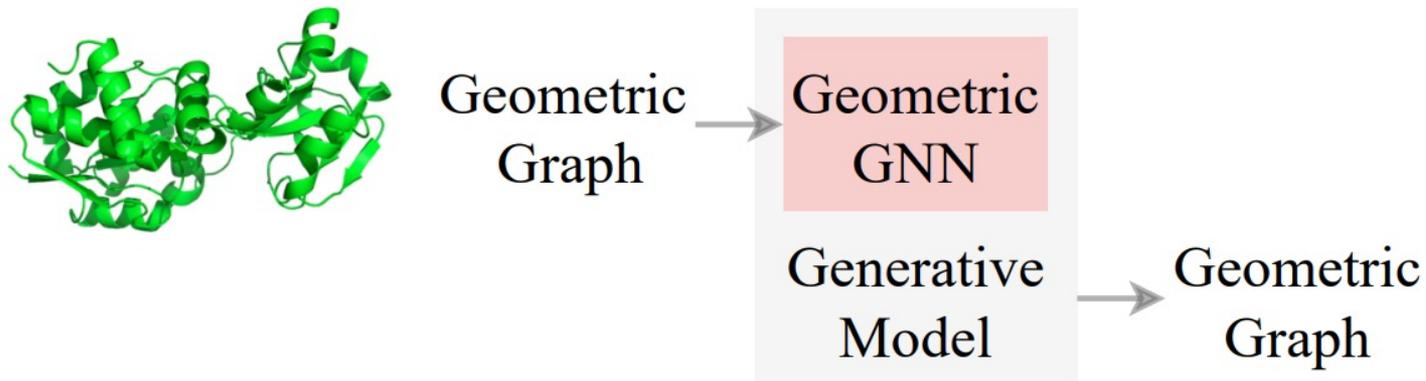
Broad Impact on Sciences

- Supervised Learning: Structured Prediction
 - Molecular Simulation



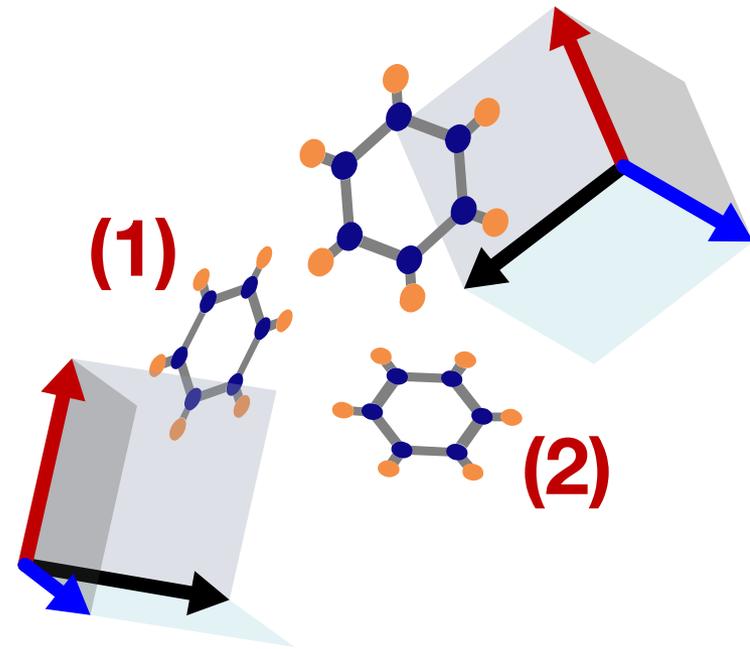
Broad Impact on Sciences

- Generative Models
 - Drug or material design



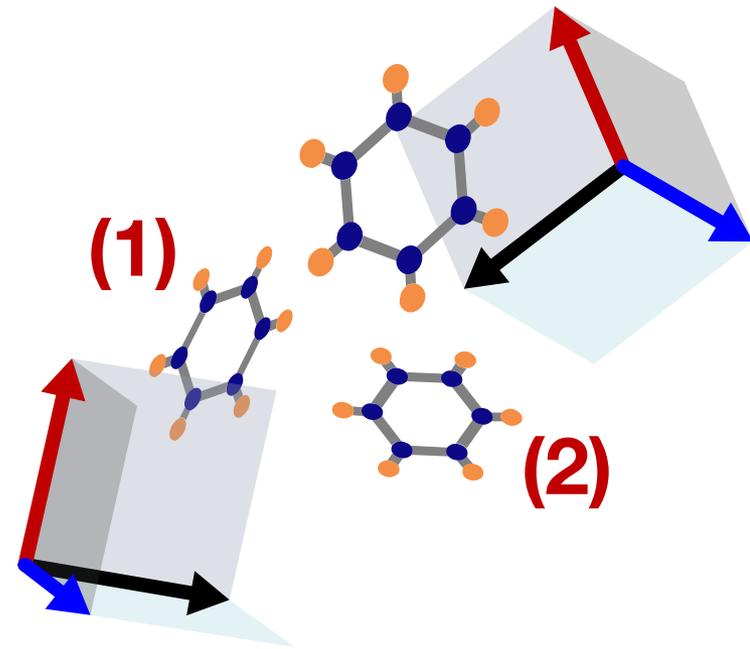
Physical Symmetry Groups

- To describe geometric graphs we use coordinate systems
 - (1) and (2) use different coordinate systems to describe the **same** molecular geometry.
- We can describe the transform between coordinate systems with symmetries of Euclidean space
 - 3D rotations, translations, reflection



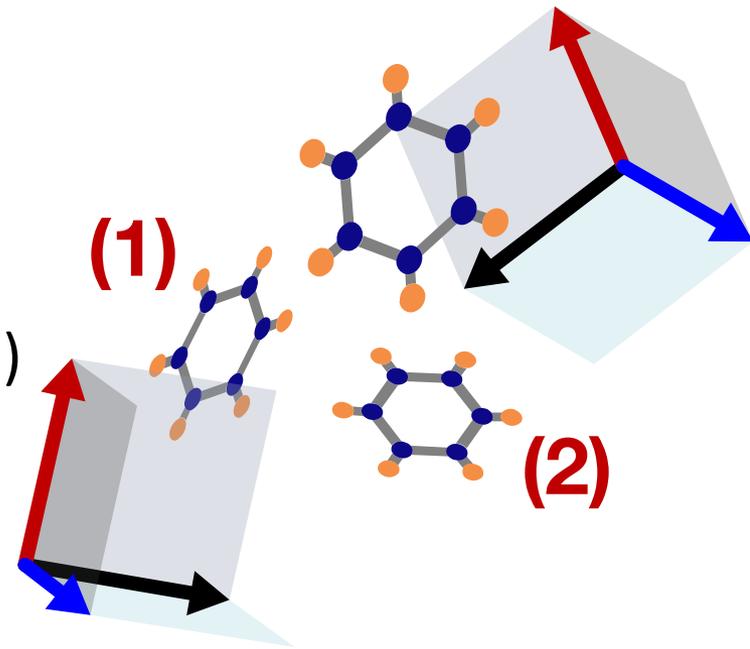
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 - 3D rotations, translations, reflection
- **However, output of traditional GNNs given (1) and (2) are completely different!**



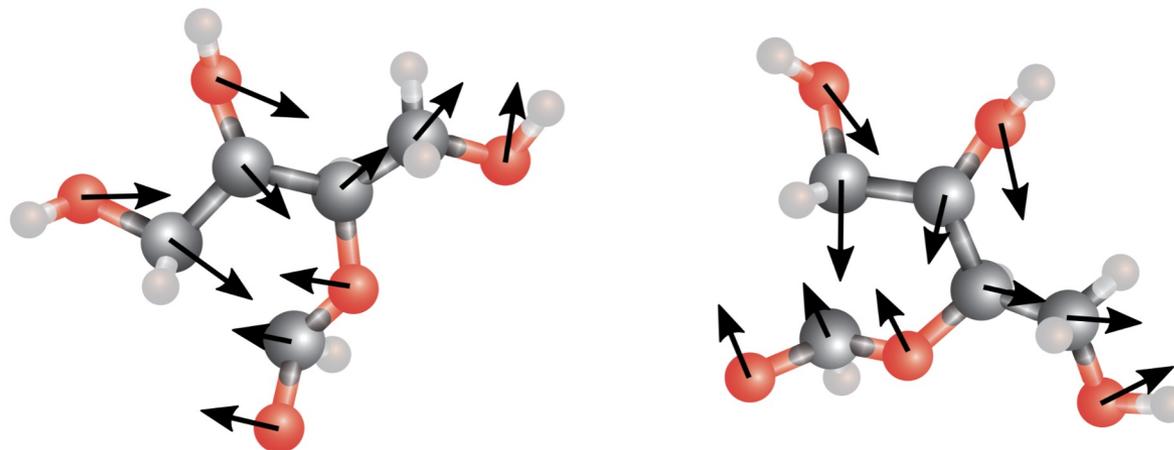
Symmetry of Inputs

- However, output of traditional GNNs given (1) and (2) as **completely different!**
- We want our GNNs can see (1) and (2) as the same system though described differently...
- I.e., we want design **Geometric GNNs aware of symmetry!**



Symmetry of Outputs

- Beyond input space, output can also be tensors
- Example: simulation (force prediction)
 - Given a molecule and a rotated copy, predicted forces should be the same up to rotation
 - (i.e., Predicted forces are **equivariant** to rotation)



Equivariance

- Formal definition of **Equivariance**:
a **function** $F: X \rightarrow Y$ is equivariant if for a transformation ρ it satisfies:

$$F \circ \rho_X(x) = \rho_Y \circ F(x)$$

- Example: ρ_X, ρ_Y are same rotation transformation

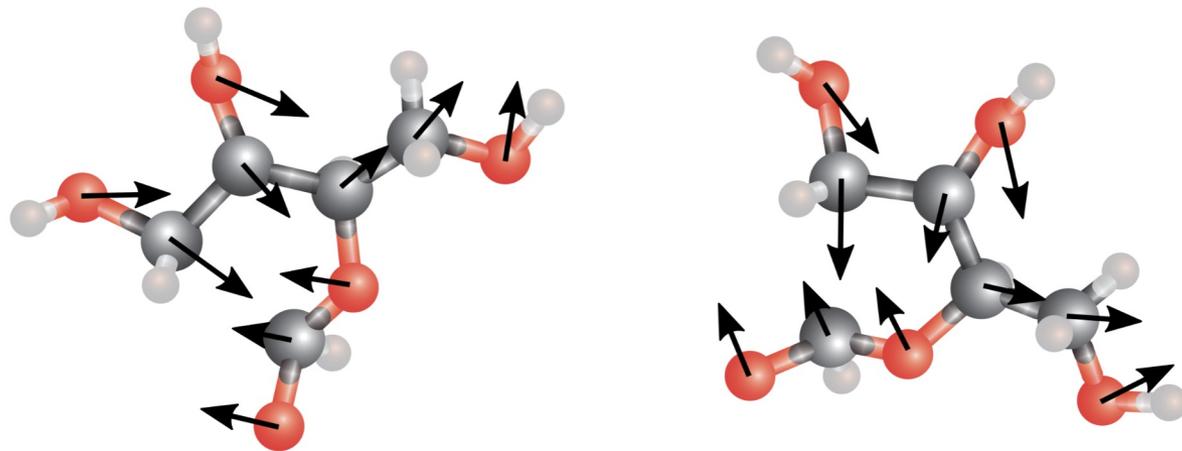
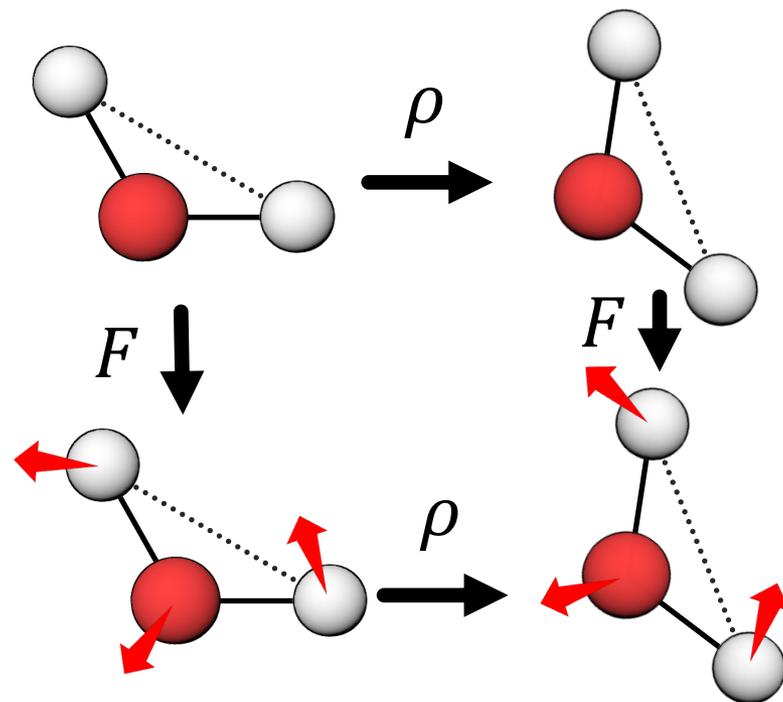
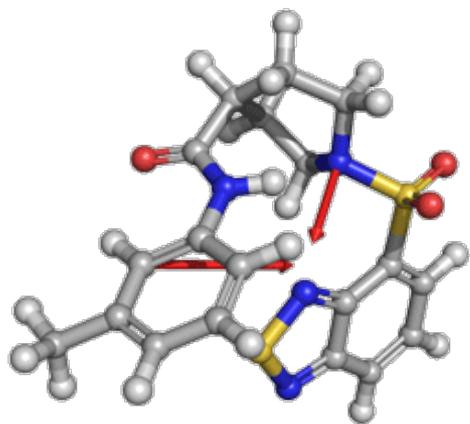


Illustration: 3D Rotation Equivariance

$$F \circ \rho(x) = \rho \circ F(x)$$

The equation says that applying the ρ on the input has the same effect as applying it to the output.



Visual explanation of the equivariance

A GIF illustrating the rotation equivariance of atomic forces.

Two red arrows stand for forces acting on atoms, which rotate together with the molecule.

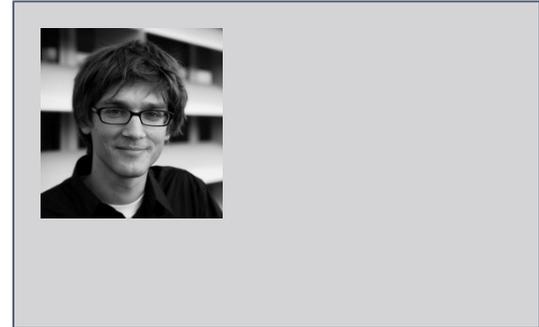
Shi, Chence, et al. "Learning gradient fields for molecular conformation generation." *International Conference on Machine Learning*. PMLR, 2021.

Invariance

- Definition of **Invariance**:
a **function** $F: X \rightarrow Y$ is
invariant if for a
transformation ρ it satisfies:

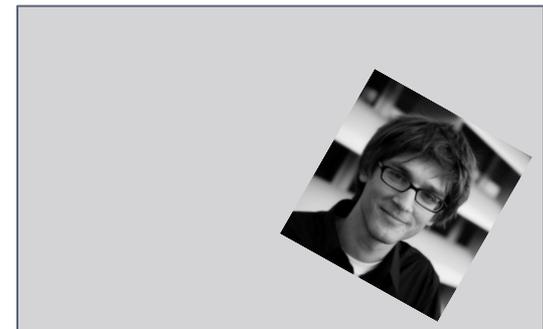
$$F \circ \rho_X(x) = F(x)$$

- **Note:** invariance is a special case of equivariance where ρ_Y is defined as no transformation.



✓ Yes, Prof. Leskovec.

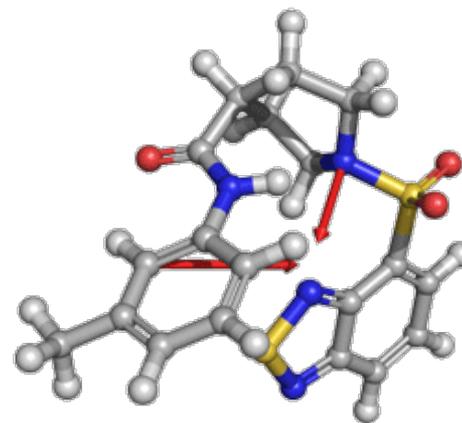
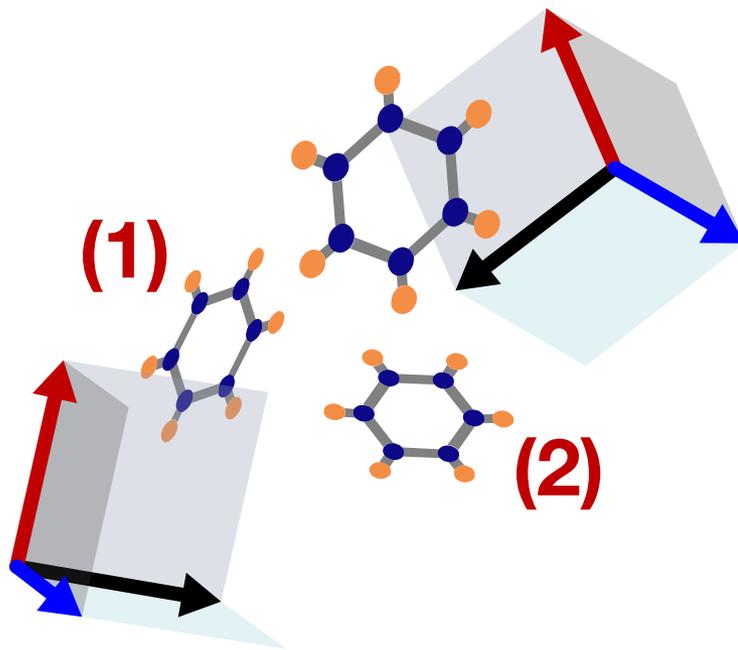
After roto-translation...



✓ Still Prof. Leskovec!

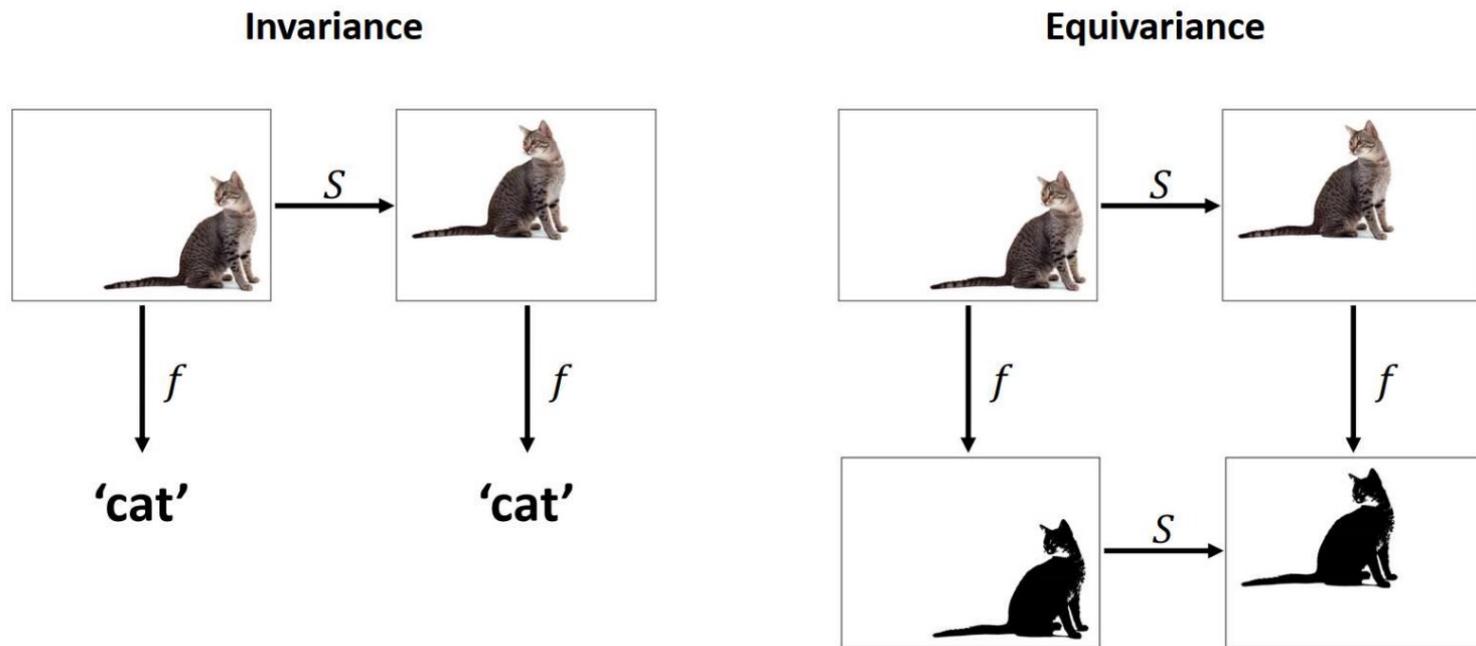
Invariance & Equivariance

- For geometric graphs, we consider 3D Special Euclidean ($SE(3)$) symmetries, e.g.:
 - structure \mathbf{x} -> force \mathbf{v} : equivariant tensors
rotation equivariant and translation invariant .
 - structure \mathbf{x} -> energy E : invariant scalars



Invariance & Equivariance

- The analogy in image domain...
 - Classification: invariant label
 - Segmentation: equivariant pixel coordinates



<https://www.doc.ic.ac.uk/~bkainz/teaching/DL/notes/equivariance.pdf>

Summarization

Neural networks are specially designed for different **data types** in order to make use of special features (symmetries) of the data.

Data type

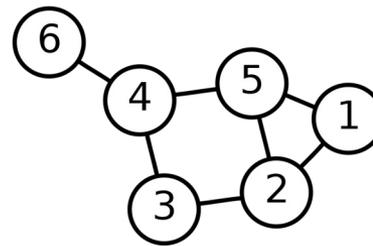
Images



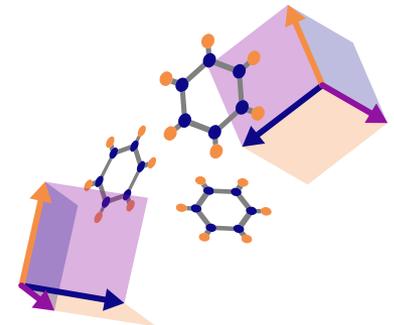
Text

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Graph



Geometric Graph in 3D



Type of neural network

Convolutional

Pixels closer together are more important to each other.

Spatial translation symmetry

Recurrent

The meaning of a current word depends on what came before.

Time translation symmetry

Graph

Data on nodes interacts via edges

Permutation symmetry

Euclidean

Geometric data “means” the same thing even when we use different coordinate systems

Euclidean symmetry

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Stanford CS224W: Geometric Graphs NNs

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

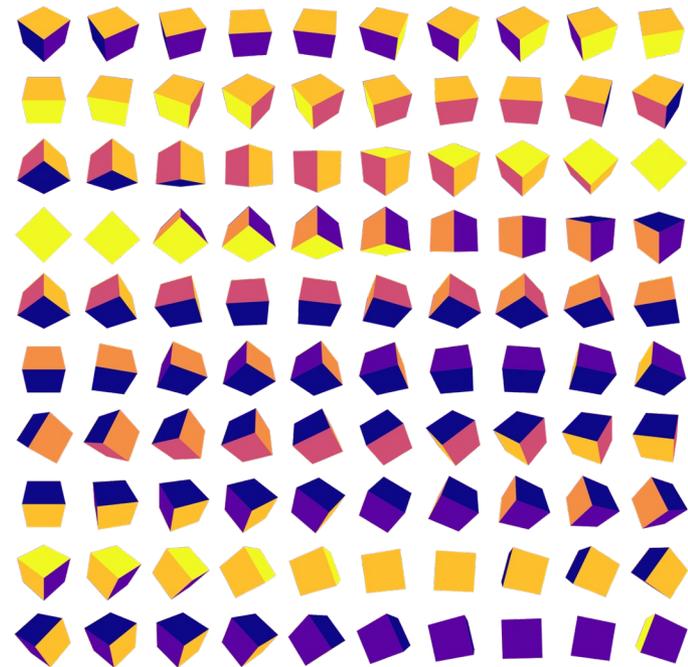
<http://cs224w.stanford.edu>



Handle Symmetry

- For ML models without handling symmetry:
expensive **data augmentation**
create more training data by augmenting original data to include all possible symmetries (rotations)
- Alternative: design Geometric GNNs!

training without rotational symmetry

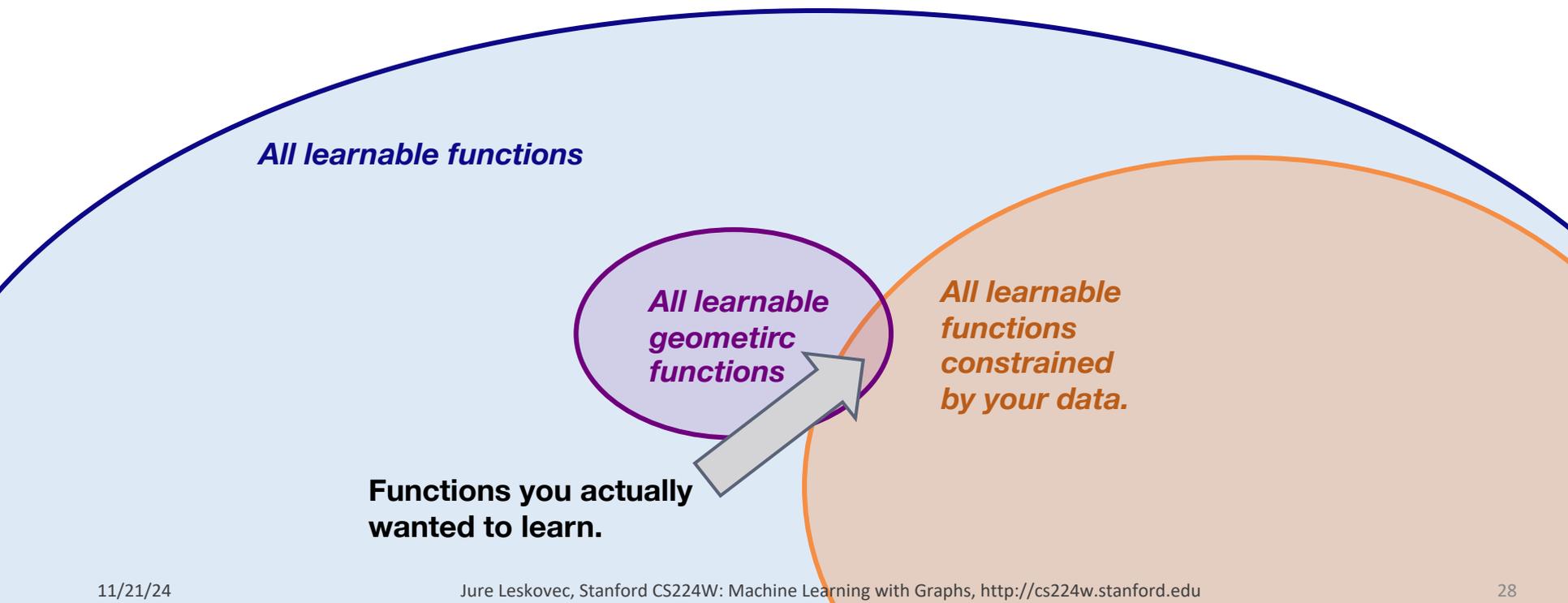


training with symmetry



Advantage

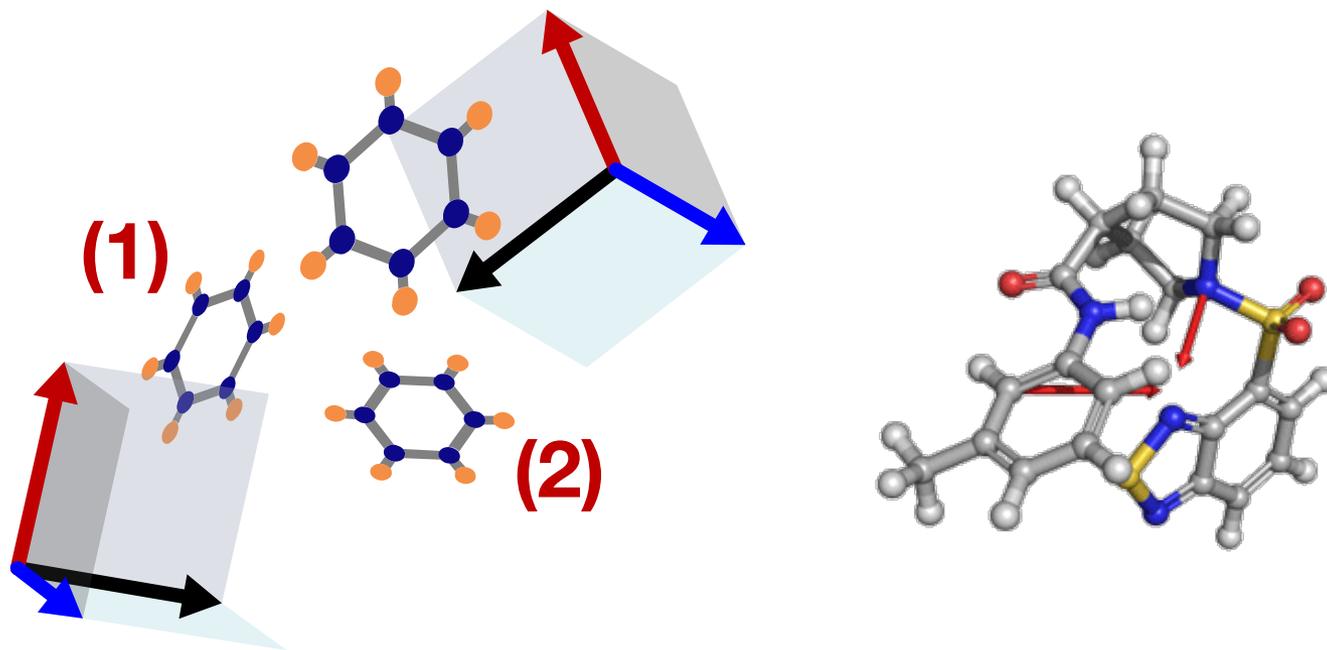
- You can substantially shrink the space of functions you need to optimize over.
- This means you need fewer data to constrain your function.



Geometric GNNs

Two classes of Geometric GNNs:

- **Invariant** GNNs for learning invariant **scalar** features
- **Equivariant** GNNs for learning equivariant **tensor** features.



Invariant functions vs. **Equivariant** functions

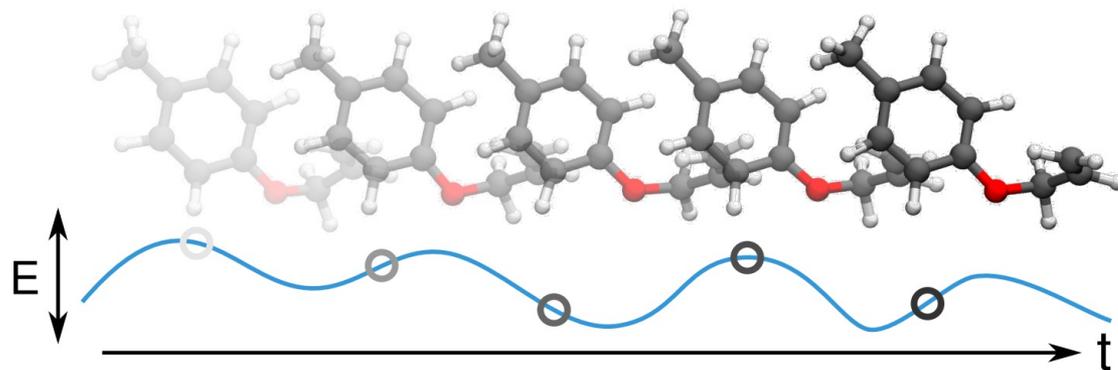
Molecular Dynamics Simulations

- For simulating the stable structure of molecular geometries: computationally costly quantum mechanical calculations

- Energy $E(\mathbf{r}_1, \dots, \mathbf{r}_n)$

- Forces

$$\mathbf{F}_i(\mathbf{r}_1, \dots, \mathbf{r}_n) = -\frac{\partial E}{\partial \mathbf{r}_i}(\mathbf{r}_1, \dots, \mathbf{r}_n).$$

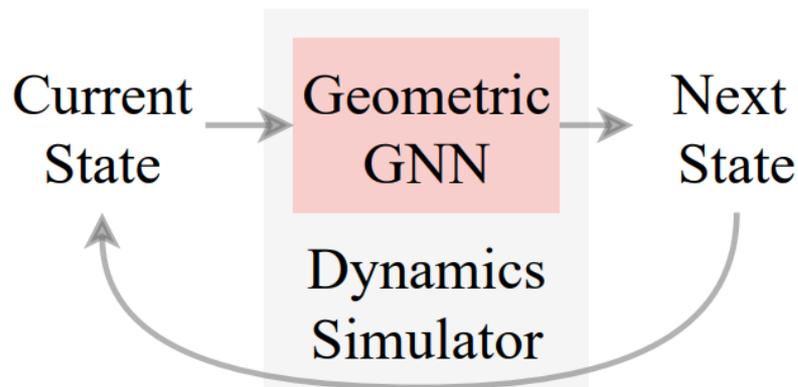
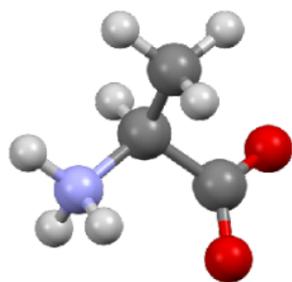


$$\hat{H}\Psi = E\Psi$$

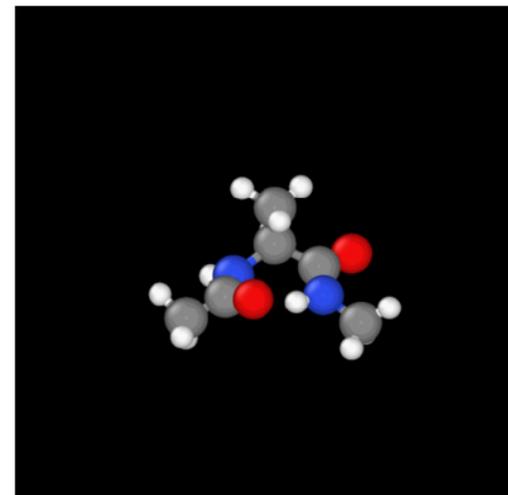
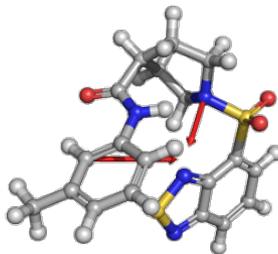
Method	Complexity
Hartree Fock	$O(n^3) - O(n^4)$
Density Functional Theory	$O(n^3) - O(n^4)$
MP2	$O(n^5)$
CCSD	$O(n^6)$
CCSD(T)	$O(n^7)$
Full CI	$O(n!)$

Molecular Dynamics Simulations

- Usage: forces can be used to optimize the structure by $X^t + F \rightarrow X^{t+1}$ (simulation)



F: forces acting on all atoms for optimizing the structures



For ML Models...

- Problem Definition

- **Inputs:**

molecular graphs with atom types $X = (x_1, \dots, x_n) \in R^d$
and positions $R = (r_1, \dots, r_n) \in R^3$

- **Predict:**

the molecular total energy $E(r_1, \dots, r_n)$ (**invariant**)
forces $F = (f_1, \dots, f_n)$ acting on each atom (**equivariant**).

- Forces are partial derivatives of energy function.

$$\mathbf{F}_i(\mathbf{r}_1, \dots, \mathbf{r}_n) = -\frac{\partial E}{\partial \mathbf{r}_i}(\mathbf{r}_1, \dots, \mathbf{r}_n).$$

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Stanford CS224W: Invariant GNNs: SchNet

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

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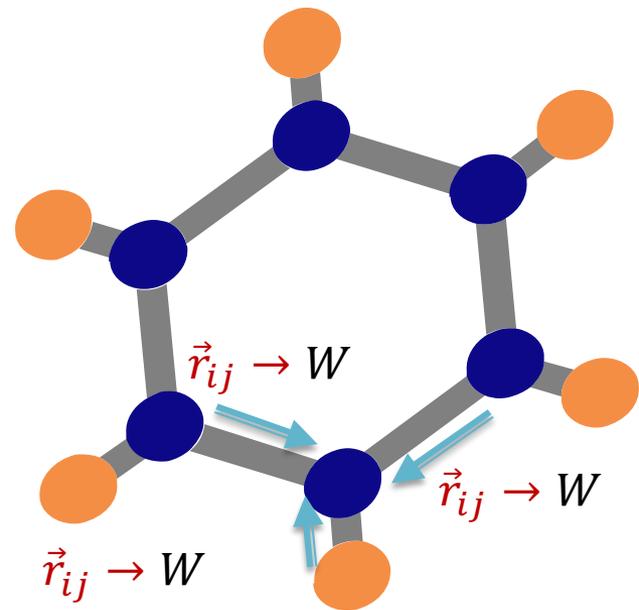


Invariant GNNs: SchNet

- **SchNet** updates the node embeddings at the l^{th} layer by message passing layers

$$\mathbf{x}_i^{l+1} = (X^l * W^l)_i = \sum_j \mathbf{x}_j^l \circ W^l(\mathbf{r}_i - \mathbf{r}_j),$$

- A weight matrix W is determined by the relative position from neighbor atoms j to i
- This kernel matrix $W: \mathbb{R}^3 \rightarrow \mathbb{R}^d$ then controls interaction from neighbor atoms by $x_j \cdot W$
- All the neighbor messages are aggregated by $\sum_j x_j \cdot W$



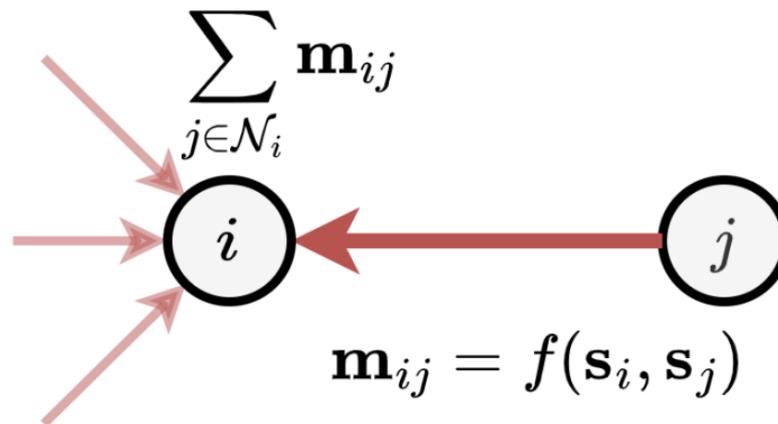
x^l : node embeddings at l layer
 r : atomic coordinates

Schütt, Kristof T., et al. "SchNet—a deep learning architecture for molecules and materials." *The Journal of Chemical Physics* 148.24 (2018): 241722.

Message Passing Neural Nets

- Node features are updated from iteration t to $t+1$ via learnable **permutation invariant** neighborhood **aggregate AGG** and **update UPD**:

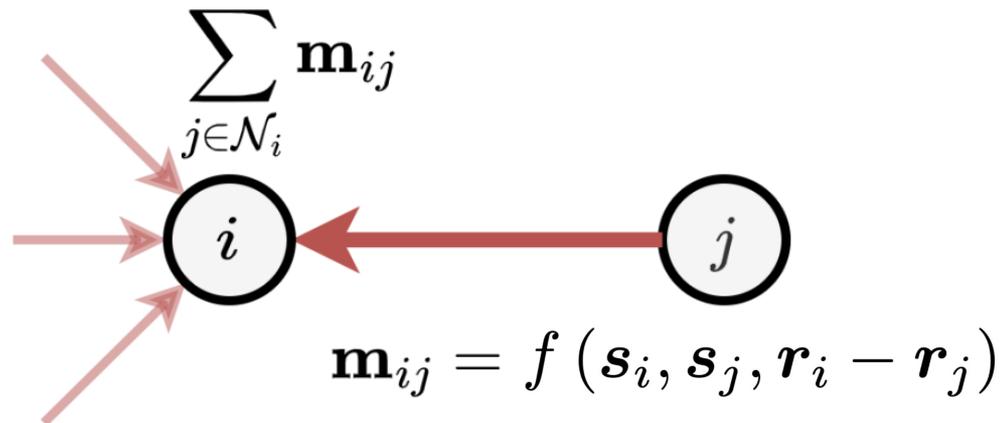
$$\mathbf{m}_i^{(t)} = \text{AGG} \left(\left\{ \left(\mathbf{s}_i^{(t)}, \mathbf{s}_j^{(t)} \right) \mid j \in \mathcal{N}_i \right\} \right)$$
$$\mathbf{s}_i^{(t+1)} = \text{UPD} \left(\mathbf{s}_i^{(t)}, \mathbf{m}_i^{(t)} \right)$$



Message Passing Neural Nets

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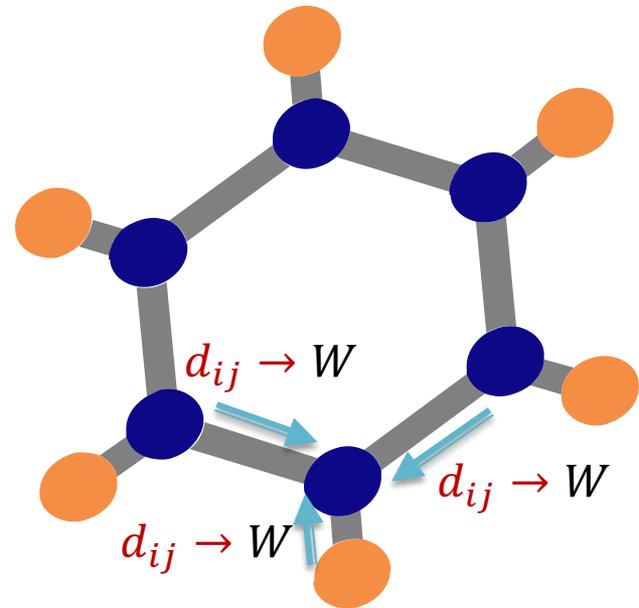


Invariant GNNs: SchNet

- **SchNet** makes W invariant by **scalarizing** relative positions \vec{r}_{ij} with **relative distances** d_{ij}
 $= \|\vec{r}_{ij}\|$:
 - $\|\vec{r}_{ij}\|$ are invariant to rotations and translations
 - \Rightarrow each message passing layer weight W is invariant

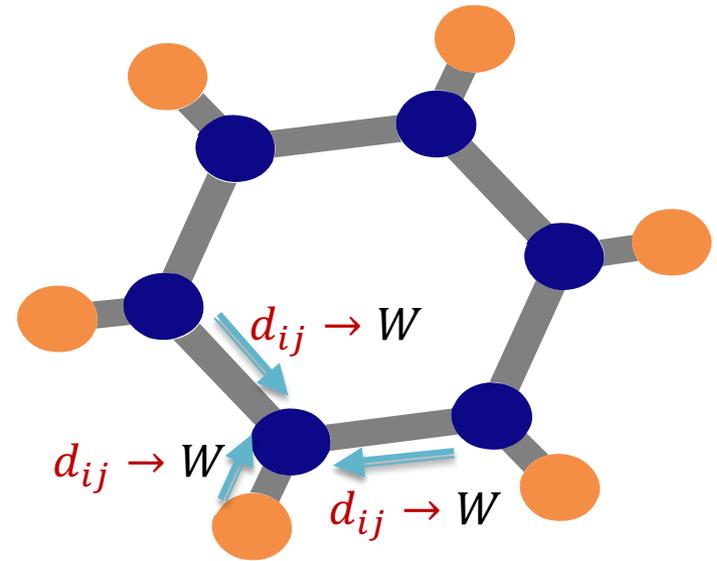
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x^l : node embeddings at l layer
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invariant GNNs: SchNet

- **SchNet** makes W invariant by **scalarizing** relative positions \vec{r}_{ij} with **relative distances** d_{ij}
 $= \|\vec{r}_{ij}\|$:
 - $\|\vec{r}_{ij}\|$ are invariant to rotations and translations
 - \Rightarrow each message passing layer weight W is invariant
 - \Rightarrow aggregated node embeddings $\sum_j x_j \cdot W$ is invariant
 - \Rightarrow therefore, whole network output is invariant!



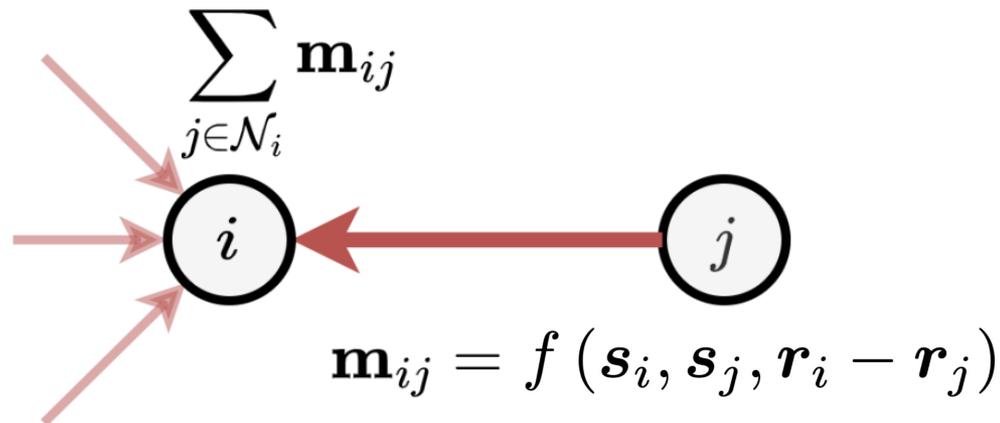
$$\mathbf{x}_i^{l+1} = (X^l * W^l)_i = \sum_j \mathbf{x}_j^l \circ W^l(\mathbf{r}_i - \mathbf{r}_j),$$

x^l : node embeddings at l layer
 r : atomic coordinates

Message Passing Neural Nets

- Node features are updated from iteration t to $t+1$ via learnable **permutation invariant** neighborhood **aggregate AGG** and **update UPD**:

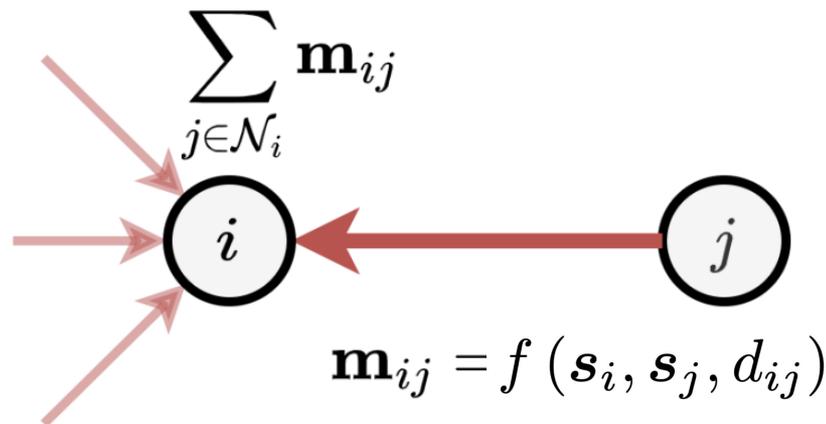
$$\mathbf{m}_i^{(t)} = \text{AGG} \left(\left\{ \left(\mathbf{s}_i^{(t)}, \mathbf{s}_j^{(t)} \right) \mid j \in \mathcal{N}_i \right\} \right)$$
$$\mathbf{s}_i^{(t+1)} = \text{UPD} \left(\mathbf{s}_i^{(t)}, \mathbf{m}_i^{(t)} \right)$$



Message Passing Neural Nets

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Stanford CS224W: Invariant GNNs: DimeNet

CS224W: Machine Learning with Graphs

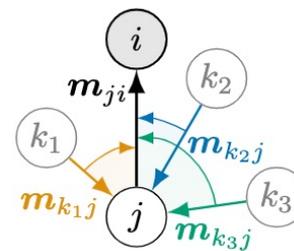
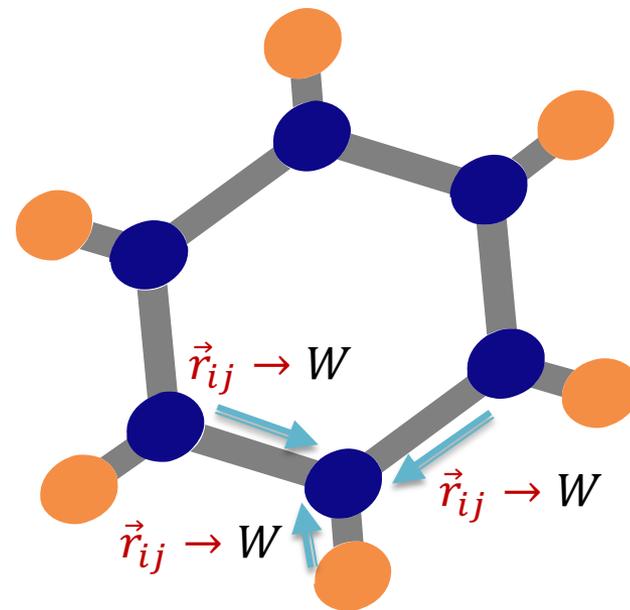
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Improved SchNet: DimeNet

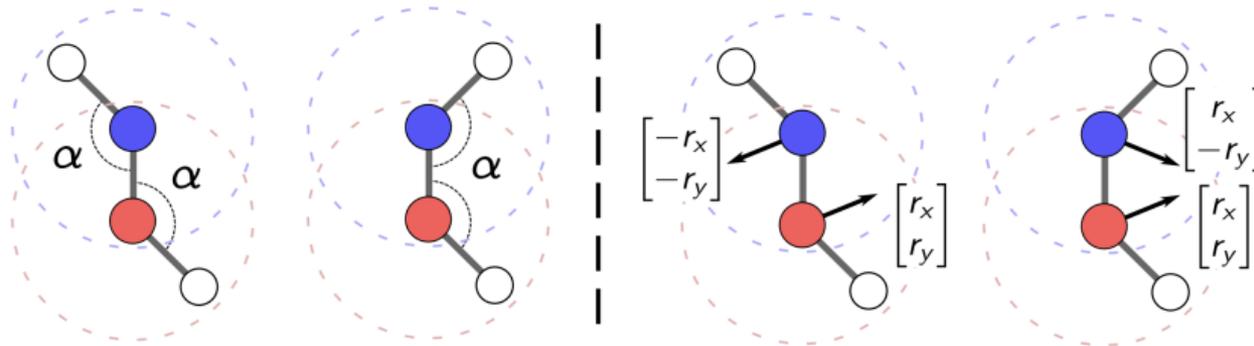
- Chemically, potential energy can be modeled as sum of four parts
$$E = E_{\text{bonds}} + E_{\text{angle}} + E_{\text{torsion}} + E_{\text{non-bonded}},$$
 - SchNet only depends on atom types and pairwise distance, **ignore** many information like angles and torsions
- DimeNet** resolves this problem by
 - Do message interaction based on
 - distance between atoms
 - angle between bonds(both of which are invariant to translation and rotation!)



Gasteiger, Johannes, Janek Groß, and Stephan Günnemann.
"Directional message passing for molecular graphs." *ICLR* (2020).

Expressiveness

- Distances/Angles are incomplete descriptors for uniquely identifying geometric structure.



- This pair of geometric graphs cannot be distinguished by **identical** scalar quantities.
- But they can be distinguished based on **directional** or **geometric** information

Limitations of invariant GNNs

- Why not limit yourself to invariant functions?
- You have to **guarantee** that your input features already contain any necessary equivariant interactions.

*All learnable
equivariant
functions*

*All invariant
functions
constrained by
your data.*

*All learnable
invariant
functions.*

**Functions you actually
wanted to learn.**

OR



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Stanford CS224W: Equivariant GNNs: PaiNN

CS224W: Machine Learning with Graphs

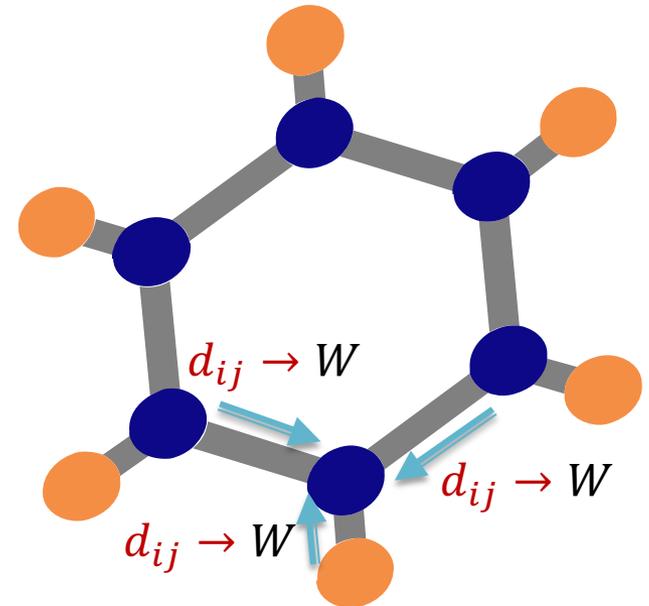
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Equivariant GNNs: PaiNN

- **PaiNN** still take learnable weights W conditioned on the **relative distance** $\|\vec{r}_{ij}\|$ to control message passing
- However, differently, in PaiNN each node has two features (both **scalar features** s_i and **vector features** v_i)



Schütt, Kristof, Oliver Unke, and Michael Gastegger. "Equivariant message passing for the prediction of tensorial properties and molecular spectra." *International Conference on Machine Learning*. PMLR, 2021.

Equivariant GNNs: PaiNN

- The two features (scalar features s_i and vector features v_i) are
 - initialed by: atom embeddings and 0 tensors
 - updated by: residual updates

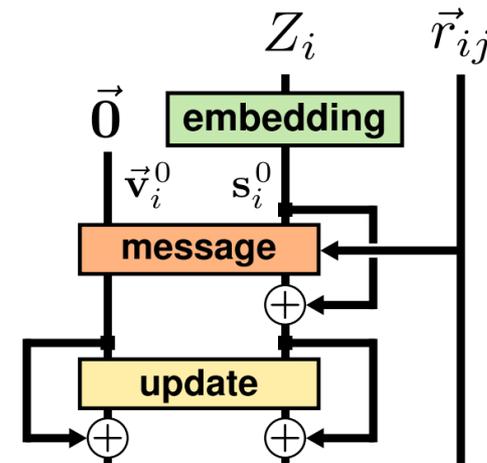
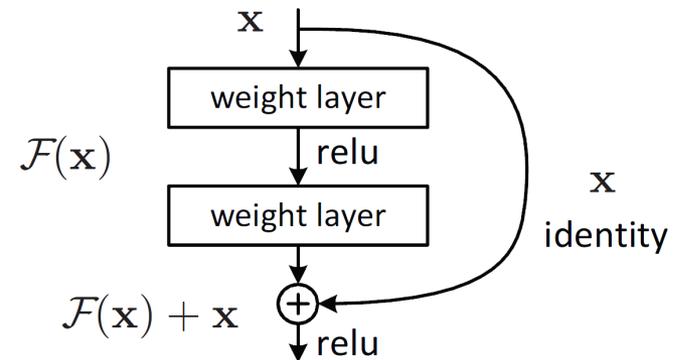
$s, \Delta s$: scalar features and its updates

$v, \Delta v$: tensor features and its updates

ϕ, W : networks

$$\begin{aligned} \Delta s_i^m &= (\phi_s(\mathbf{s}) * \mathcal{W}_s)_i \\ &= \sum_j \phi_s(\mathbf{s}_j) \circ \mathcal{W}_s(\|\vec{r}_{ij}\|) \end{aligned}$$

$$\begin{aligned} \Delta \vec{v}_i^m &= \sum_j \vec{v}_j \circ \phi_{vv}(\mathbf{s}_j) \circ \mathcal{W}_{vv}(\|\vec{r}_{ij}\|) \\ &+ \sum_j \phi_{vs}(\mathbf{s}_j) \circ \mathcal{W}'_{vs}(\|\vec{r}_{ij}\|) \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|} \end{aligned}$$

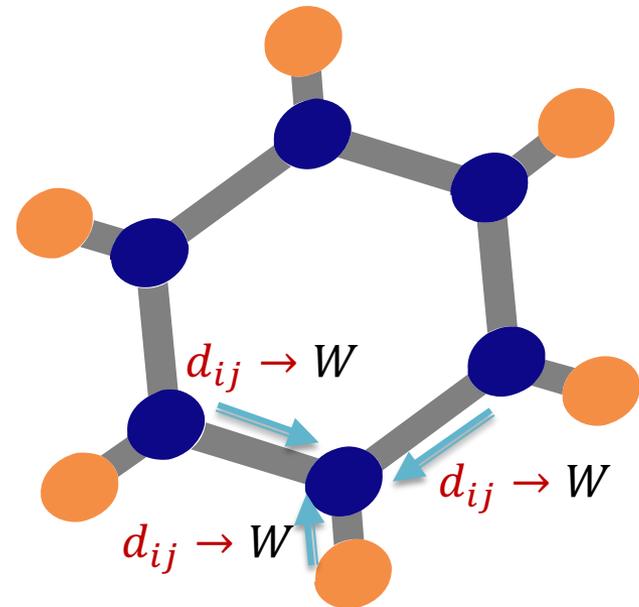


Equivariant GNNs: PaiNN

- scalar features s_i update for atom i :

$$\begin{aligned}\Delta \mathbf{s}_i^m &= (\phi_s(\mathbf{s}) * \mathcal{W}_s)_i \\ &= \sum_j \phi_s(\mathbf{s}_j) \circ \mathcal{W}_s(\|\vec{r}_{ij}\|)\end{aligned}$$

- ϕ_s, \mathcal{W}_s are neural networks
- Similar to SchNet
 - invariant weights \mathcal{W}_s by $\|\vec{r}_{ij}\|$
 - => scalar neighbor messages $\phi_s \cdot \mathcal{W}_s$
 - => inv sum over messages

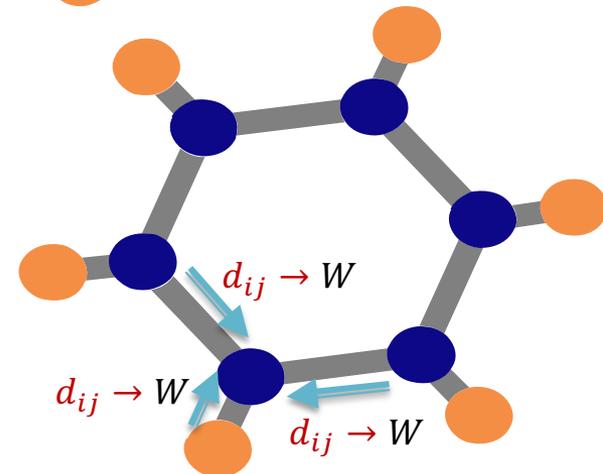
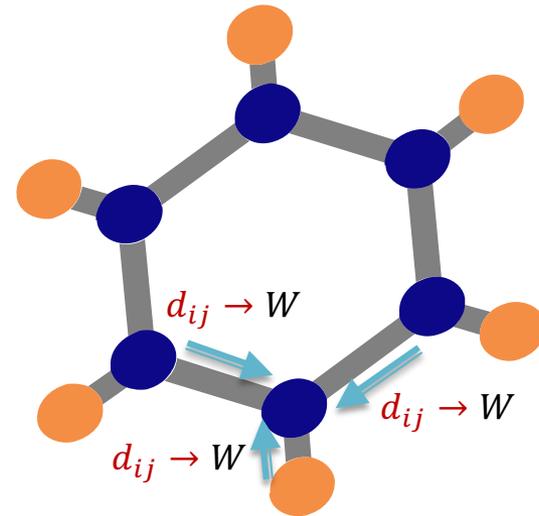


Equivariant GNNs: PaiNN

- vector features v_i update:

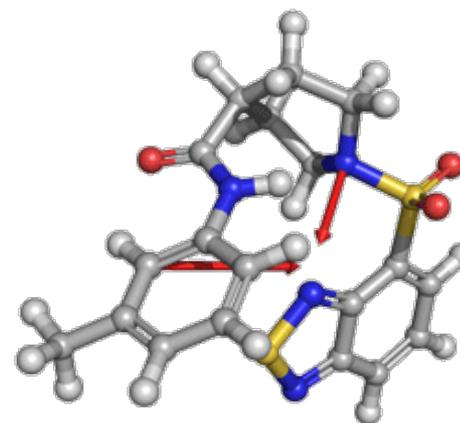
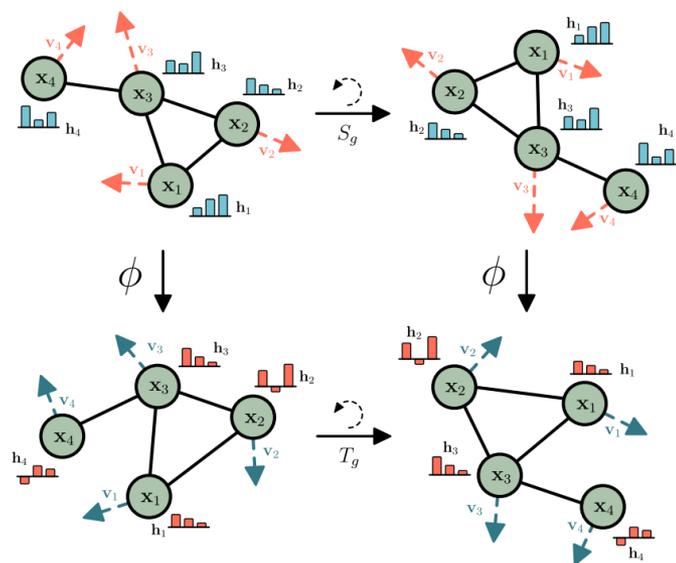
$$\Delta \vec{v}_i^m = \sum_j \vec{v}_j \circ \phi_{vv}(\mathbf{s}_j) \circ \mathcal{W}_{vv}(\|\vec{r}_{ij}\|) + \sum_j \phi_{vs}(\mathbf{s}_j) \circ \mathcal{W}'_{vs}(\|\vec{r}_{ij}\|) \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|}$$

- ϕ, W are all neural networks
- Different to SchNet
 - invariant weights W_s by $\|\vec{r}_{ij}\|$
 - => tensor neighbor messages $\phi_s \cdot W_s \cdot \vec{r}_{ij}$
 - => weighted sum of relative directions \vec{r}_{ij} , thus keeping the equivariant properties!



Equivariant GNNs: PaiNN

- By stacking multiple PaiNN layers...
- **vector features** v_i after final layer are tensor features equivariant w.r.t to input coordinates
- are therefore can be directly used as force prediction



Satorras, Victor Garcia, Emiel Hoogetboom, and Max Welling. "E(n) equivariant graph neural networks." *International conference on machine learning*. PMLR, 2021.

Summary of Geometric GNNs

- Geometric GNNs need to capture sufficient information of geometries
- SchNet (and DimeNet) achieve **invariance** by only learning over scalarized **invariant features** (distances, angles, ...)
- PaiNN designs both **scalar** and **tensor** features, where the tensor features are **equivariant** with input coordinates
- Applications: equivariant output can be used as force prediction for molecular simulation

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Stanford CS224W: Geometric Generative Models

CS224W: Machine Learning with Graphs

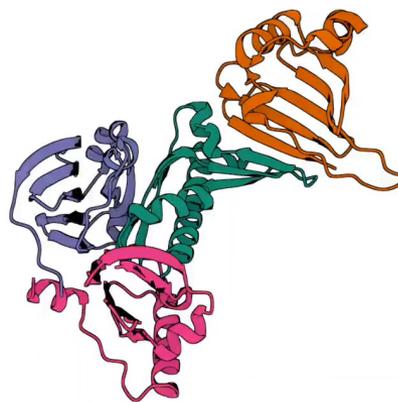
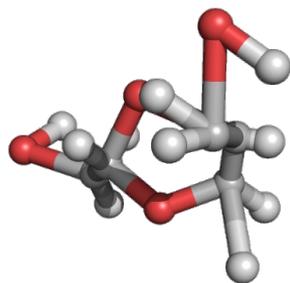
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Broad Applications

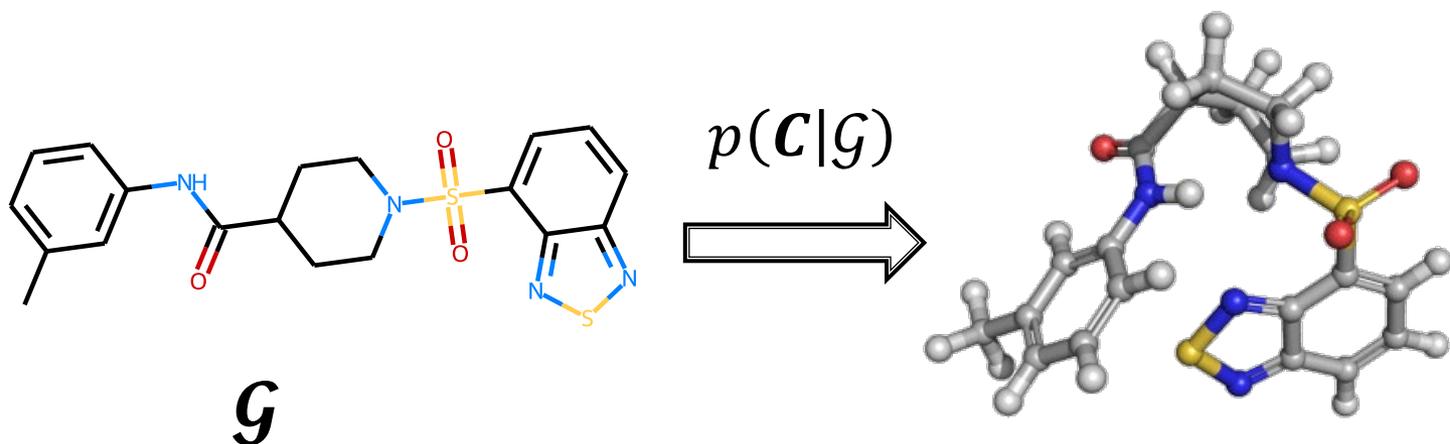
- Accelerate scientific simulation
 - Molecule/Protein Design
 - Biomolecule structure prediction
 - Protein-molecule interaction
 - Molecular simulation



<https://generatebiomedicines.com/chroma>

Molecular Conformation Generation

- Generate stable conformations from molecular graph
 - Molecular graph \mathcal{G} : 2D atom-bond graph
 - Conformation \mathcal{C} : atomic 3D coordinates
 - One molecule can have multiple possible conformations, which follows a distribution conditioned on temperature T

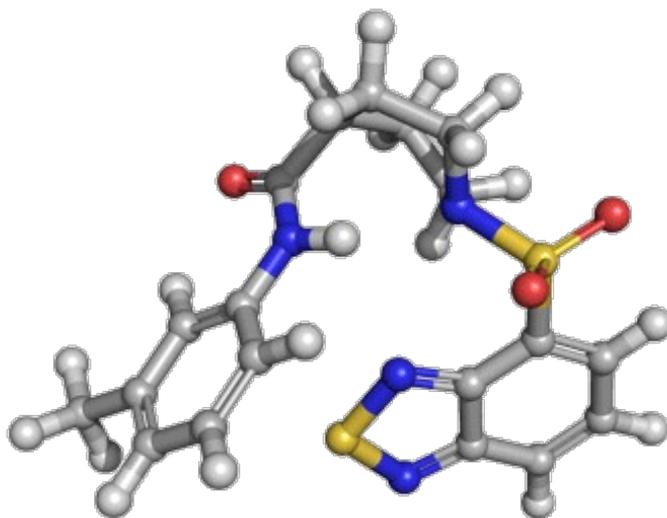


$$\mathcal{C} \propto \exp(-E(\mathcal{C})/T)$$

Boltzmann distribution

Challenges

- Generative models learn the data distribution
- Similar to the learning algorithm, generation process should also capture the physical symmetry groups, i.e., **equivariant to roto-translation**

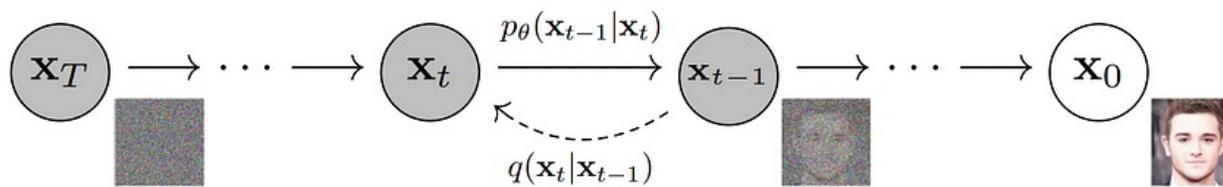


$$C \propto \exp(-E(C)/T)$$

Boltzmann distribution

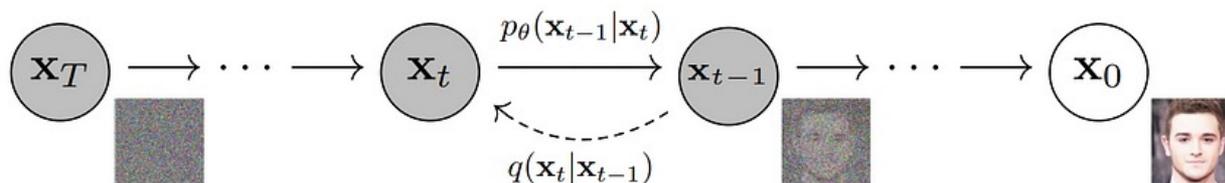
Background: Diffusion Models

- Define **forward diffusion process** to destroy data into different noisy-level samples
- Learn **reverse models** to **generate by denoising**



Ho, Jonathan, Ajay Jain, and Pieter Abbeel. "Denoising diffusion probabilistic models." *Advances in Neural Information Processing Systems* 33 (2020): 6840-6851.

Background: Diffusion Models



■ Training:

- Sample random noise ϵ
- Destroy the data by $x_t = \mu_t x + \sigma_t \epsilon$ at every t
 - μ_t, σ_t, t are pre defined
- Learn models $f_\theta(x_t)$ to predict the noise ϵ

t : timestep
 μ : means to shrink data
 σ : variance as noise level

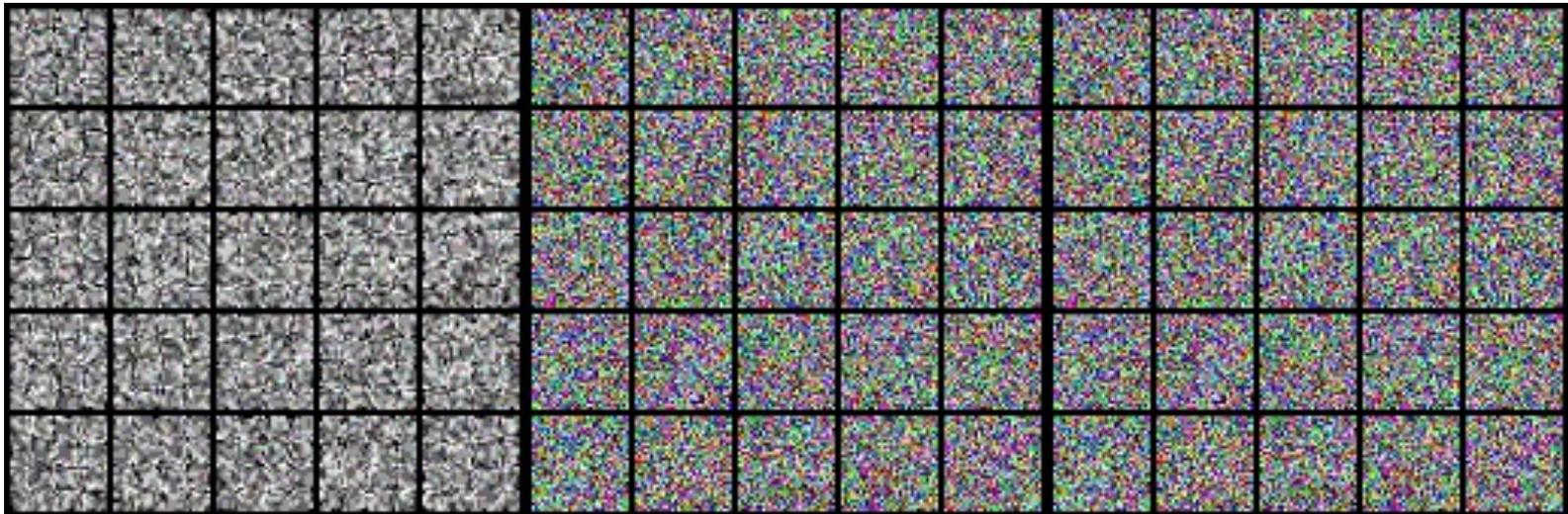
higher t -> smaller μ and larger σ

■ Sampling:

- Sample $x_T \sim N(0, I)$ from Gaussian random noise
- Generate x by repeatedly predicting and subtracting the noise
- Recover clean data

Background: Diffusion Models

- Then the learned **reverse** model can be used to generate data by **progressively denoising**

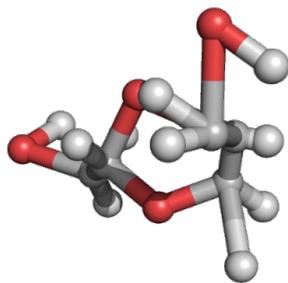


Song, Yang, and Stefano Ermon. "Generative modeling by estimating gradients of the data distribution." *Advances in neural information processing systems* 32 (2019).

Geometric Diffusion

- We bring the idea into molecule generation!
- *Minkai Xu, Lantao Yu, Yang Song, Chence Shi, Stefano Ermon, and Jian Tang. "GeoDiff: A Geometric Diffusion Model for Molecular Conformation Generation." In International Conference on Learning Representations. 2021.*
- Top 50 most cited AI papers in 2022

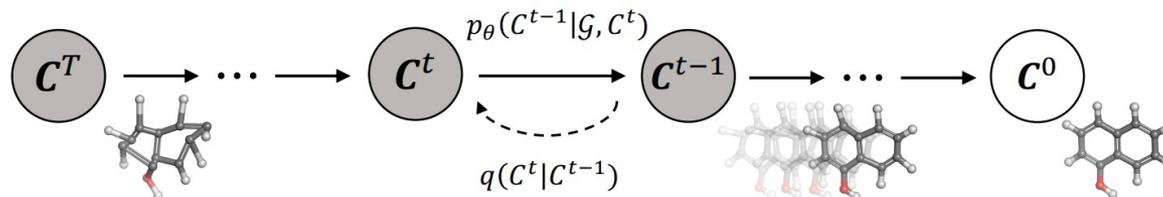
42	Conditional Prompt Learning for Vision-Language Models
43	Fine-Tuning can Distort Pretrained Features and Underperform Out-of-Distribution
44	Measuring and Improving the Use of Graph Information in Graph Neural Networks
45	Exploring Plain Vision Transformer Backbones for Object Detection
46	GeoDiff: a Geometric Diffusion Model for Molecular Conformation Generation
47	OFA: Unifying Architectures, Tasks, and Modalities Through a Simple Sequence-to-Sequence Learning Framework
48	Block-NeRF: Scalable Large Scene Neural View Synthesis



<https://www.zeta-alpha.com/post/must-read-the-100-most-cited-ai-papers-in-2022>

Geometric Diffusion

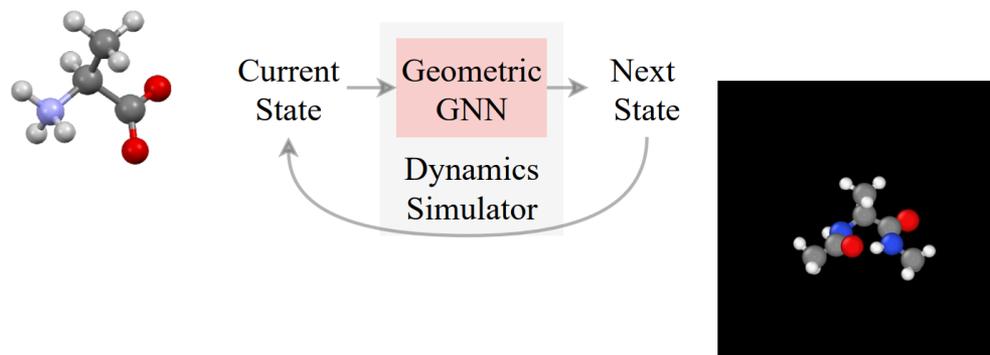
- GeoDiff (Geometric Diffusion)



- Diffusion process gradually perturb the molecular geometry until the conformation is destroyed.
- Symmetrically, we want to learn the **reverse generative process** to progressively refined a random noisy geometry

Minkai Xu, Lantao Yu, Yang Song, Chence Shi, Stefano Ermon, and Jian Tang.
"GeoDiff: A Geometric Diffusion Model for Molecular Conformation Generation."
In *International Conference on Learning Representations*. 2021.

Geometric Diffusion

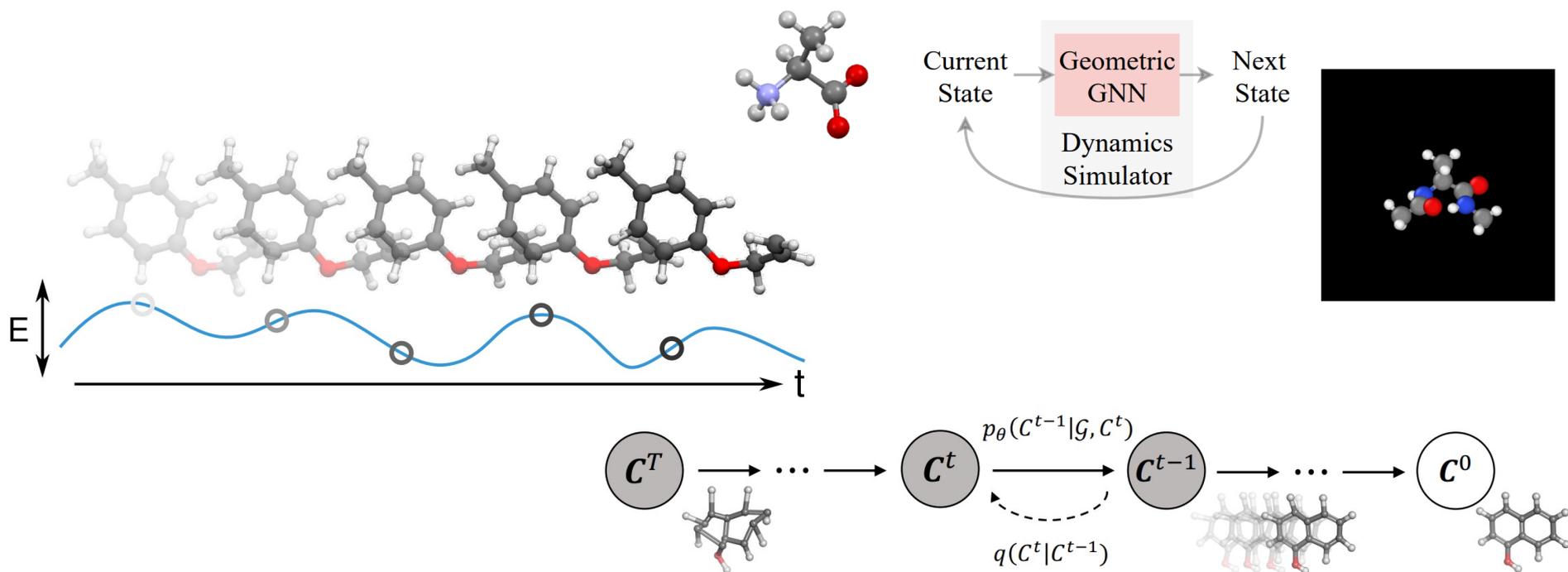


- **Diffusion process** gradually perturb the molecular geometry until the conformation is destroyed.
- Symmetrically, we want to learn the **reverse generative process** to progressively refined a random noisy geometry
- **Wait!** This is kind of similar to molecular simulation!

Minkai Xu, Lantao Yu, Yang Song, Chence Shi, Stefano Ermon, and Jian Tang. "GeoDiff: A Geometric Diffusion Model for Molecular Conformation Generation." In *International Conference on Learning Representations*. 2021.

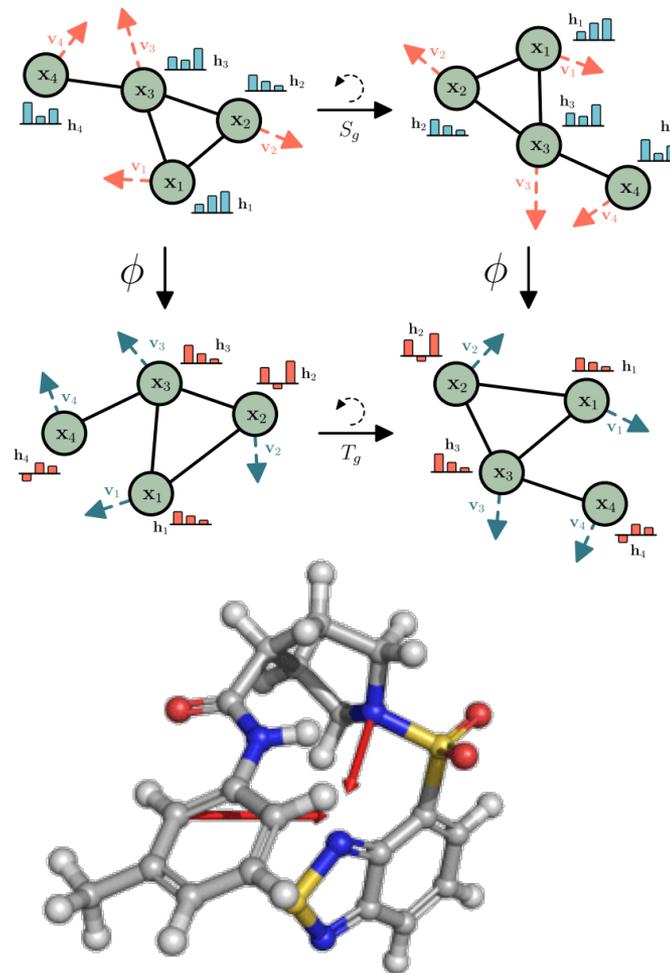
Geometric Diffusion

- To optimize structures to better states:
 - Simulation: learns to predict force
 - Diffusion models: learns to predict noise



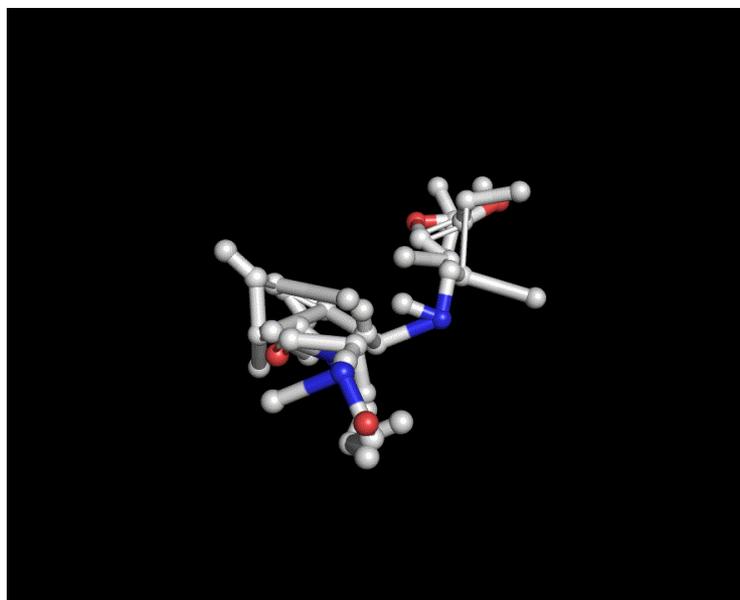
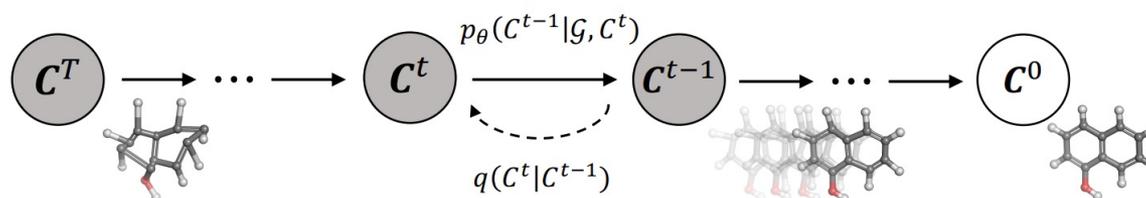
Geometric Diffusion

- Insight!
Similar to force, denoising direction should be equivariant with the molecular coordinates!
- Solution:
Parameterizing the denoising network with equivariant GNN 😊



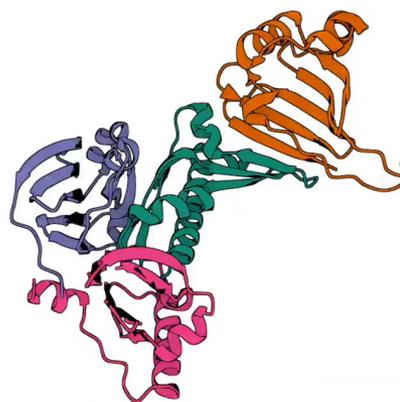
GeoDiff: Sampling

- Sampling by an **equivariant denoising** procedure:

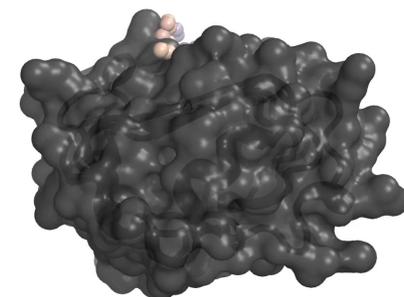
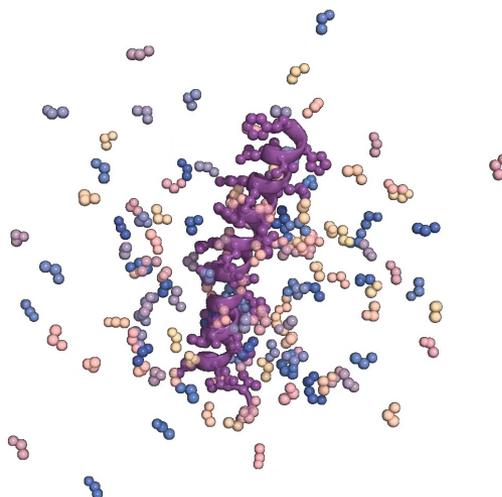


Most Recent Progress

- *Illuminating protein space with a programmable generative model*
John Ingraham, Max Baranov, Zak Costello, Vincent Frappier, et al



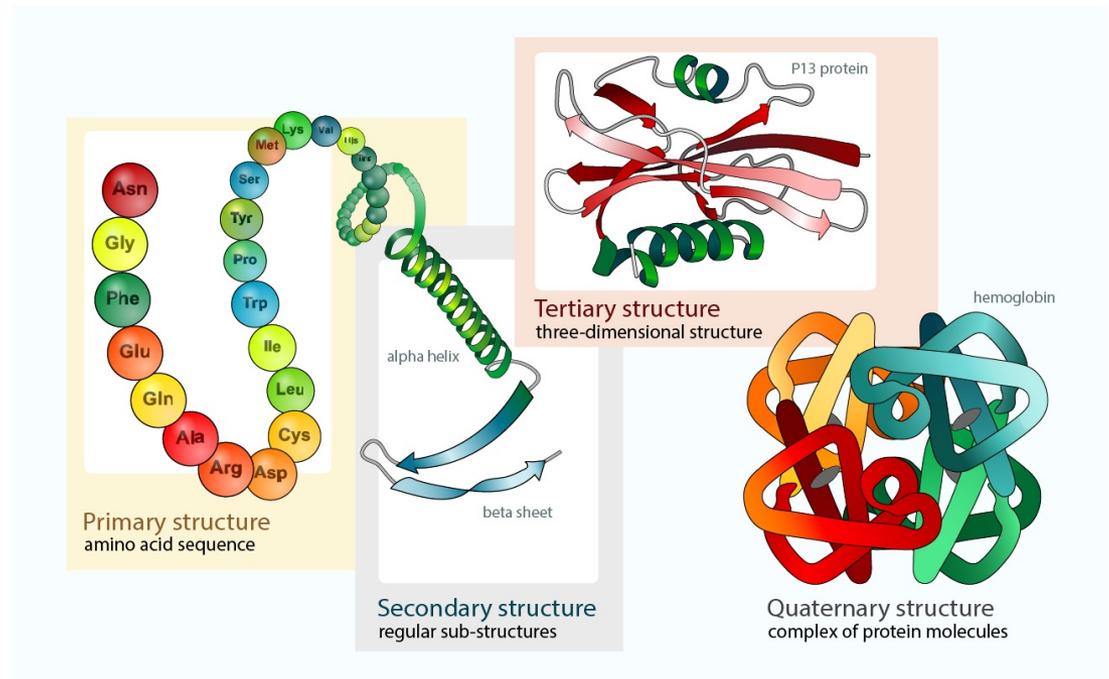
- *Broadly applicable and accurate protein design by integrating structure prediction networks and diffusion generative models*
Joseph L Watson, David Juergens, Nathaniel R Bennett, Brian L Trippe, Jason Yim, Helen E Eisenach, Woody Ahern, et al



Protein Folding

What is Protein Folding?

- Process by which chain of amino acids adopts three-dimensional conformation.
- Objective: minimize free energy.



Summary

- Geometric Graphs
- Geometric Graph NNs
 - Invariant GNNs
 - Equivariant GNNs
- Geometric Generative Models
 - Diffusion Models