


Reasoning over Knowledge Graphs

CS224W: Machine Learning with Graphs
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<http://cs224w.stanford.edu>

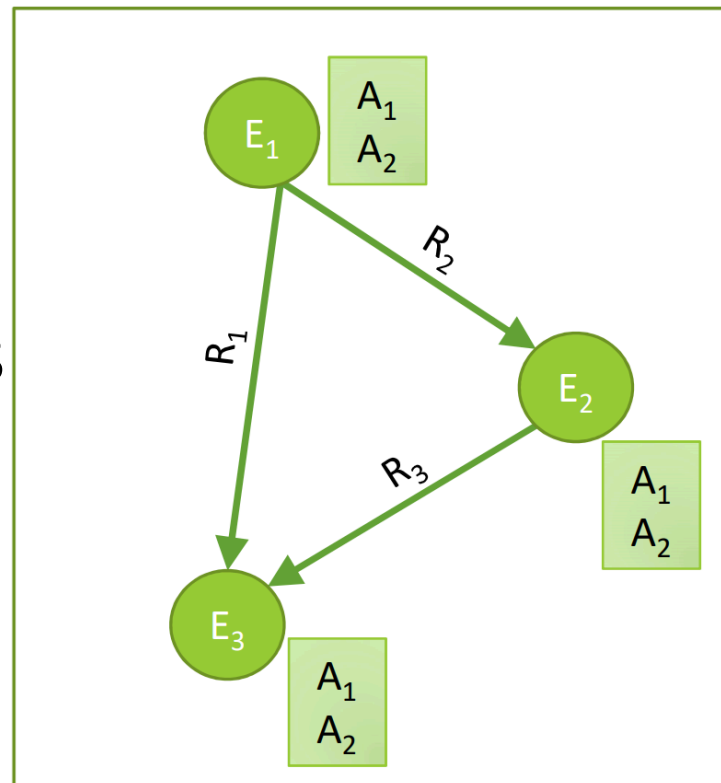


Outline of Today's Lecture

1. Introduction to Knowledge Graphs 
2. Knowledge Graph completion
3. Path Queries
4. Conjunctive Queries
5. Query2Box: Reasoning with Box Embeddings

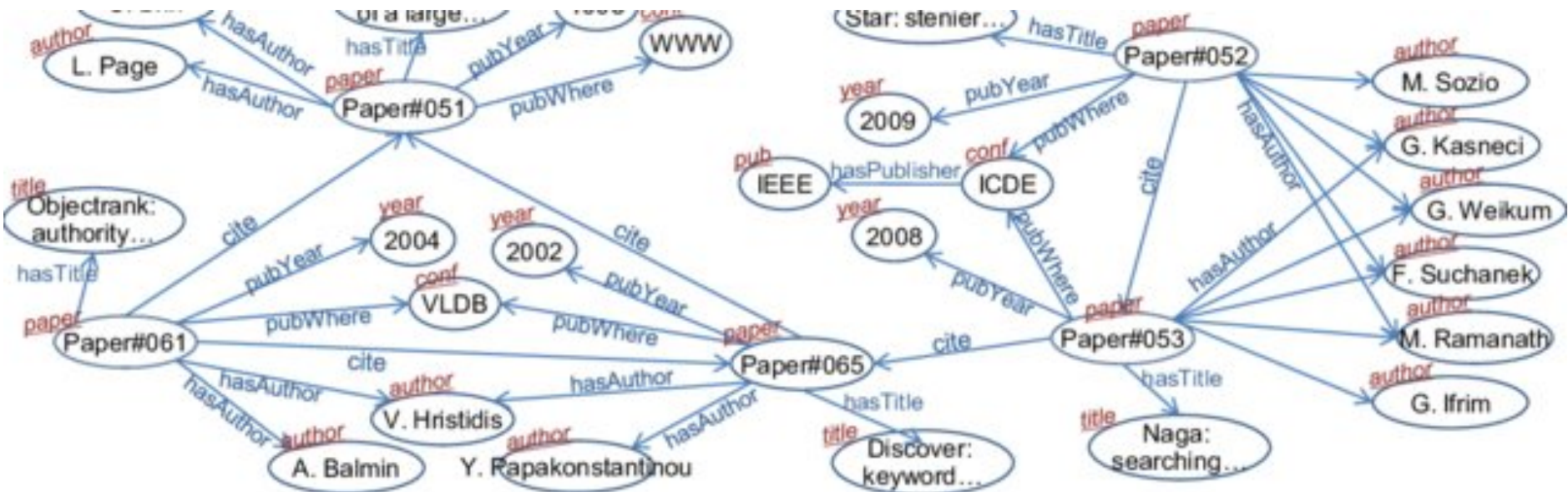
Knowledge Graphs

- Knowledge in graph form
 - Capture entities, types, and relationships
- Nodes are **entities**
- Nodes are labeled with their **types**
- Edges between two nodes capture **relationships** between entities



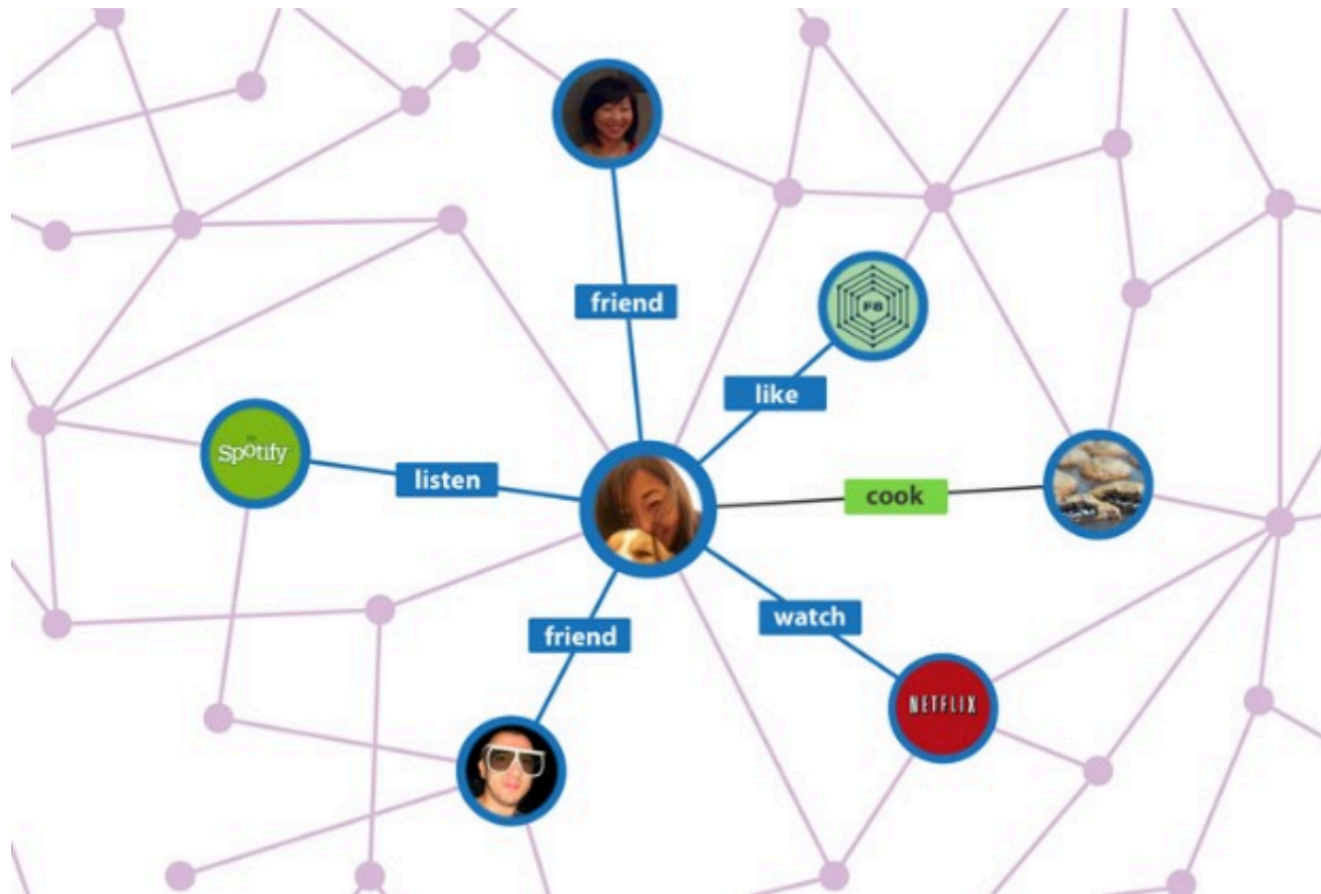
Example: Bibliographic networks

- **Node types:** paper, title, author, conference, year
- **Relation types:** pubWhere, pubYear, hasTitle, hasAuthor, cite

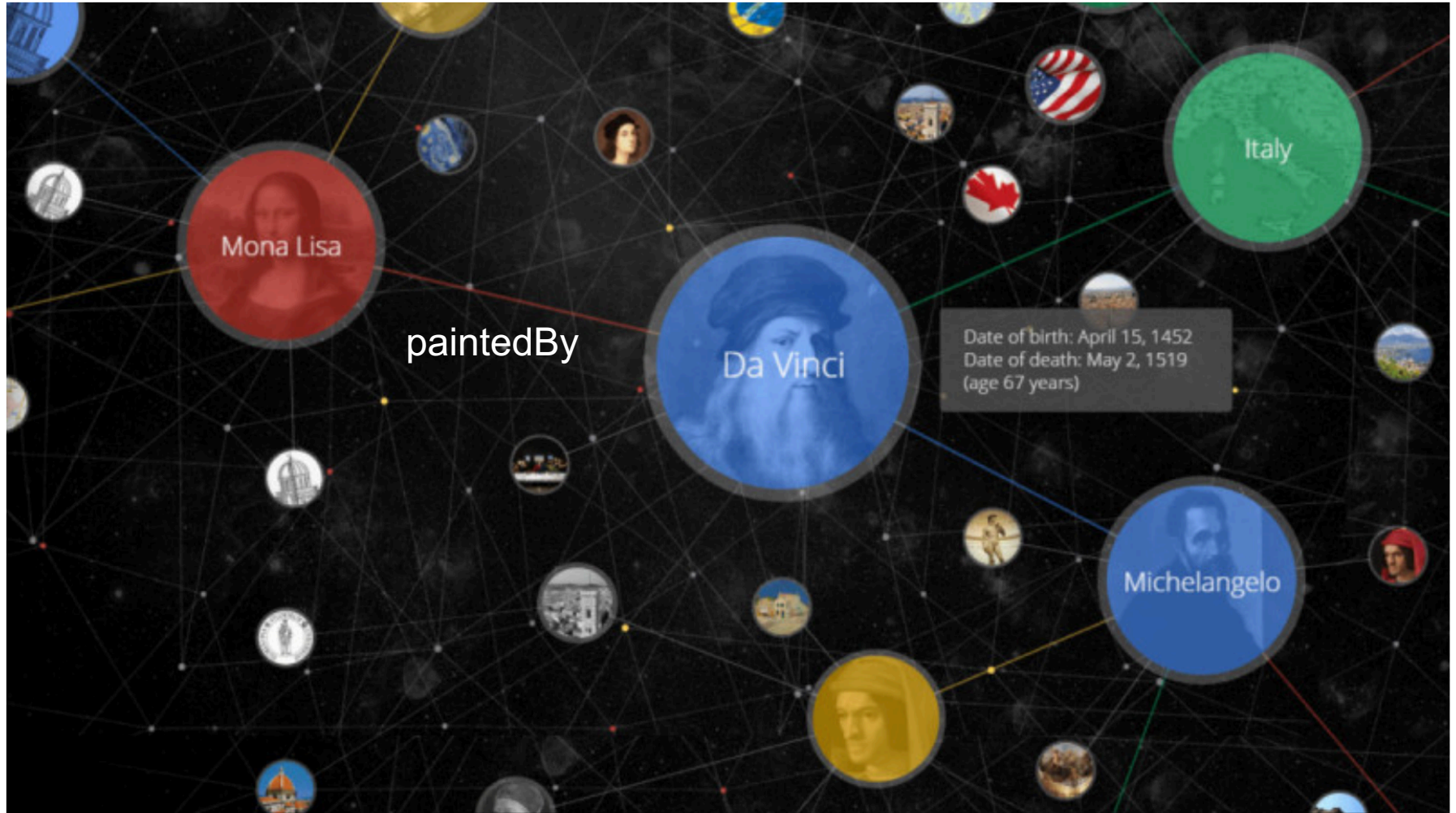


Example: Social networks

- **Node types:** account, song, post, food, channel
- **Relation types:** friend, like, cook, watch, listen

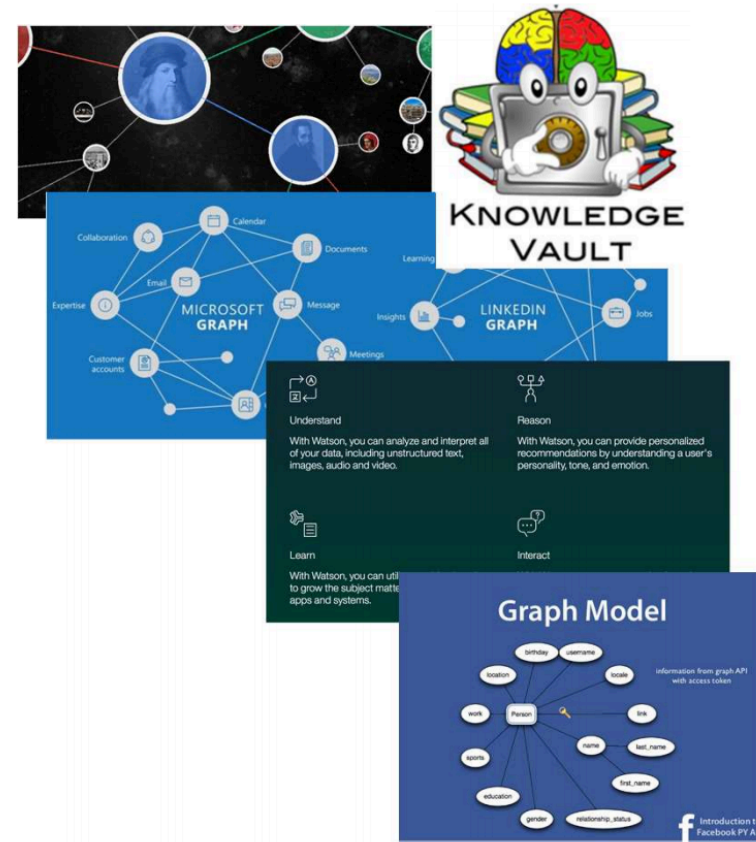


Example: Google Knowledge Graph



Knowledge Graphs in Practice



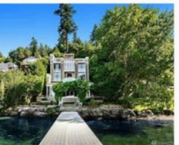



- Google Knowledge Graph
- Amazon Product Graph
- Facebook Graph API
- IBM Watson
- Microsoft Satori
- Project Hanover/Literome
- LinkedIn Knowledge Graph
- Yandex Object Answer



Applications of Knowledge Graphs

■ Serving information





Homes for sale in Bellevue >\$4M Any beds Any baths Any year Compare

| | | | | | |
|---|---|---|--|---|---|
|  <p>809 97th Ave SE, Bellevue, WA 98004 \$4,580,000 4 bed · 4.75 bath · 6,220 sq ft</p> <input type="checkbox"/> Compare |  <p>719 96th Ave SE, Bellevue, WA 98004 \$9,988,000 5 bed · 5.75 bath · 14,140 sq ft</p> <input type="checkbox"/> Compare |  <p>355 Shoreland Dr SE, Bellevue, WA 98004 \$4,988,000 5 bed · 4.75 bath · 6,500 sq ft</p> <input type="checkbox"/> Compare |  <p>12210 NE 33rd St, Bellevue, WA 98005 \$6,888,000 6 bed · 6.5 bath · 10,088 sq ft</p> <input type="checkbox"/> Compare |  <p>24 Columbia Ky, Bellevue, WA 98006 5 bed · 4 bath · 5,090 sq ft</p> <input type="checkbox"/> Compare |  <p>4648 NE 95th Ave, Bellevue, WA 98004 \$9,400,000 4 bed · 5.5 bath · 6,100 sq ft</p> <input type="checkbox"/> Compare |
|---|---|---|--|---|---|

latest films by the director of titanic

All Images Videos Maps News Shop | My saves

Latest films by the director of Titanic

| | | | | | | | |
|--|---|---|---|---|---|---|--|
|  <p>Avatar 4 Dec 20, 2024 (...)</p> |  <p>Avatar 3 Dec 17, 2021 (...)</p> |  <p>Avatar 2 Dec 18, 2020 (...)</p> |  <p>Avatar Dec 18, 2009 (...)</p> |  <p>Aliens of the Deep Jan 28, 2005 (...)</p> |  <p>Ghosts of the Abyss Mar 31, 2003 (...)</p> |  <p>Expedition: Bismarck Dec 8, 2002 (U...)</p> |  <p>Titanic Dec 19, 1997 (...)</p> |
|--|---|---|---|---|---|---|--|

Applications of Knowledge Graphs

■ Question answering and conversation agents

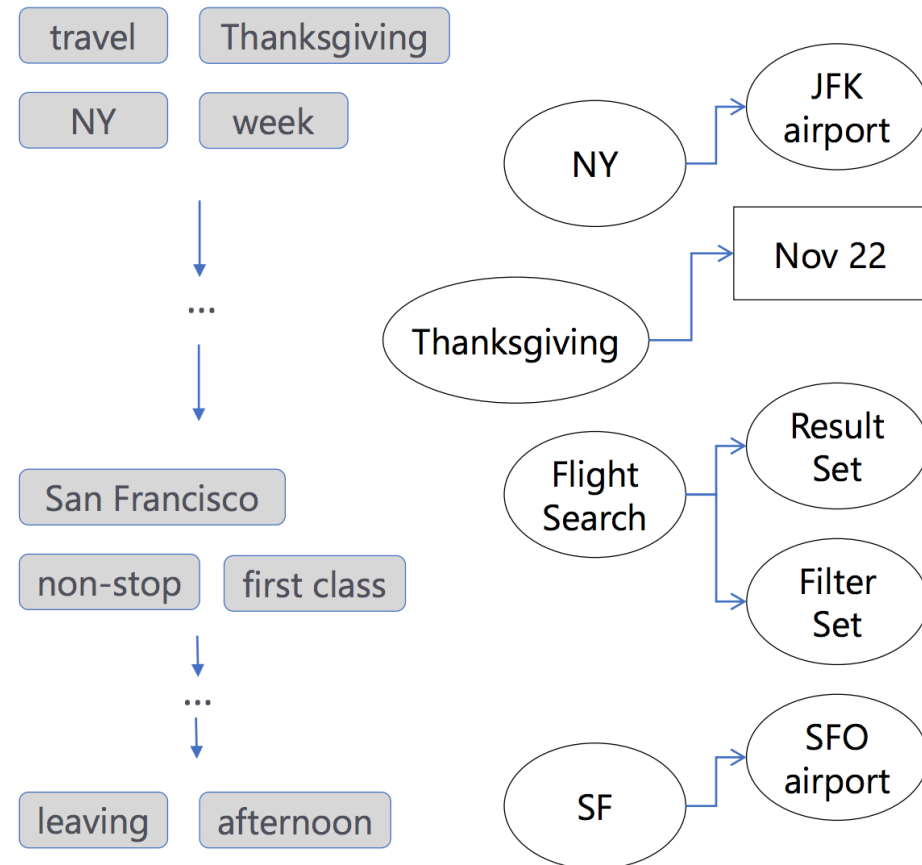
I want to travel to NY 2 days before Thanksgiving, staying for a week

Okay, booking a flight to JFK from November 20 to November 27. Where will you be flying from?


From San Francisco, and also non-stop in first class

Got it, I've found some flights for you ...

How about leaving in the afternoon



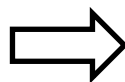
Outline

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2. Knowledge Graph completion 
3. Path Queries
4. Conjunctive Queries
5. Query2Box: Reasoning with Box Embeddings

Knowledge Graph Datasets

- Publicly available KGs:
 - FreeBase, Wikidata, Dbpedia, YAGO, NELL, etc.
- Common characteristics:
 - **Massive**: millions of nodes and edges
 - **Incomplete**: many true edges are missing

Given a massive KG,
enumerating all the
possible facts is
intractable!



Can we predict plausible
BUT missing links?

Example: Freebase



- Freebase

- ~50 million **entities**
- ~38K **relation types** ←
- ~3 billion **facts/triples**

93.8% of persons from Freebase have no place of birth and 78.5% have no nationality!

- FB15k/FB15k-237

- A **complete** subset of Freebase, used by researchers to learn KG models

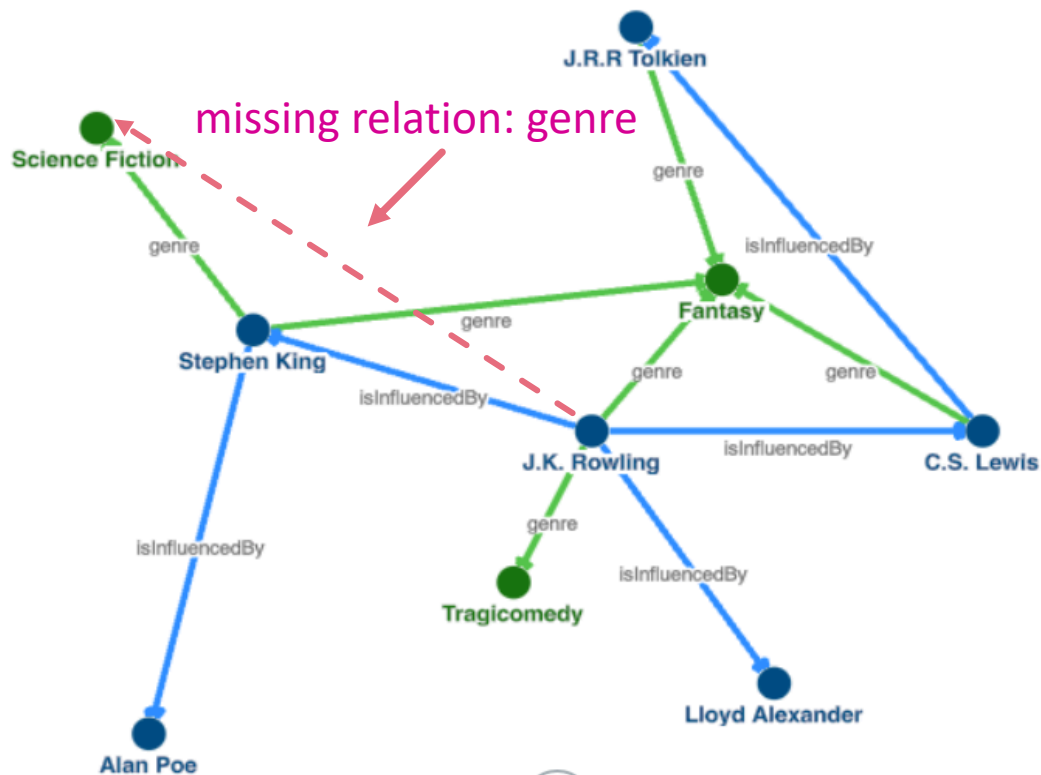
| Dataset | Entities | Relations | Total Edges |
|-----------|----------|-----------|-------------|
| FB15k | 14,951 | 1,345 | 592,213 |
| FB15k-237 | 14,505 | 237 | 310,079 |

[1] Paulheim, Heiko. "Knowledge graph refinement: A survey of approaches and evaluation methods." *Semantic web* 8.3 (2017): 489-508.

[2] Min, Bonan, et al. "Distant supervision for relation extraction with an incomplete knowledge base." *Proceedings of the 2013 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*. 2013.

KG Completion

- Given an enormous KG, can we complete the KG / predict missing relations?
 - links + type



KG Representation

- Edges in KG are represented as **triples** (h, r, t)
 - **head** (h) has **relation** (r) with **tail** (t).
- Key Idea:
 - Model entities and relations in the embedding/vector space \mathbb{R}^d .
 - Given a true triple (h, r, t) , the goal is that the **embedding of (h, r)** **should be close** to the **embedding of t** .
 - How to embed (h, r) ?
 - How to define closeness?

Relation Patterns

- **Symmetric** Relations:

$$r(h, t) \Rightarrow r(t, h) \quad \forall h, t$$

- **Example:** Family, Roommate

- **Composition** Relations:

$$r_1(x, y) \wedge r_2(y, z) \Rightarrow r_3(x, z) \quad \forall x, y, z$$

- **Example:** My mother's husband is my father.

- **1-to-N, N-to-1** relations:

$r(h, t_1), r(h, t_2), \dots, r(h, t_n)$ are all True.

- **Example:** r is "StudentsOf"

TransE

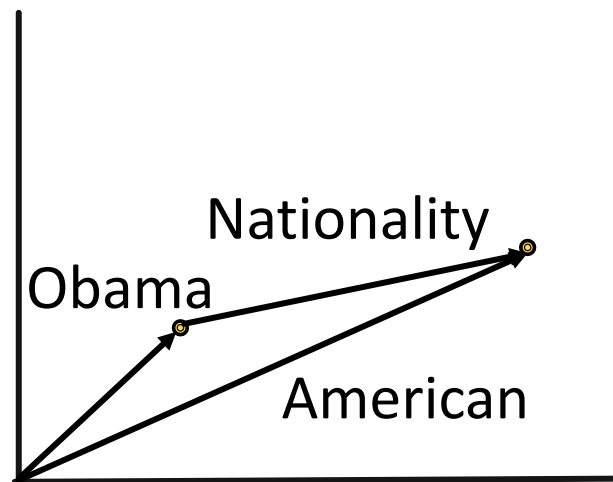
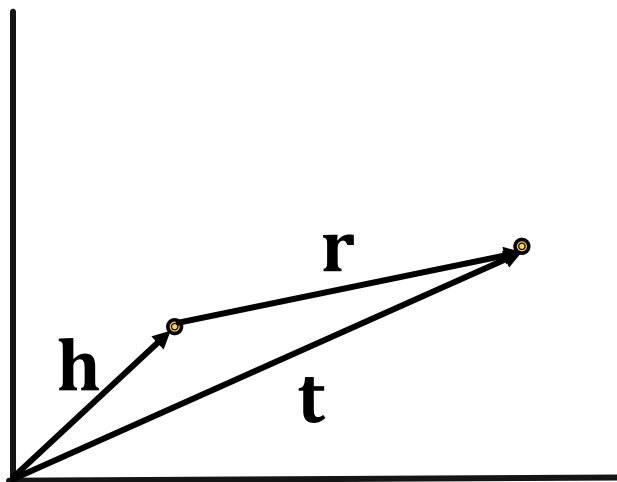
- **Translation Intuition:**

For a triple (h, r, t) , $\mathbf{h}, \mathbf{r}, \mathbf{t} \in \mathbb{R}^d$,

$$\mathbf{h} + \mathbf{r} = \mathbf{t}$$

NOTATION:
embedding
vectors will
appear in
boldface

Score function: $f_r(h, t) = ||h + r - t||$



TransE Training

- **Translation Intuition:** for a triple (h, r, t) ,
 $\mathbf{h} + \mathbf{r} = \mathbf{t}$

Max margin loss:

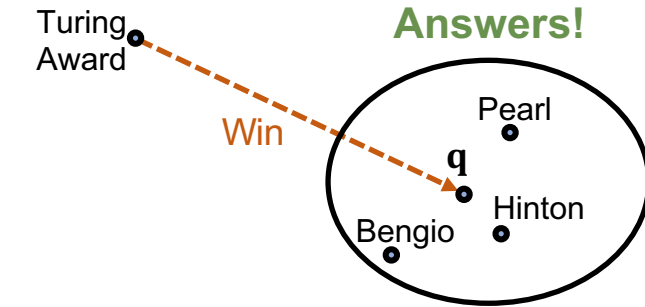
$$\mathcal{L} = \sum_{(h,r,t) \in G, (h,r,t') \notin G} [\gamma + \underbrace{f_r(h, t)}_{\text{Valid triple}} - \underbrace{f_r(h, t')}_{\text{Corrupted triple}}]_+$$

where γ is the margin, i.e., the smallest distance tolerated by the model between a valid triple and a corrupted one.

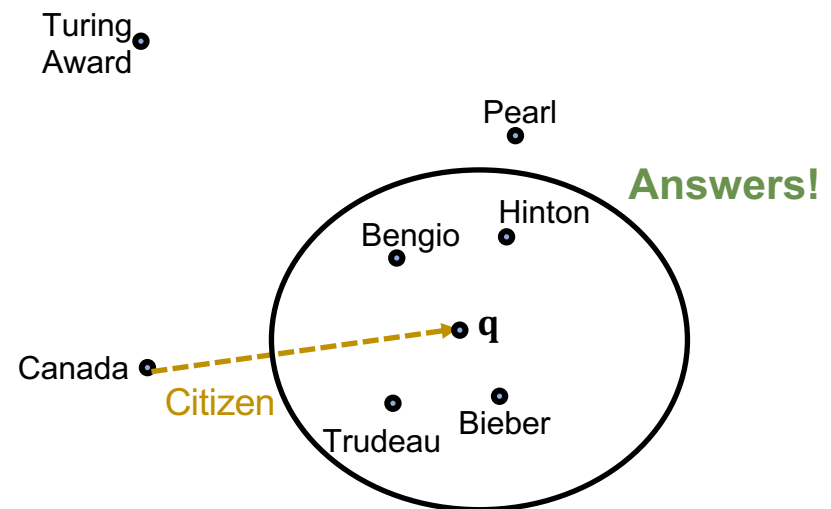
NOTE: check lecture 7 for a more in-depth discussion of TransE!

Link Prediction in a KG using TransE

- Who has won the Turing award?



- Who is a Canadian citizen?



Composition in TransE

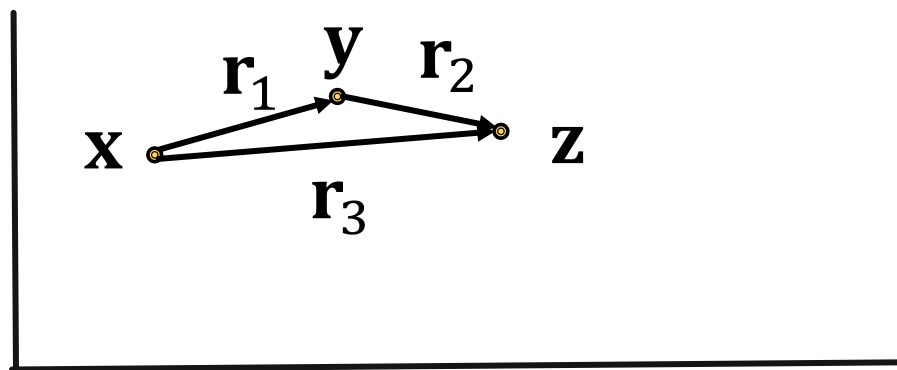
- **Composition Relations:**

$$r_1(x, y) \wedge r_2(y, z) \Rightarrow r_3(x, z) \quad \forall x, y, z$$

- **Example:** My mother's husband is my father.

- In TransE:

$$r_3 = r_1 + r_2 \quad \checkmark$$

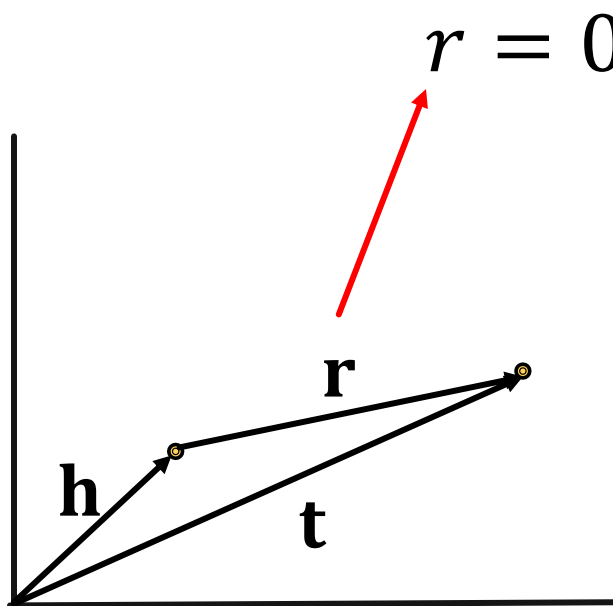


Limitation: Symmetric Relations

- **Symmetric** Relations:

$$r(h, t) \Rightarrow r(t, h) \quad \forall h, t$$

- **Example:** Family, Roommate
- In TransE:



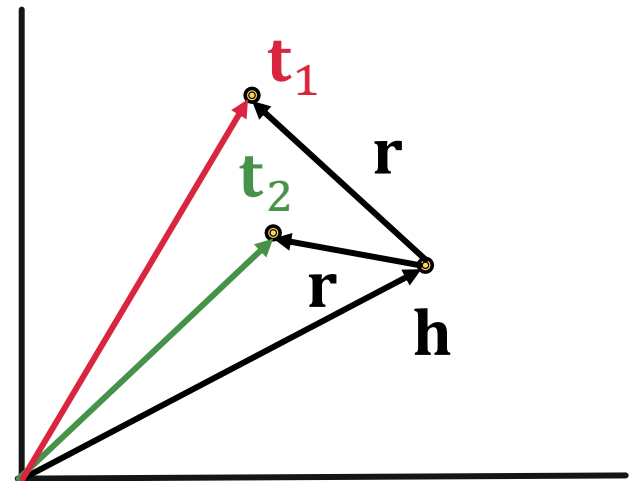
If we want TransE to handle symmetric relations r , for all h, t that satisfy $r(h, t)$, $r(t, h)$ is also True, which means $\|h + r - t\| = 0$ and $\|t + r - h\| = 0$. Then $r = 0$ and $h = t$, however h and t are two different entities and should be mapped to different locations.

Limitation: N-ary Relations

- 1-to-N, N-to-1, N-to-N relations.
- **Example:** (h, r, t_1) and (h, r, t_2) both exist in the knowledge graph, e.g., r is “StudentsOf”

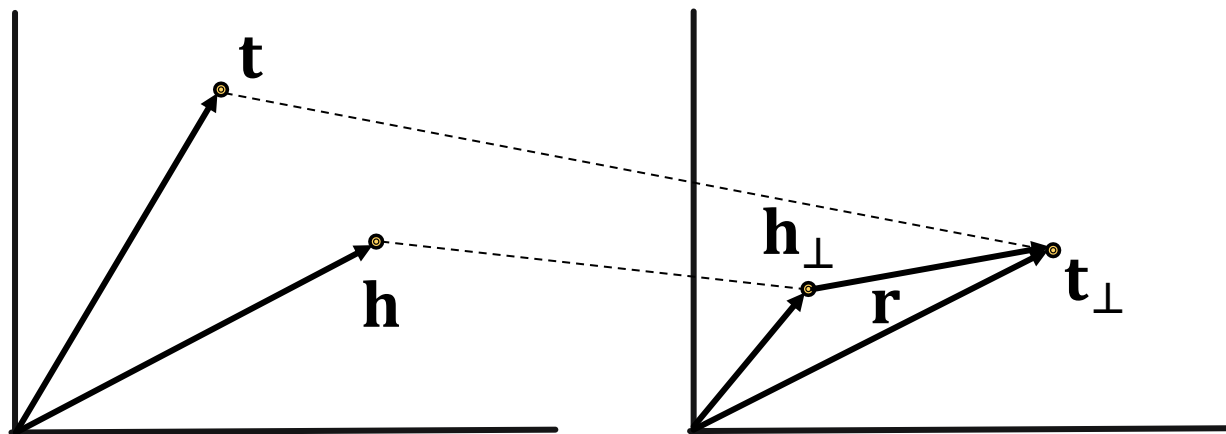
With TransE, t_1 and t_2 will map to the same vector, although they are different entities.

- $t_1 = h + r = t_2$
- $t_1 \neq t_2$ **contradictory!**



TransR

- TransR: model entities as vectors in the entity space \mathbb{R}^d and **model each relation as vector r in relation space \mathbb{R}^k** with $\mathbf{M}_r \in \mathbb{R}^{k \times d}$ as the projection matrix.
- $h_{\perp} = M_r h, t_{\perp} = M_r t$
- $f_r(h, t) = ||h_{\perp} + r - t_{\perp}||$



Lin, Yankai, et al. "Learning entity and relation embeddings for knowledge graph completion." AAAI. 2015.

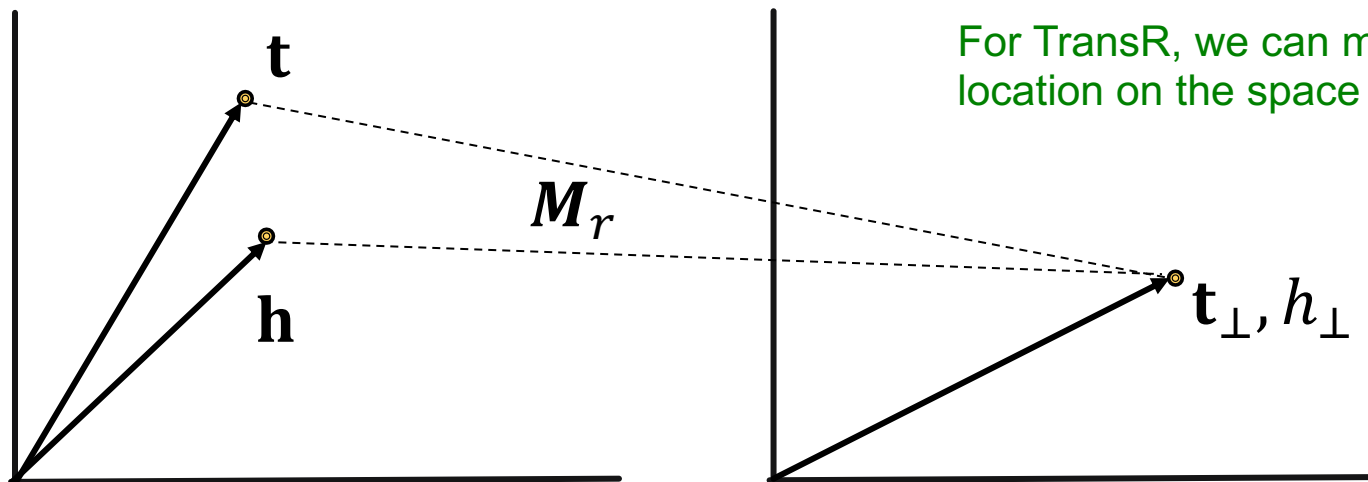
Symmetric Relations in TransR

- Symmetric Relations:

$$r(h, t) \Rightarrow r(t, h) \quad \forall h, t$$

- **Example:** Family, Roommate

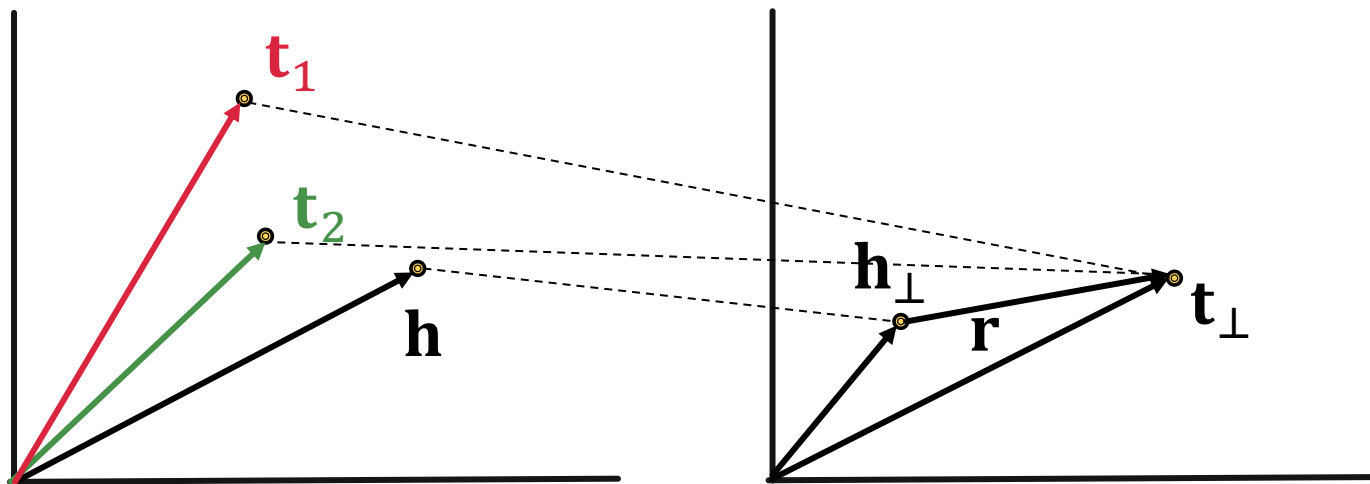
$$r = 0, \quad h_{\perp} = M_r h = M_r t = t_{\perp} \checkmark$$



N-ary Relations in TransR

- 1-to-N, N-to-1, N-to-N relations
- **Example:** If (h, r, t_1) and (h, r, t_2) exist in the knowledge graph.

We can learn M_r so that $t_{\perp} = M_r t_1 = M_r t_2$, note that t_1 does not need to be equal to t_2 !



Limitation: Composition in TransR

- **Composition Relations:**

$$r_1(x, y) \wedge r_2(y, z) \Rightarrow r_3(x, z) \quad \forall x, y, z$$

- **Example:** My mother's husband is my father.

Each relation has different space.


It is **not naturally compositional** for multiple relations! **x**

Translation-Based Embedding

| Embedding | Entity | Relation | $f_r(h, t)$ |
|-----------|-------------------------|---|-------------------------|
| TransE | $h, t \in \mathbb{R}^d$ | $r \in \mathbb{R}^d$ | $\ h + r - t\ $ |
| TransR | $h, t \in \mathbb{R}^d$ | $r \in \mathbb{R}^k, M_r \in \mathbb{R}^{k \times d}$ | $\ M_r h + r - M_r t\ $ |

| Embedding | Symmetry | Composition | One-to-many |
|-----------|----------|-------------|-------------|
| TransE | ✗ | ✓ | ✗ |
| TransR | ✓ | ✗ | ✓ |

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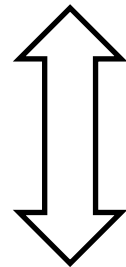
Query Types on KG

- Can we do multi-hop reasoning, i.e., **answer complex queries efficiently** on an **incomplete, massive KG**?

| Query Types | Examples |
|---------------------|--|
| One-hop Queries | Where did Hinton graduate? |
| Path Queries | Where did Turing Award winners graduate? |
| Conjunctive Queries | Where did Canadians with Turing Award graduate? |
| EPFO Queries | Where did Canadians with Turing Award or Nobel graduate? |

One-hop Queries

- We can formulate link prediction problems as answering one-hop queries.
- **Link prediction**: Is link (h, r, t) True?
- **One-hop query**: Is t an answer to query (h, r) ?



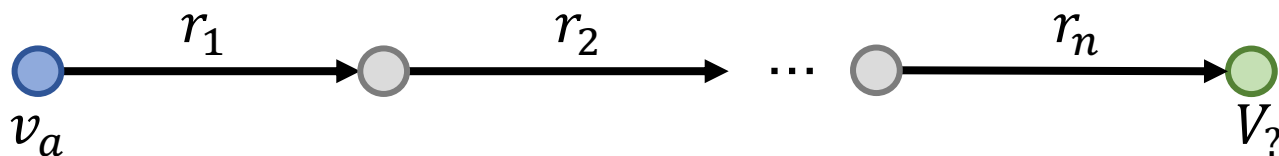
Path Queries

- Generalize one-hop queries to path queries by adding more relations on the path.
- Path queries can be represented by

$$q = (v_a, r_1, \dots, r_n)$$

v_a is a constant node, answers are denoted by $\llbracket q \rrbracket$.

Computation graph of q :

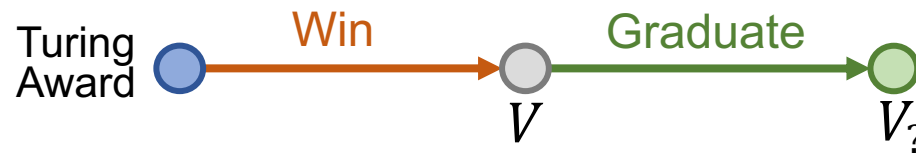


Computation graph of path queries is a chain.

Path Queries

“Where did Turing Award winners graduate?”

- v_a is “Turing Award”.
- (r_1, r_2) is (“win”, “graduate”).



Given a KG, **how to answer the query?**

Traversing Knowledge Graphs

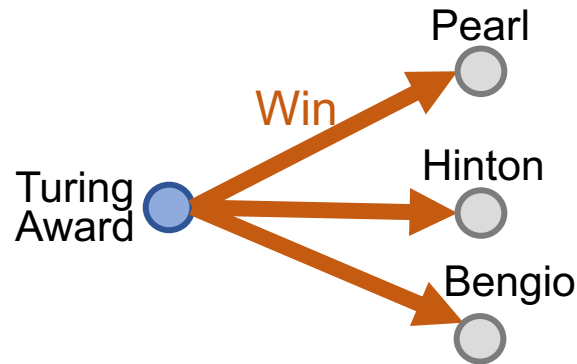
- Answer path queries by traversing the KG.
“Where did Turing Award winners graduate?”

Turing
Award 

The anchor node is Turing Award.

Traversing Knowledge Graphs

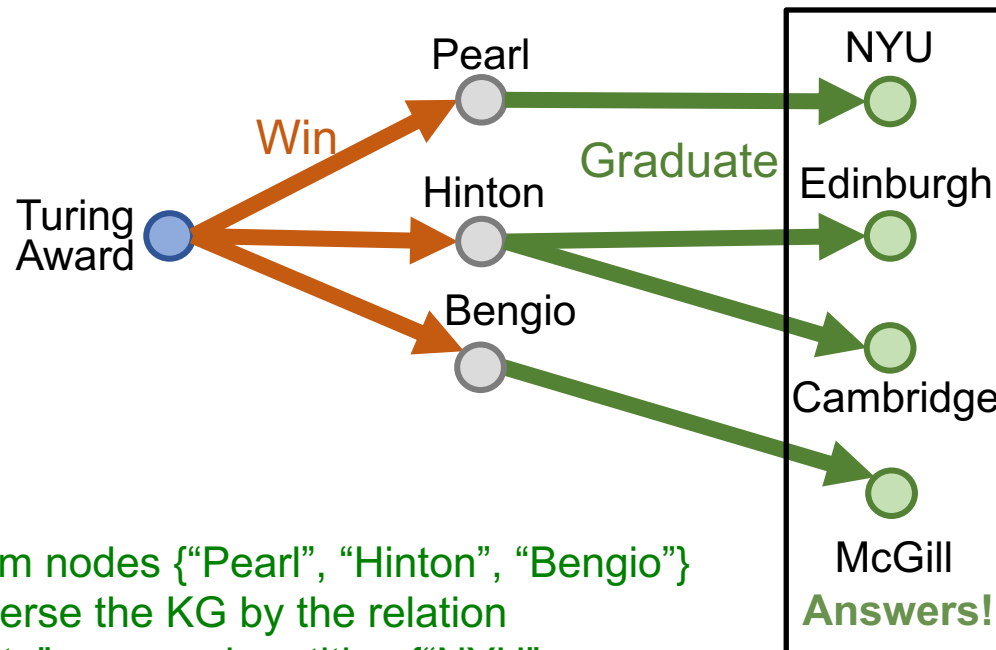
- Answer path queries by traversing the KG.
“Where did Turing Award winners graduate?”



Start from the anchor node “Turing Award” and traverse the KG by the relation “Win”, we reach entities {“Pearl”, “Hinton”, “Bengio”}.

Traversing Knowledge Graphs

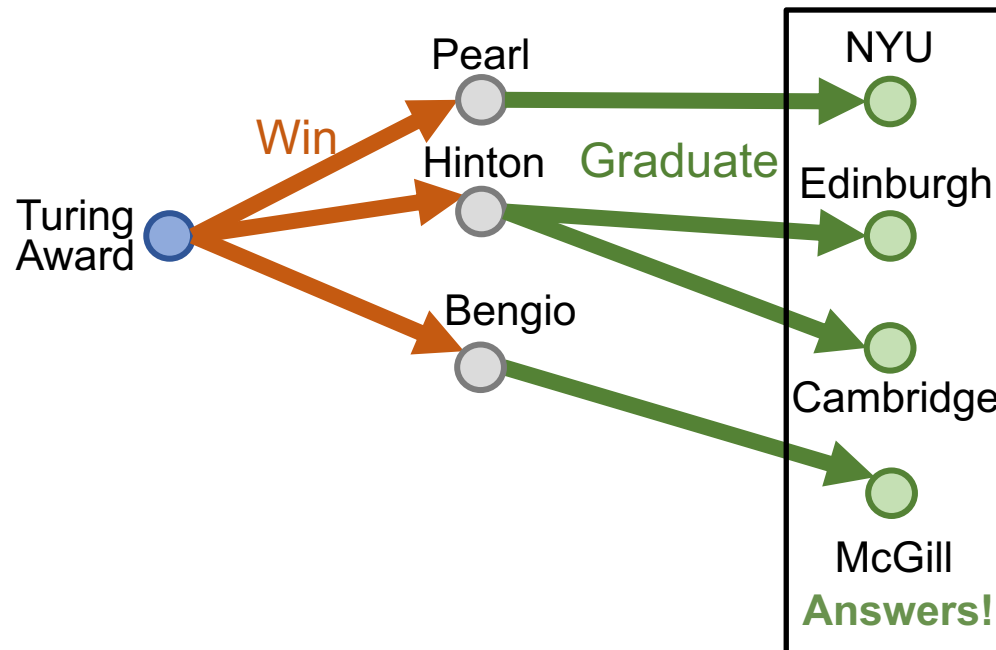
- Answer path queries by traversing the KG.
“Where did Turing Award winners graduate?”



Start from nodes {“Pearl”, “Hinton”, “Bengio”} and traverse the KG by the relation “Graduate”, we reach entities {“NYU”, “Edinburgh”, “Cambridge”, “McGill”}. These are the answers to the query!

Traversing Knowledge Graphs

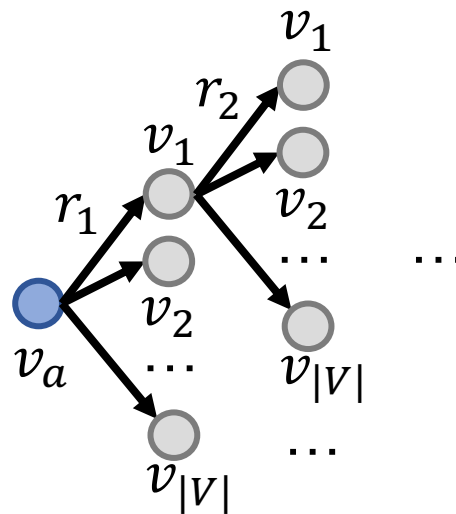
- Answer path queries by traversing the KG.
“Where did Turing Award winners graduate?”



What if KG is incomplete?

Answering Path Queries

- Can we first do link prediction and then traverse the completed (probabilistic) KG?
- **No!** The completed KG is a **dense graph!**
- **Time complexity of traversing a dense KG** with $|V|$ entities to answer (v_a, r_1, \dots, r_n) of length n is $\mathcal{O}(|V|^n)$.

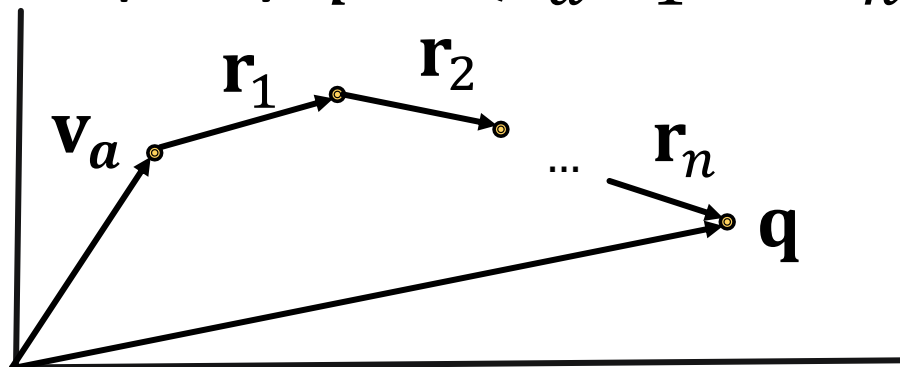


Traversing KG in Vector Space

- **Key idea: embed queries!**

- Generalize TransE to multi-hop reasoning.

Given a path query $q = (v_a, r_1, \dots, r_n)$,



$$\mathbf{q} = \mathbf{v}_a + \mathbf{r}_1 + \dots + \mathbf{r}_n$$

- Is v an answer to q ?

- Do a nearest neighbor search for all v based on $f_q(v) = \|\mathbf{q} - \mathbf{v}\|$, time complexity is $\mathcal{O}(V)$.


Traversing KG in Vector Space

- Embed path queries in vector space.

“Where did Turing Award winners graduate?”

Follow the computation graph:

Computation Graph

Turing
Award 

Embedding Space

Turing
Award 

Traversing KG in Vector Space

- Embed path queries in vector space.

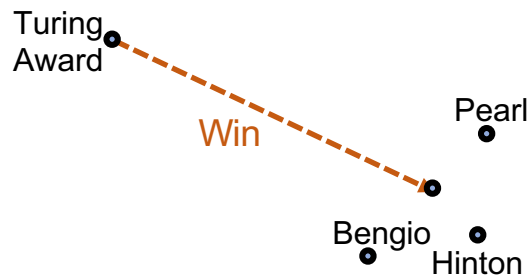
“Where did Turing Award winners graduate?”

Follow the computation graph:

Computation Graph



Embedding Space



Traversing KG in Vector Space

- Embed path queries in vector space.

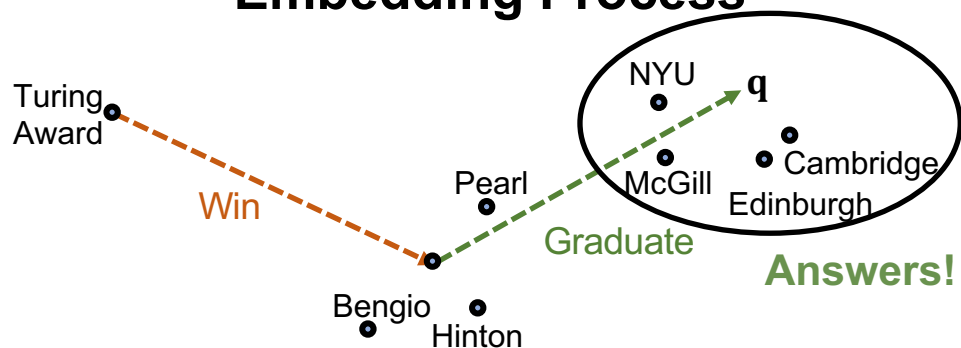
“Where did Turing Award winners graduate?”

Follow the computation graph:


Computation Graph



Embedding Process



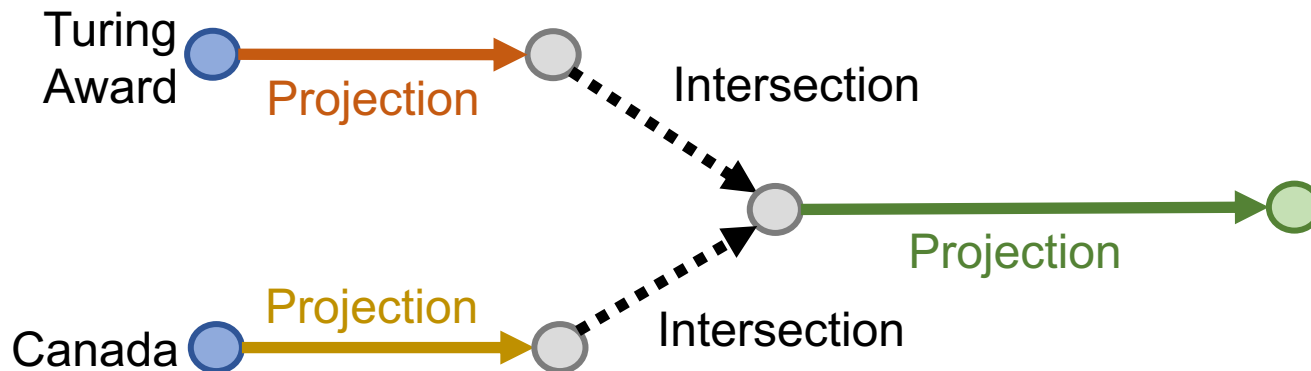
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Conjunctive Queries

- Can we answer more complex queries?
- What if we start from multiple anchor nodes?
“Where did Canadian citizens with Turing Award graduate?”

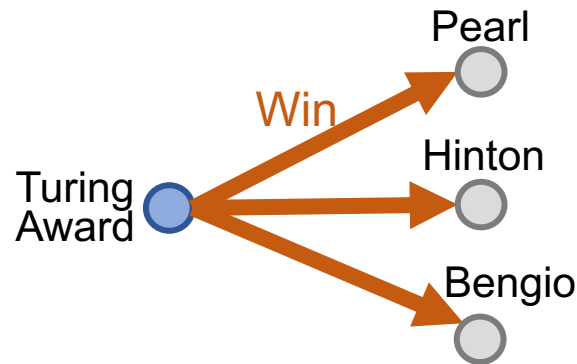
Computation graph of q :



Conjunctive Queries

- Can we answer even more complex queries?
“Where did Canadian citizens with Turing Award graduate?”

Two anchor nodes: Canada and Turing Award.

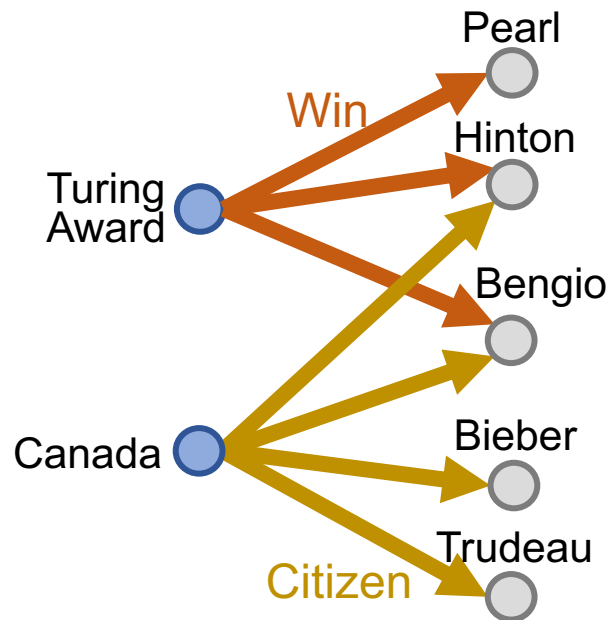


Start from the first anchor node “Turing Award”, and traverse by relation “Win”, we reach {“Pearl”, “Hinton”, “Bengio”}.

Conjunctive Queries

- Can we answer even more complex queries?
“Where did Canadian citizens with Turing Award graduate?”

Two anchor nodes: Canada and Turing Award.

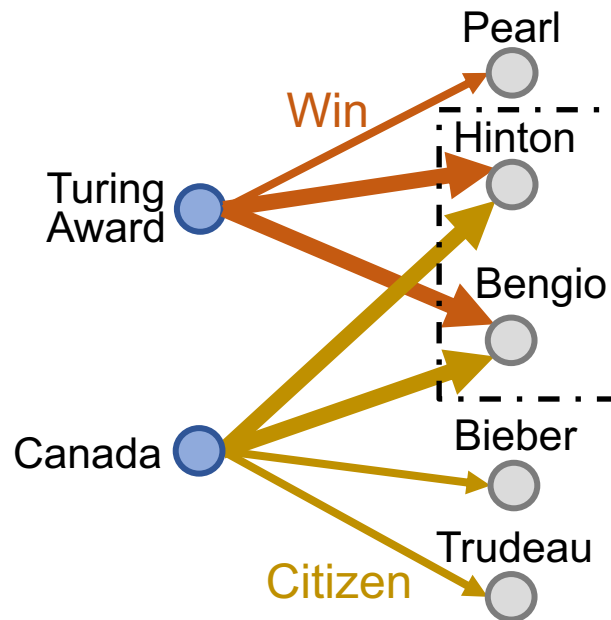


Start from the second anchor node “Canada”, and traverse by relation “citizen”, we reach { “Hinton”, “Bengio”, “Bieber”, “Trudeau” }

Conjunctive Queries

- Can we answer even more complex queries?
“Where did Canadian citizens with Turing Award graduate?”

Two anchor nodes: Canada and Turing Award.

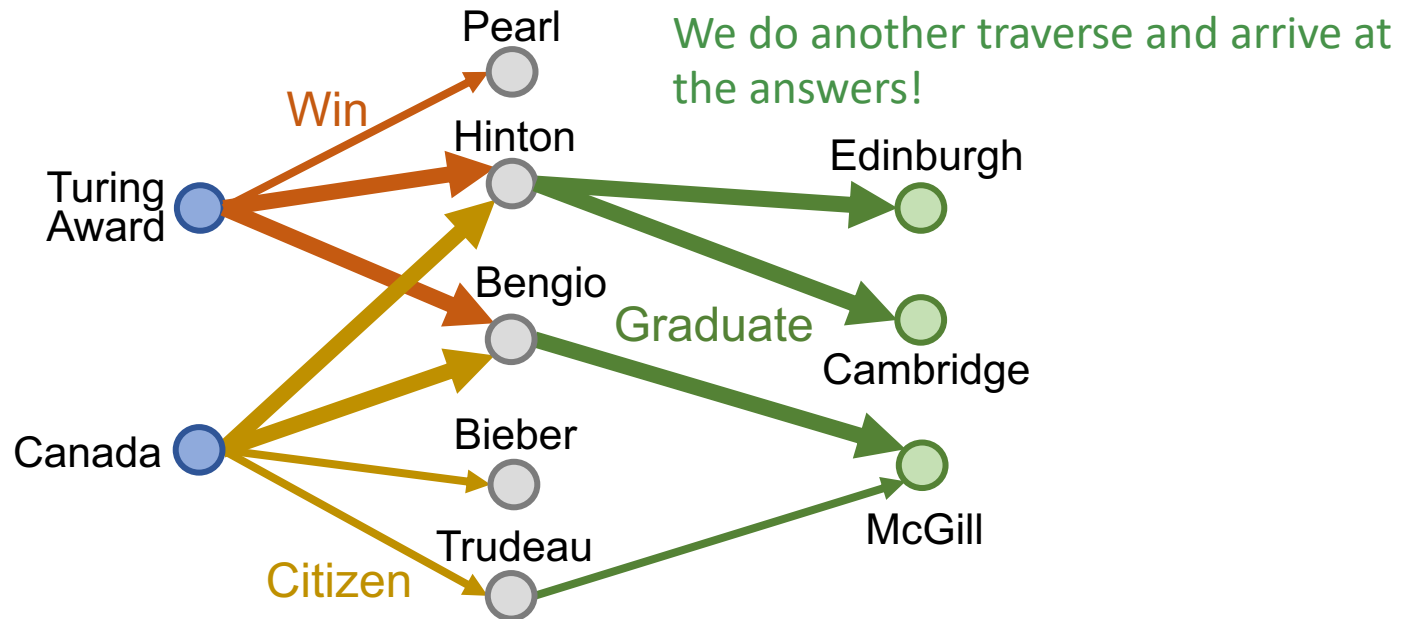


Then, we take intersection of the two sets and achieve {'Hinton', 'Bengio'}

Conjunctive Queries

- Can we answer even more complex queries?
“Where did Canadian citizens with Turing Award graduate?”

Two anchor nodes: Canada and Turing Award.



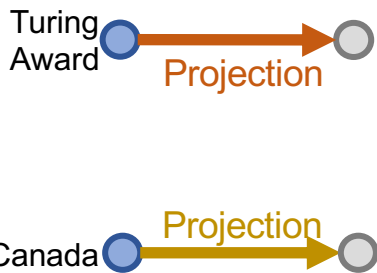
Traversing KG in Vector Space

- **Key Idea: embed queries in vector space**

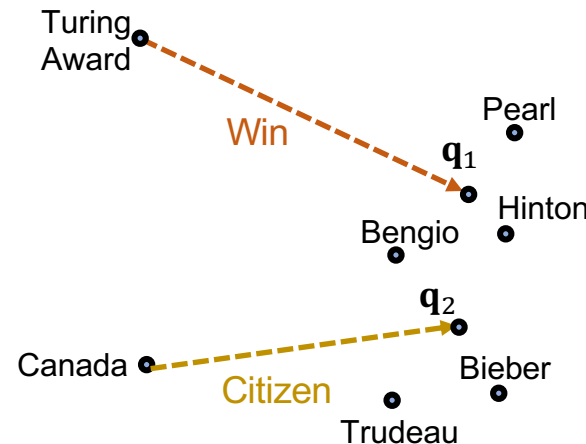
“Where did Canadian citizens with Turing Award graduate?”

Follow the computation graph:

Computation Graph



Embedding Space

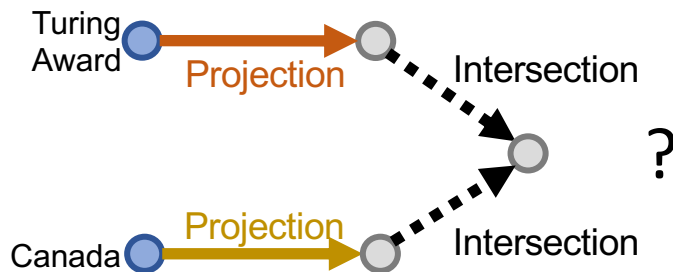


Traversing KG in Vector Space

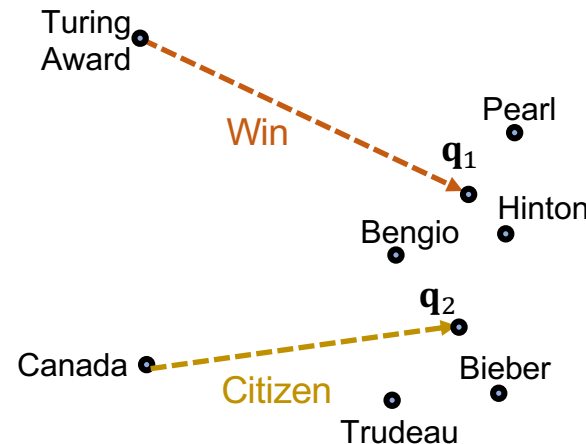
- Key Idea: embed queries in vector space
“Where did Canadian citizens with Turing Award graduate?”

Follow the computation graph:

Computation Graph



Embedding Process



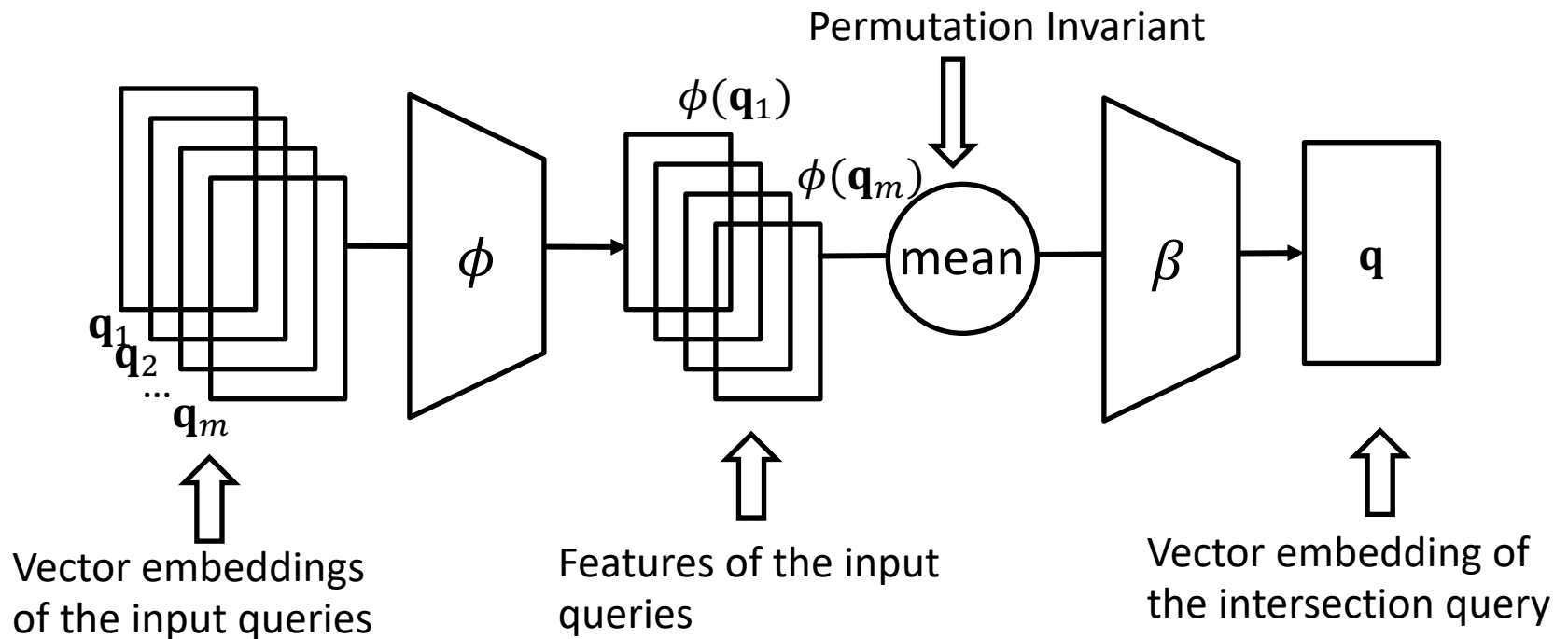
Neural Intersection Operator

- How do we take intersection of several vectors in the embedding space?
- Design a neural intersection operator \mathcal{J}
 - Input: current query embeddings $\mathbf{q}_1, \dots, \mathbf{q}_m$
 - Output: **intersection** query embedding \mathbf{q}
 - \mathcal{J} should be **permutation invariant**:
$$\mathcal{J}(\mathbf{q}_1, \dots, \mathbf{q}_m) = \mathcal{J}(\mathbf{q}_{p(1)}, \dots, \mathbf{q}_{p(m)})$$

$[p(1), \dots, p(m)]$ is any permutation of $[1, \dots, m]$

Neural Intersection Operator

- DeepSets architecture

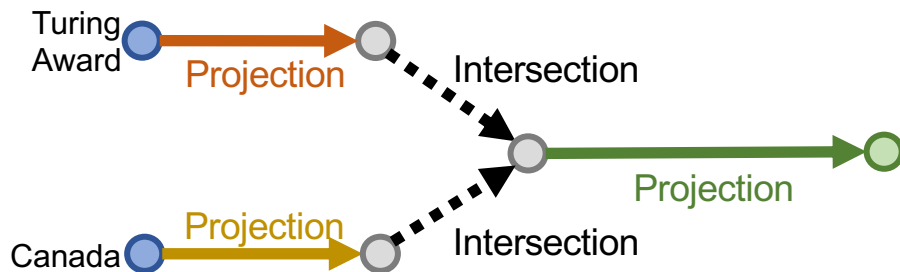


Traversing KG in Vector Space

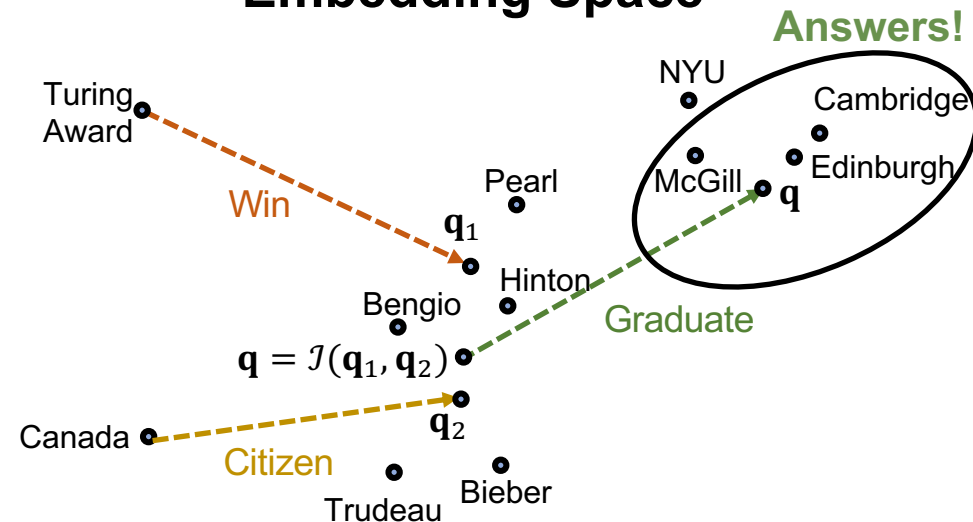
- Key Idea: embed queries in vector space
“Where did Canadian citizens with Turing Award graduate?”

Follow the computation graph:

Computation Graph



Embedding Space



Training

- Given an entity embedding \mathbf{v} and a query embedding \mathbf{q} , the distance is $f_q(v) = \|\mathbf{q} - \mathbf{v}\|$.
- **Trainable parameters:**
 - entity embeddings: $d|V|$
 - relation embeddings: $d|R|$
 - intersection operator ϕ, β : number of parameters does not depend on graph size
- **Same training strategy as TransE**

Whole Process

- **Training:**

1. Sample a query q , answer v , negative sample v' .
2. Embed the query \mathbf{q} .
3. Calculate the distance $f_q(v)$ and $f_q(v')$.
4. Optimize the loss \mathcal{L} .

- **Query evaluation:**

1. Given a test query q , embed the query \mathbf{q} .
2. For all v in KG, calculate $f_q(v)$.
3. Sort the distance and rank all v .

Limitations

- Taking the intersection between two vectors is an operation that does **not follow intuition**.
- When we traverse the KG to achieve the answers, each step produces a set of reachable entities. **How can we better model these sets?**
- Can we define a **more expressive geometry** to embed the queries?

Outline

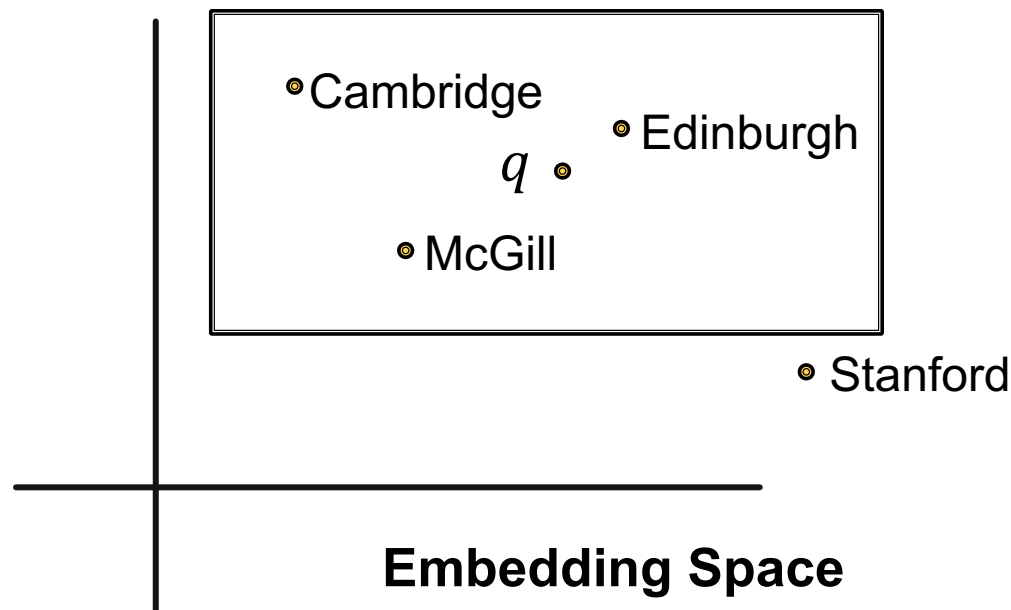
1. Introduction to Knowledge Graphs
2. Knowledge Graph completion
3. Path Queries
4. Conjunctive Queries
5. Query2Box: Reasoning with Box Embeddings



Box Embeddings

- Embed queries with **hyper-rectangles (boxes)**

$$\mathbf{q} = (\text{Center}(q), \text{Offset}(q))$$



Addressing Limitations

- Taking intersection between two vectors is an operation that does **not follow intuition**.
 - Intersection of boxes is well-defined!
- When we traverse the KG to achieve the answers, each step produces a set of reachable entities. How can we better model these sets?
 - Boxes are a **powerful abstraction**, as we can project the center and control the offset to model the set of entities enclosed in the box.

Embed with Box Embeddings

- Parameters:
 - entity embeddings: $d|V|$
 - entities are seen as zero-volume boxes
 - relation embeddings: $2d|R|$
 - augment each relation with an offset
 - intersection operator ϕ, β : number of parameters does not depend on graph size
 - New operator, inputs are boxes and output is a box

Embed with Box Embedding

- Embed queries in vector space

“Where did Canadian citizens with Turing Award graduate?”

Note that computation graph stays the same!

Follow the computation graph:

Computation Graph

Turing
Award 

Canada 

Embedding Space

Turing
Award 

Canada 

Embed with Box Embedding

- Embed queries in vector space

“Where did Canadian citizens with Turing Award graduate?”

Note that computation graph stays the same!

Follow the computation graph:

Computation Graph

Embedding Process



?

Turing
Award



Canada

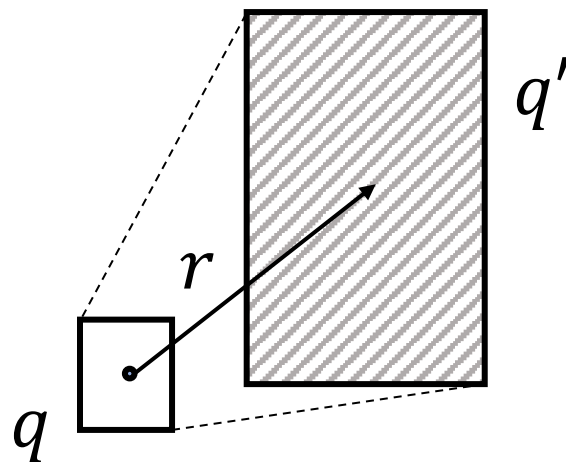
Projection Operator

- Geometric Projection Operator \mathcal{P}

- $\mathcal{P} : \text{Box} \times \text{Relation} \rightarrow \text{Box}$

$$\text{Cen}(q') = \text{Cen}(q) + \text{Cen}(r)$$

$$\text{Off}(q') = \text{Off}(q) + \text{Off}(r)$$



Embed with Box Embedding

- Embed queries in vector space

“Where did Canadian citizens with Turing Award graduate?”

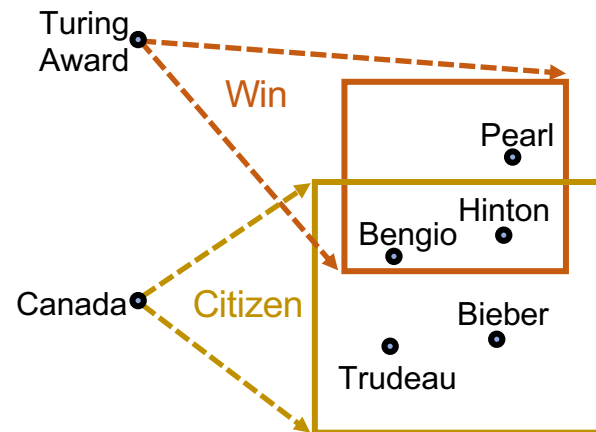
Note that computation graph stays the same!

Follow the computation graph:

Computation Graph



Embedding Space



Embed with Box Embedding

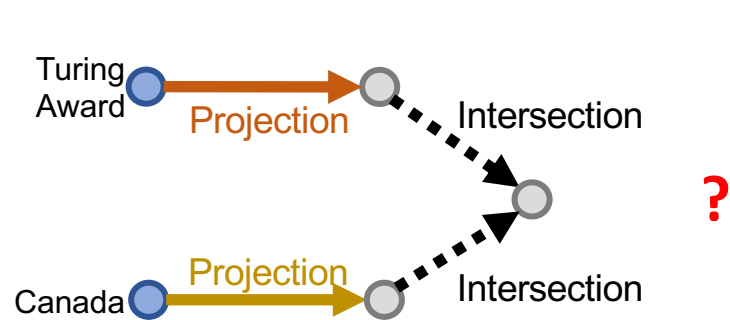
- Embed queries in vector space

“Where did Canadian citizens with Turing Award graduate?”

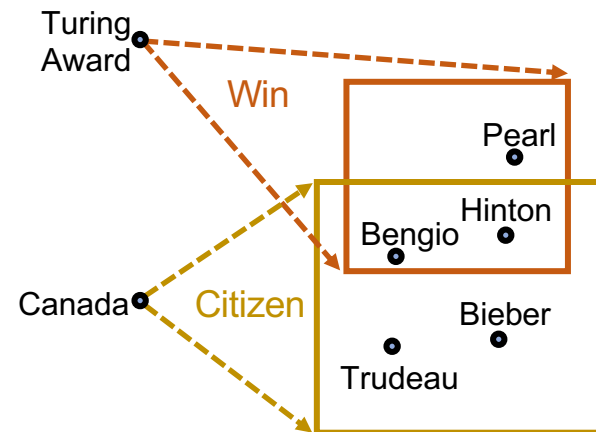
Note that computation graph stays the same!

Follow the computation graph:

Computation Graph

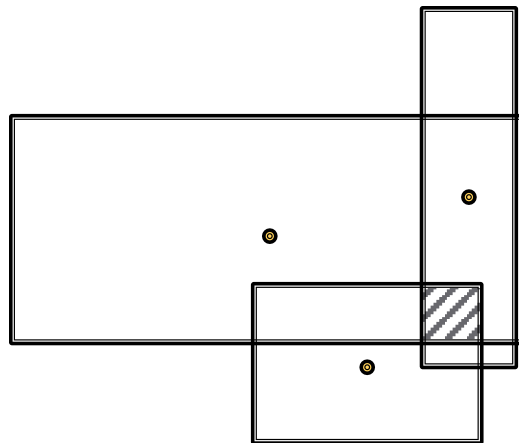


Embedding Space



Intersection Operator

- Geometric Intersection Operator \mathcal{J}
- $\mathcal{J} : \text{Box} \times \cdots \times \text{Box} \rightarrow \text{Box}$
 - The new center is a weighted average.
 - The new offset shrinks.



Intersection Operator

- Geometric Intersection Operator \mathcal{J}

- $\mathcal{J} : \text{Box} \times \cdots \times \text{Box} \rightarrow \text{Box}$

$$\text{Cen}(q_{inter}) = \sum_i w_i \odot \text{Cen}(q_i)$$

dimension-wise product

weight

$$\begin{aligned} \text{Off}(q_{inter}) &= \min(\text{Off}(q_1), \dots, \text{Off}(q_n)) \\ &\odot \sigma(\text{Deepsets}(\mathbf{q}_1, \dots, \mathbf{q}_n)) \end{aligned}$$

guarantees shrinking

Sigmoid function:
squashes output in (0,1)

Embed with Box Embedding

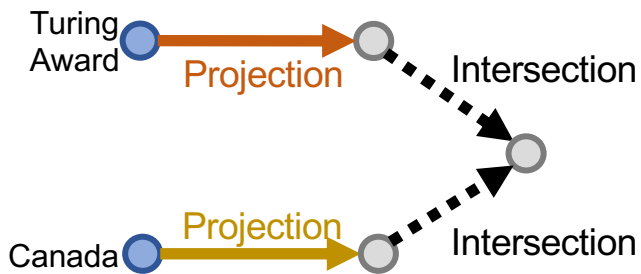
- Embed queries in vector space

“Where did Canadian citizens with Turing Award graduate?”

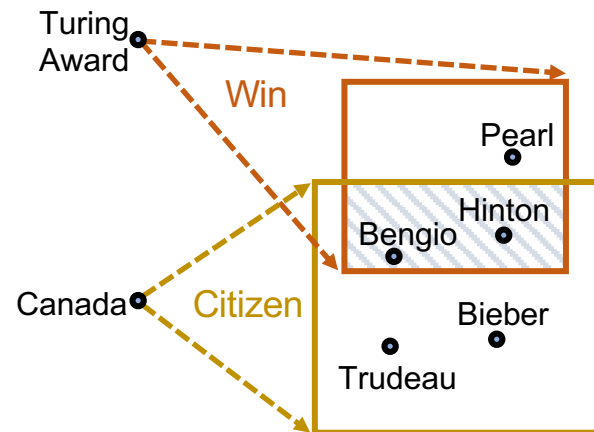
Note that computation graph stays the same!

Follow the computation graph:

Computation Graph



Embedding Space



Embed with Box Embedding

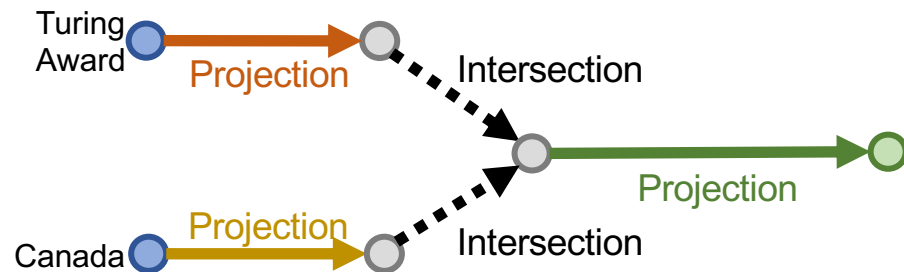
- Embed queries in vector space

“Where did Canadian citizens with Turing Award graduate?”

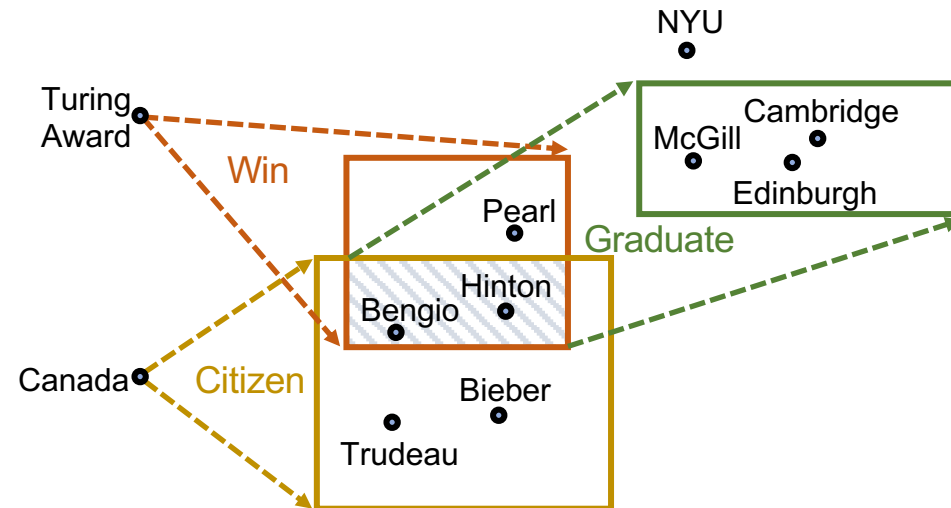
Note that computation graph stays the same!

Follow the computation graph:

Computation Graph



Embedding Space

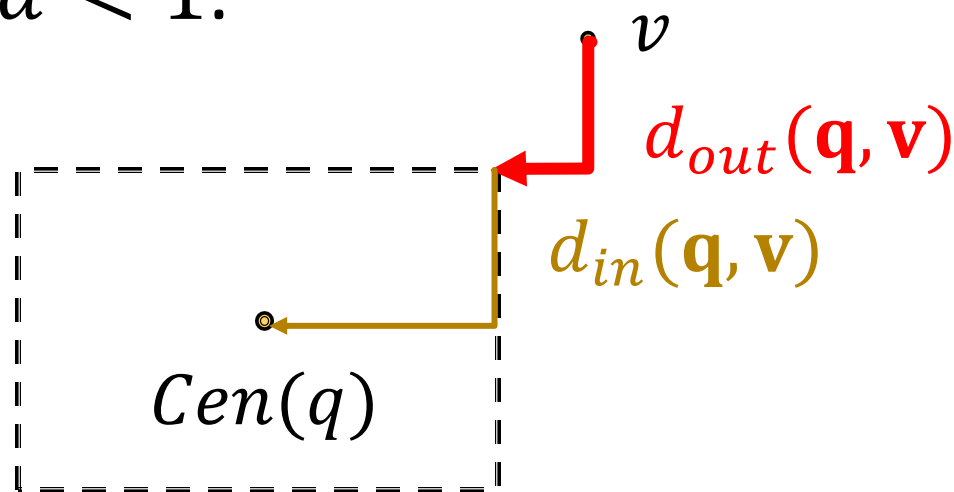


Entity-to-Box Distance

- Given a query box \mathbf{q} and entity vector \mathbf{v} ,

$$d_{box}(\mathbf{q}, \mathbf{v}) = d_{out}(\mathbf{q}, \mathbf{v}) + \alpha \cdot d_{in}(\mathbf{q}, \mathbf{v})$$

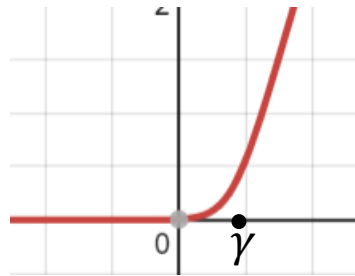
where $0 < \alpha < 1$.



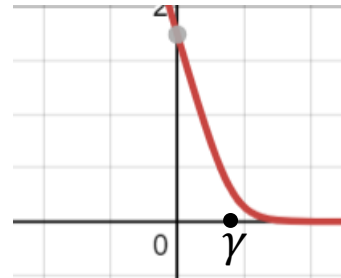
Training Query2box

- Given a set of queries and answers,

$$\mathcal{L} = -\log \sigma(\gamma - d_{box}(q, v)) - \log \sigma(d_{box}(q, v'_i) - \gamma)$$



$-\log \sigma(\gamma - d_{box}(q, v))$
minimize loss \rightarrow minimize $d_{box}(q, v)$



$-\log \sigma(d_{box}(q, v') - \gamma)$
minimize loss \rightarrow maximize $d_{box}(q, v')$

Relation Patterns

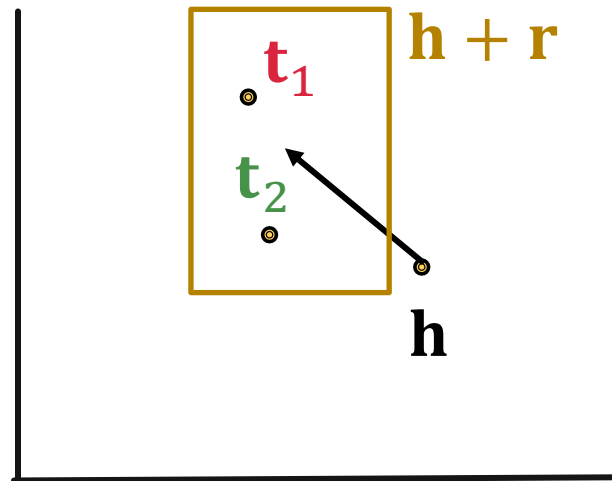
- Can query2box handle different relation patterns?

| Embedding | Symmetry | Composition | One-to-many |
|-----------|----------|-------------|-------------|
| TransE | ✗ | ✓ | ✗ |
| TransH | ✓ | ✗ | ✓ |
| Query2Box | ✓ | ✓ | ✓ |

For details please check the paper <https://openreview.net/forum?id=BJgr4kSFDS>

N-ary Relations in query2box

- 1-to-N, N-to-1, N-to-N relations.
- **Example:** Both (h, r, t_1) and (h, r, t_2) exist.
- Box Embedding can handle since t_1 and t_2 will be mapped to different locations in the box of (h, r) . ✓



Symmetric Relations in query2box

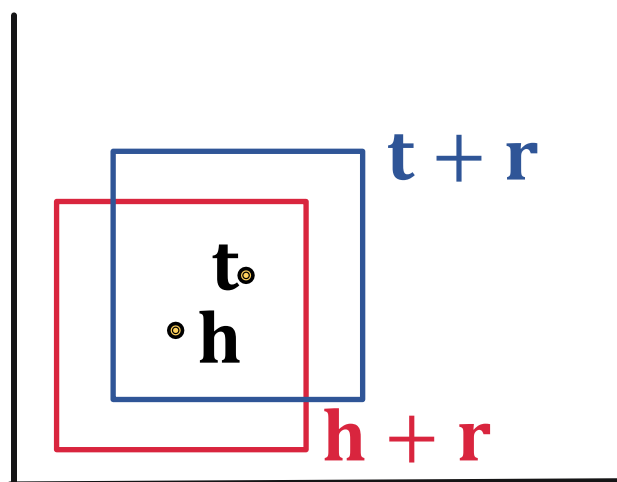
- Symmetric Relations:

$$r(h, t) \Rightarrow r(t, h) \quad \forall h, t$$

- **Example:** Family, Roommate

- Box Embedding

$$Cen(r) = 0 \quad \checkmark$$



For symmetric relations r , we could assign $Cen(r) = 0$. In this case, as long as t is in the box of (h, r) , it is guaranteed that h is in the box of (t, r) . So we have $r(h, t) \Rightarrow r(t, h)$

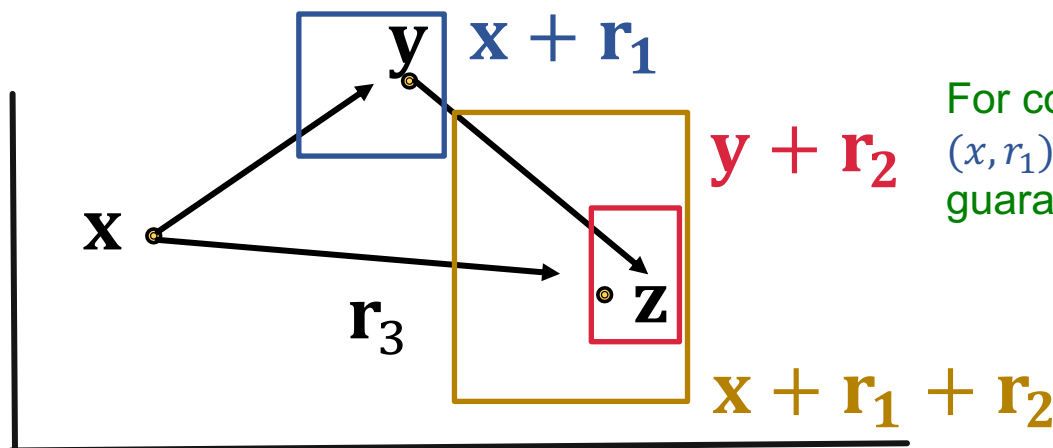
Composition Relations in query2box

- **Composition Relations:**

$$r_1(x, y) \wedge r_2(y, z) \Rightarrow r_3(x, z) \quad \forall x, y, z$$

- **Example:** My mother's husband is my father.
- **Box Embedding**

$$\mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_2 \quad \checkmark$$



For composition relations, if y is in the box of (x, r_1) and z is in the box of (y, r_2) , it is guaranteed that z is in the box of $(x, r_1 + r_2)$.

EPFO queries

- Can we embed even more complex queries?
“Where did Canadians with Turing Award *or* Nobel graduate?”
- **Conjunctive queries + disjunction** is called Existential Positive First-order (**EPFO**) queries.
- Can we also design a disjunction operator and embed EPFO queries in low-dimensional vector space? **YES!**

For details please check the paper <https://openreview.net/forum?id=BJgr4kSFDS>

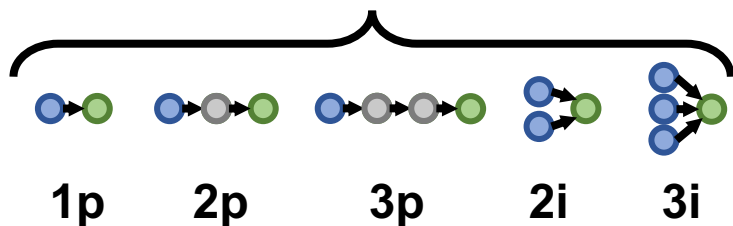
Experiments

- Datasets: FB15K, FB15K-237

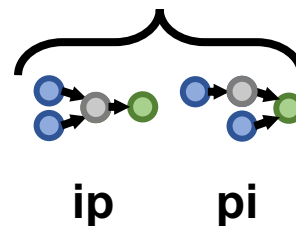
| Dataset | Entities | Relations | Training Edges | Validation Edges | Test Edges | Total Edges |
|-----------|----------|-----------|----------------|------------------|------------|-------------|
| FB15k | 14,951 | 1,345 | 483,142 | 50,000 | 59,071 | 592,213 |
| FB15k-237 | 14,505 | 237 | 272,115 | 17,526 | 20,438 | 310,079 |

- Goal: can the model discover true answers that cannot be achieved by traversing the KG?
 - Training KG: Training Edges
 - Validation KG: Training Edges + Validation Edges
 - Test KG: Training Edges + Validation Edges + Test Edges
- Queries:

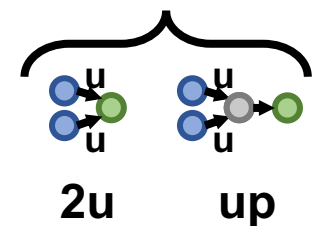
Training Conjunctive Queries



Unseen Conjunctive Queries

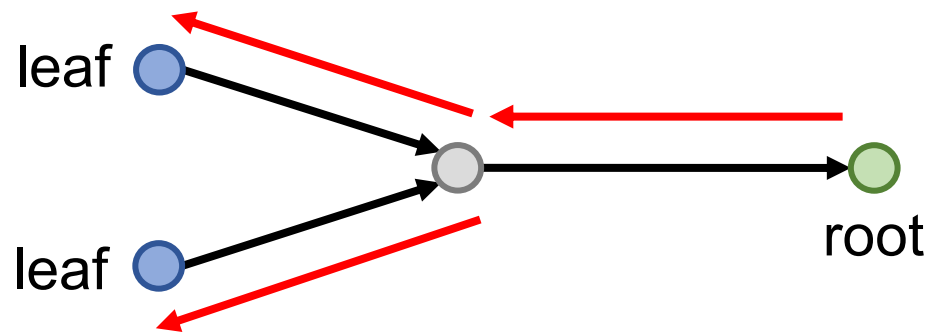


Union Queries

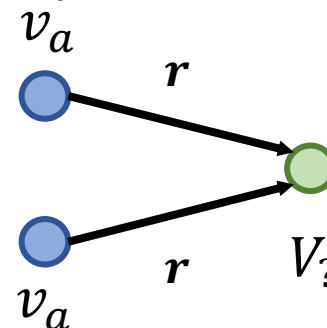
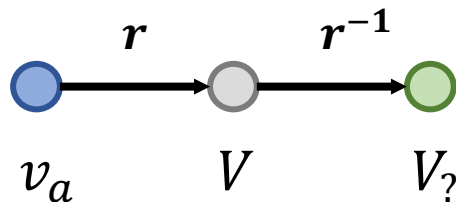


Query Generation

- Given a query structure, use pre-order traversal (traverse from root to leaves) to assign an entity/relation for every node/edge.

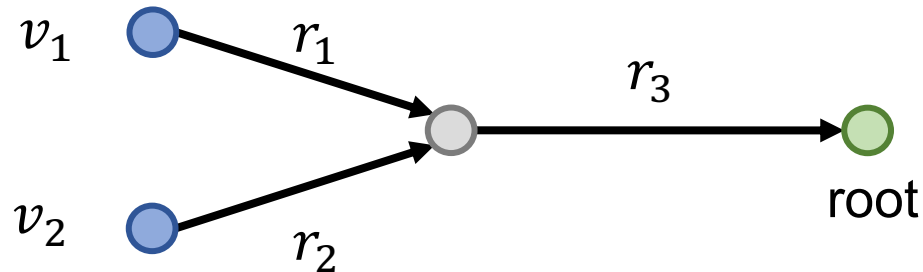


- We explicitly rule out degenerated queries.



Query Generation

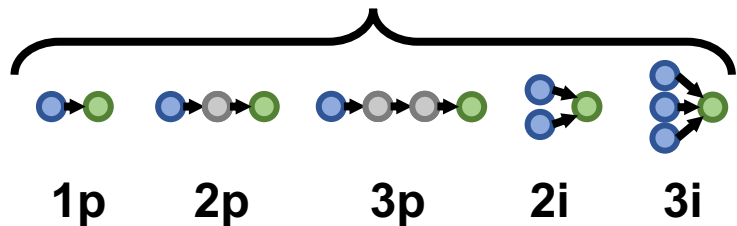
- After instantiation, run post-order traversal (traverse from leaves v_1, v_2 to root) to achieve all answers.



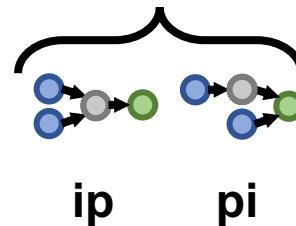
- For test queries, we guarantee that they cannot be fully answered on training/validation KG.

Query Statistics

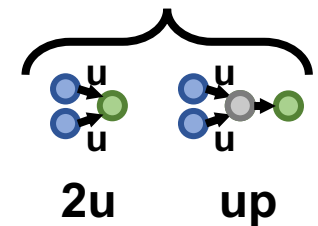
Training Conjunctive Queries



Unseen Conjunctive Queries



Union Queries



| Queries | Training | | Validation | | Test | |
|-----------|----------|---------|------------|--------|--------|--------|
| Dataset | 1p | others | 1p | others | 1p | others |
| FB15k | 273,710 | 273,710 | 59,097 | 8,000 | 67,016 | 8,000 |
| FB15k-237 | 149,689 | 149,689 | 20,101 | 5,000 | 22,812 | 5,000 |

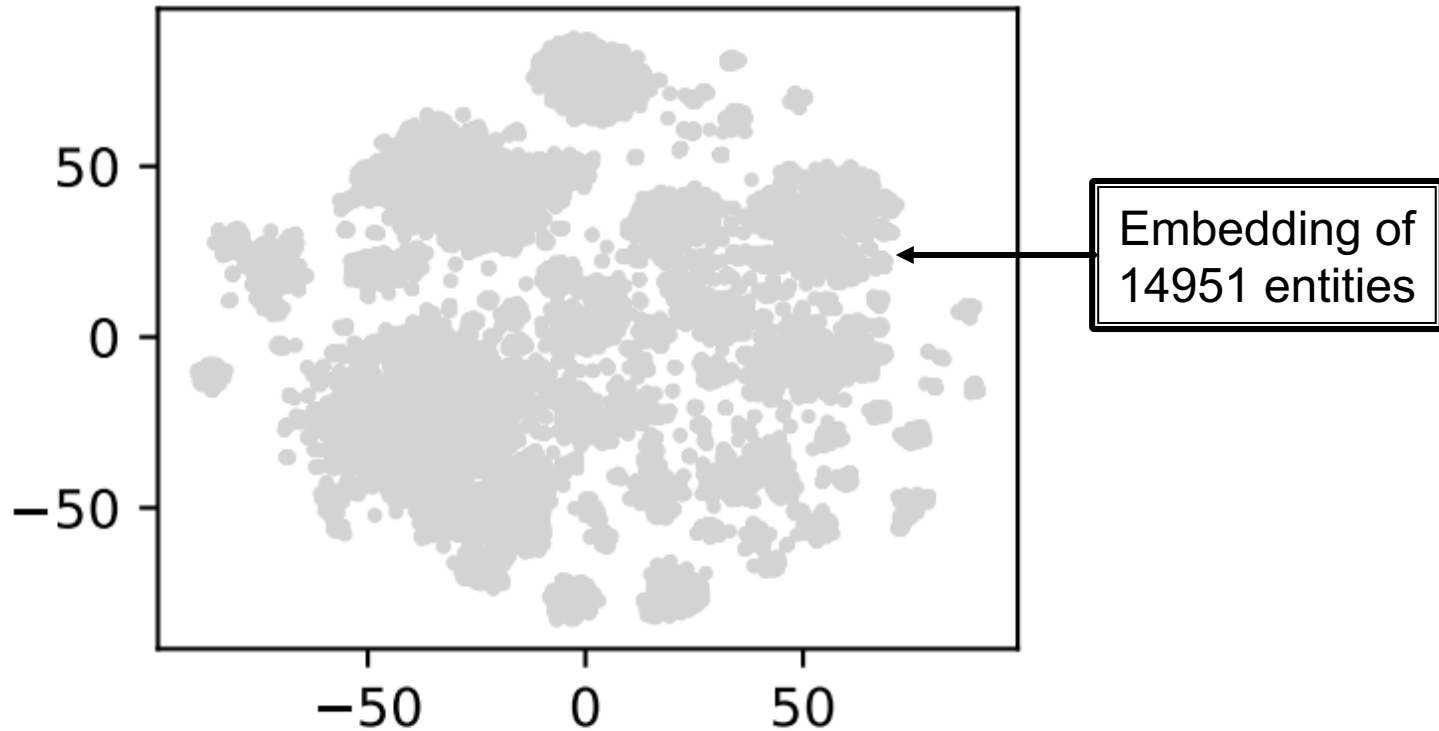
Visualization

- What does query2box actually learn?

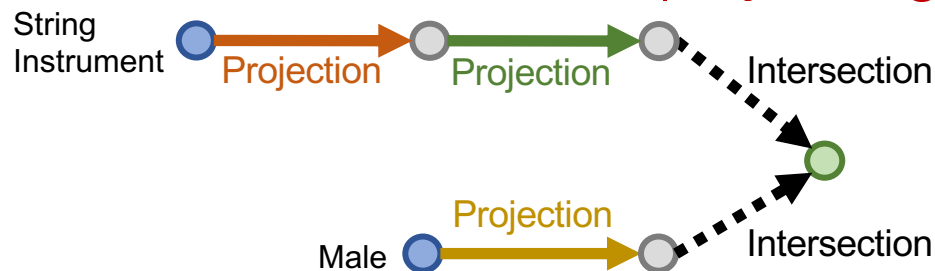
Example: *“List male instrumentalists who play string instruments”*

- We use T-SNE to reduce the embedding space to a 2-dimensional space, in order to **visualize the query results**

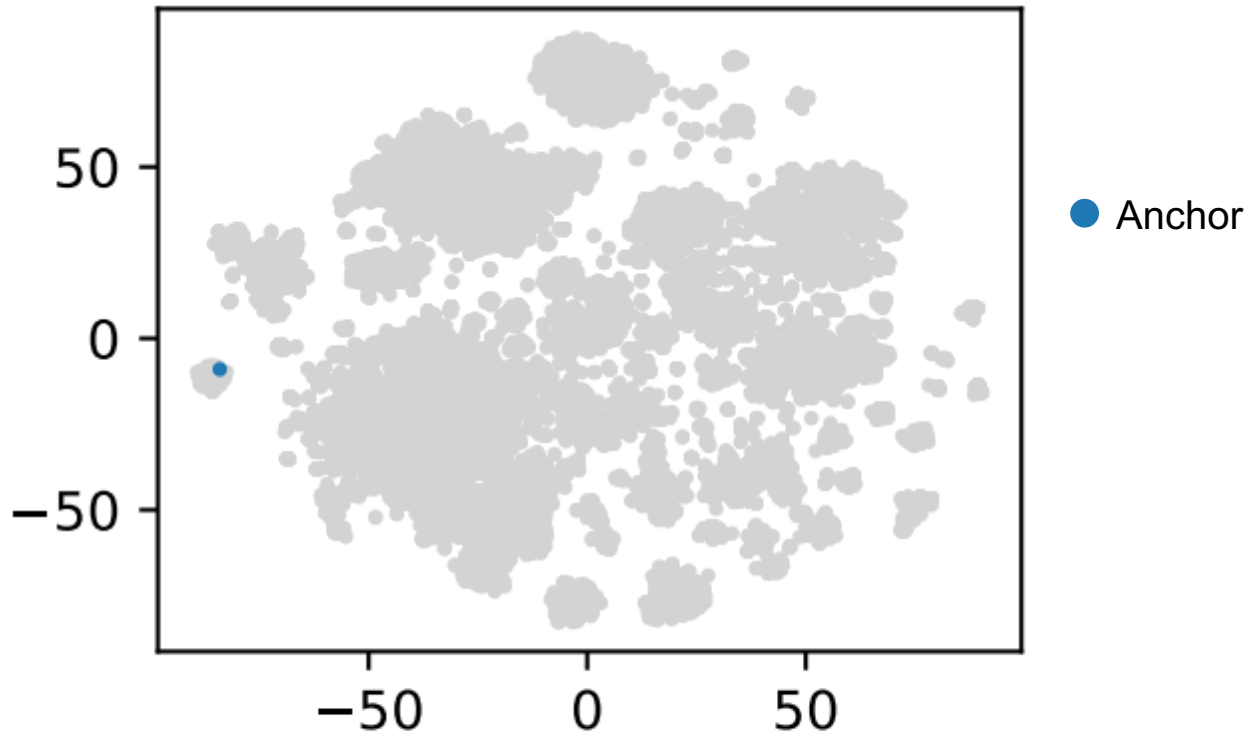
Embedding Space



“List male instrumentalists who play string instruments”



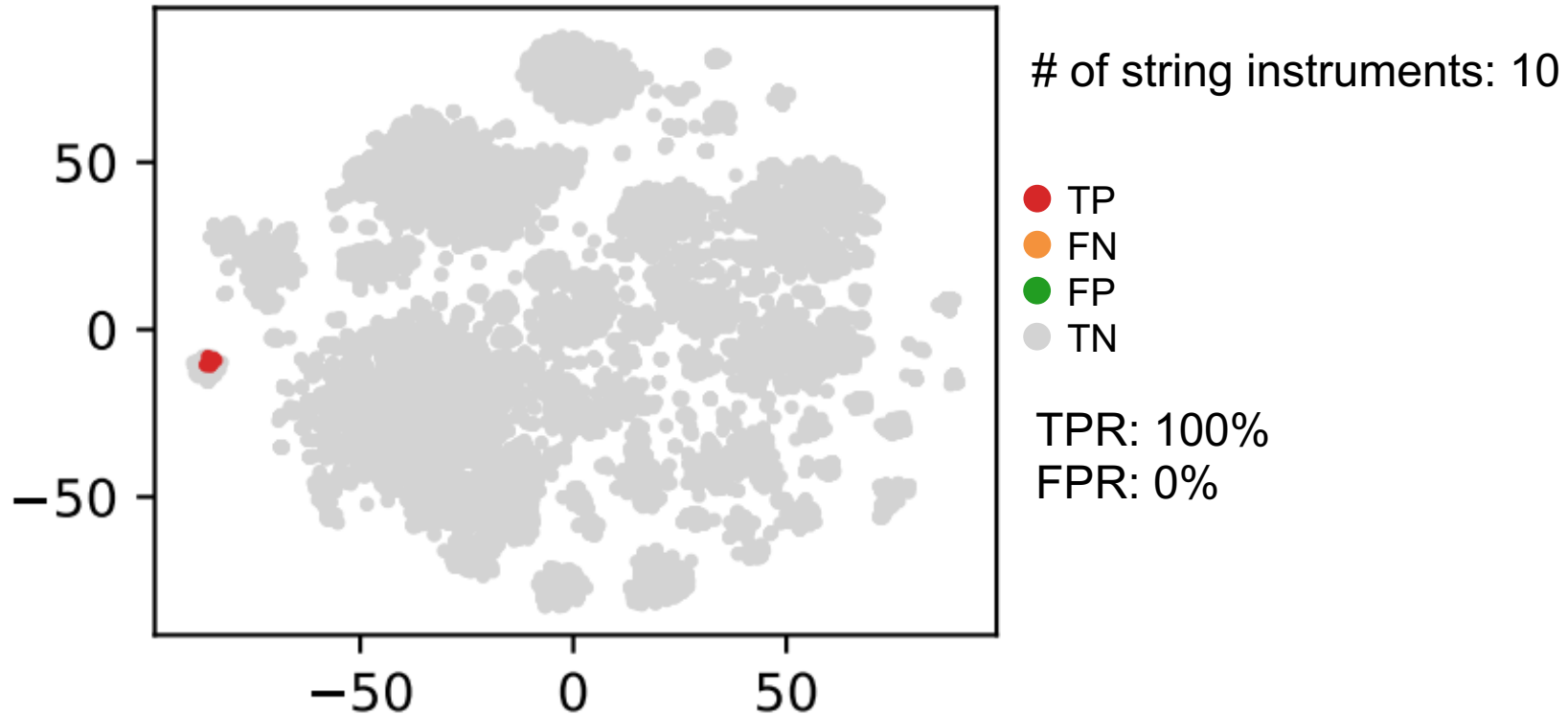
Embedding Space



“List male instrumentalists who play string instruments”

String
Instrument 

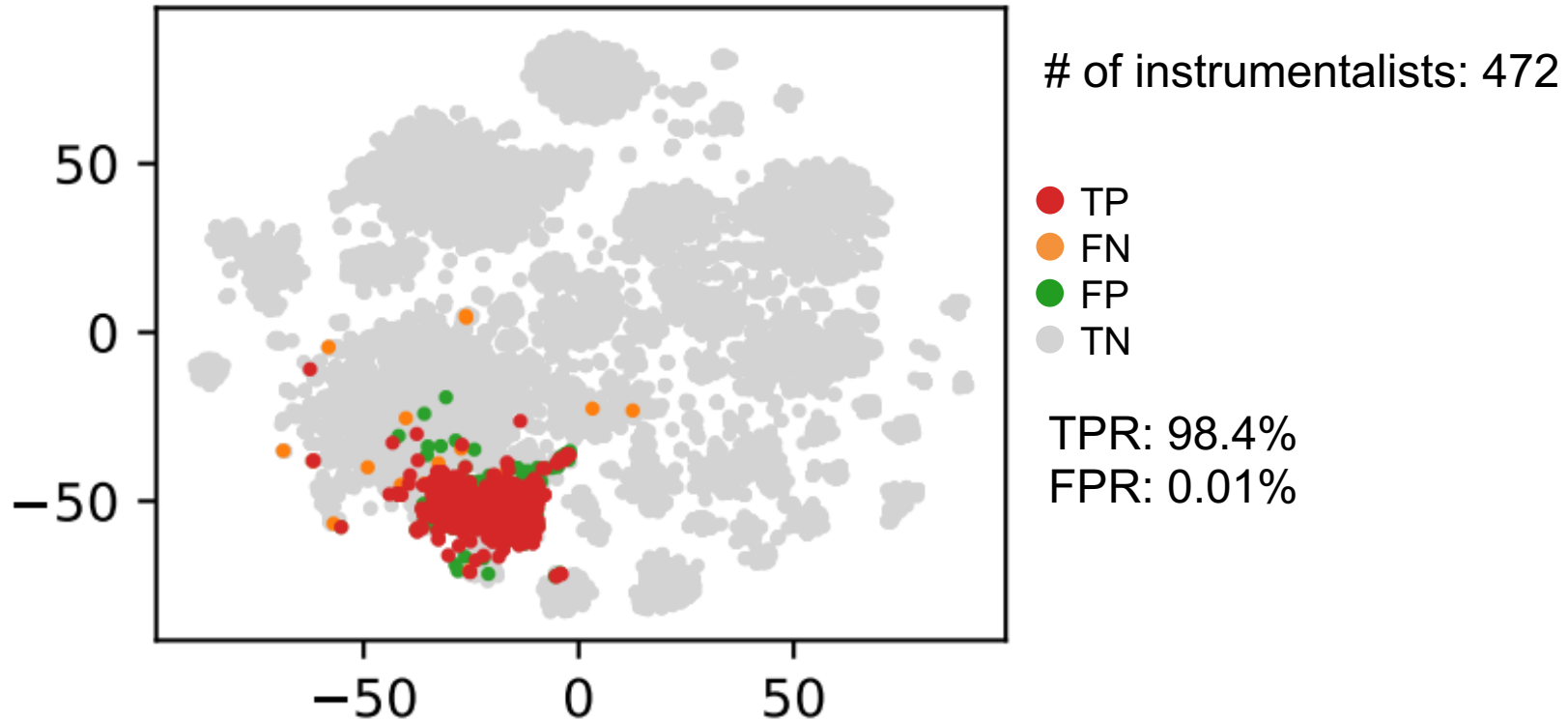
Embedding Space



“List male instrumentalists who play string instruments”



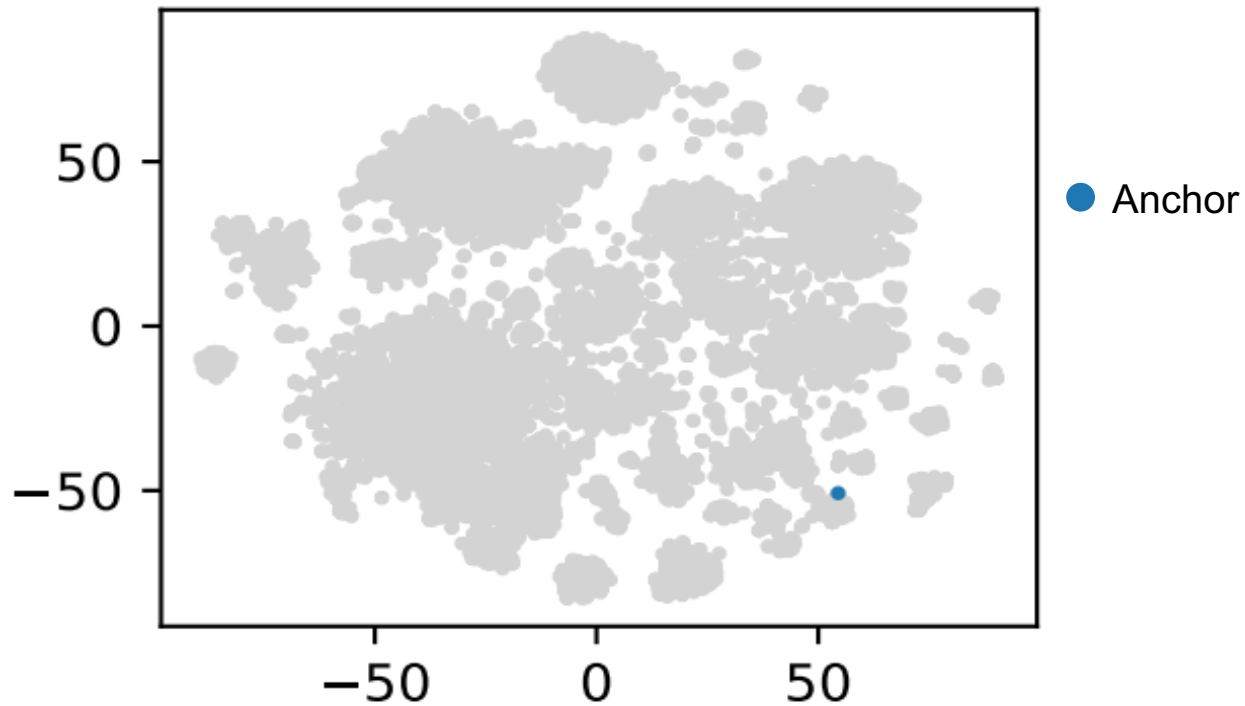
Embedding Space



“List male instrumentalists who play string instruments”



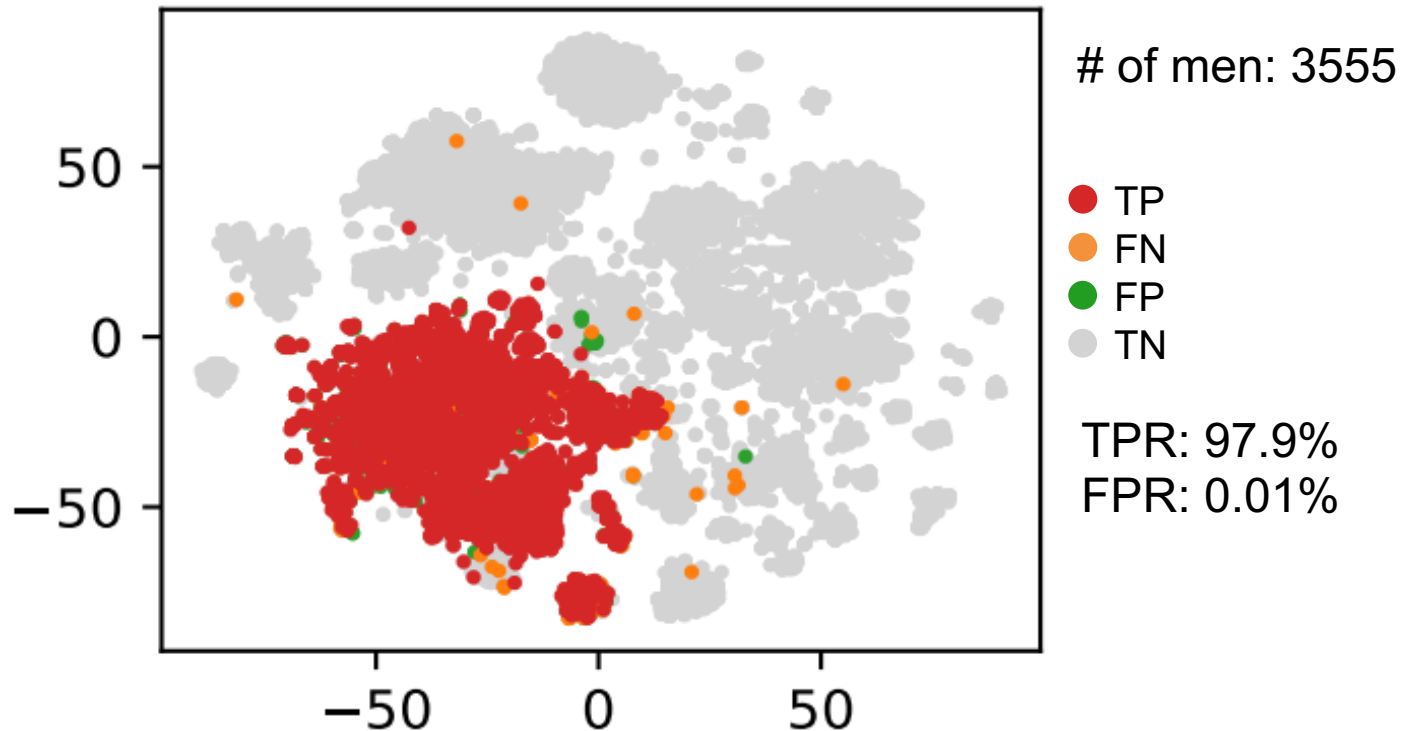
Embedding Space



“List male instrumentalists who play string instruments”

Male 

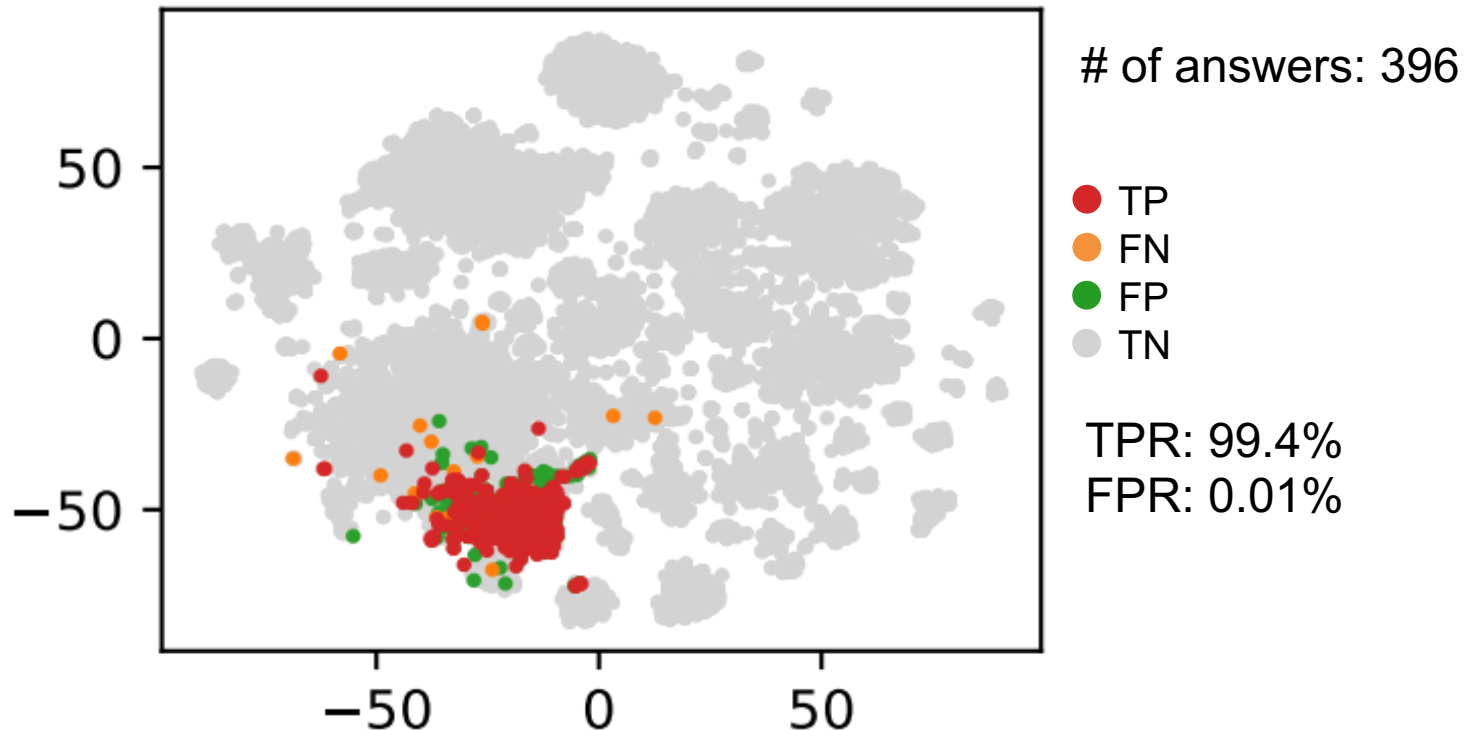
Embedding Space



“List male instrumentalists who play string instruments”



Embedding Space



“List male instrumentalists who play string instruments”

