The Network Analysis in the Funding of the Financial System

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1 Abstract

In real world, financial institutions make decisions of their funding and lending structure on the daily basis. The exchange of credits among the financial institute is so frequent and sensible to the economic environment that an increase on the reference interest rate could modify the inter-banking funding network. It results important to central authorities to be aware of the network structure and the possible implications that a monetary policy could have over the network. In this project we present the Mexican bank network structure of two financial markets (repo and inter-banking market) from July 2015 to August 2017. A static network analysis focusing on outbreak detection is performed to identify the crucial institutions that the central authorities need to monitor for potential crisis. We also study the dynamics of the network focusing on adoption of the interest rate when the reference interest rate is modified and community formation.

2 Introduction

Central authorities play an important role on maintaining the stability of the financial system. We study both the *static structure* and the *evolution* of the network to further inform central authorities on their policy making.

One main goal of the central authorities is to regulate the activities of the financial institutions. Prevention and early detection of potential financial crisis is a major responsibility. Hence, establishing good monitor position inside the network to effectively oversee network activities is essential to the central authorities.

Another main goal of central banks is to maintain a price stability (low inflation rates). This is achieved by different channels like injecting or extracting liquidity from the market, and modifying the reference funding rate to adjust the demand of money on the market, among others[8]. It becomes crucial that central authorities have deep understanding of the structure of the market, e.g. how it has evolve and how a decision of either increasing or decreasing the reference interest rate impacts the funding decisions between institutions.

The paper is structured as follows. We first review existing papers in literature of the financial networks are usually studied in Section. 3, especially focus on existing method of studying network dynamics and network hierarchy. The dataset we use is described in Section. 4. Our study begins with the most recent snapshot of the network to study its static structure in Section. 5 and performs an outbreak study for early crisis detection in Section. 6. Then the paper proceeds to study the dynamics evolution of the network focusing on the interest rate adoption, and community formation (Section. 7).

3 Related Work

Intuitively, the financial system can be modeled by networks. Financial institutions can be viewed as nodes and the activities like loaning and borrowing between different institutions can be treated as directed edges. We find in the literature that both the *network dynamics* and *network hierarchy* have well established modellings that yield sensible results in economics terms.

3.1 Network Dynamics

In Bräuning et al[1] and Billio et al[2], for example, credit spill overs are used as weighted edges at a given time stamp. Credit spill overs is an economics term quantifying the net amount of transactions between two nodes. The credit spill over from node A to B is the amount of credit A holds against B. With the credit spill over network, both papers performed pair-wise Granger-causality tests for the entire graph. The tests are performed over the time-series, i.e. from past to recent snapshots of the network, to find causal effects of one node on another. Specifically, the papers treat the nodes status R as time-series. Given two nodes i, j, for simplicity, a linear causal test is defined as:

$$R^i_{t+1} = a^i R^i_t + b^{ij} R^j_t + e^i_{t+1} \tag{1}$$

$$R^j_{t+1} = a^j R^j_t + b^{ji} R^i_t + e^j_{t+1} \tag{2}$$

If non-zero a and/or b values are obtained from regression the nodes are believed to be dependent on the status of its past and/or its neighbor's past status. Both papers have shown that if the model is given all the historical data of all market participants should be able to forecast the future risk. Particularly the Billio et al[2] approach has shown to be predictive for instance over the period of financial crisis.

However, the methods used by Bräuning et al[1] and Billio et al[2] have short-comings. As one can see from Eq.1 and 2, the time-series analysis relies heavily on the full history of the network to be collected in order to construct meaningful model over the course of time evolution. If some entities joined the network very recently, and its interaction history with other nodes is not long enough, then the model may not be as predictive since the causal relation cannot be formalized. In other words, these models are not robust against the network structure change. Also the pair-wise tests examines only the casual relationship between two immediate neighbors and tend to ignore the long range and secondary correlations of the nodes, which might not be negligible in case like one node's neighbor's neighbor being the most influential entity in the network.

3.2 Network Hierarchy

In literature, we also see many examples of how to establish the hierarchy for financial networks. Conventional notations like centrality and betweeness are adopted for instance in Martinez-Jaramillo et al[4]. With these notions, one could then answer the question if the central nodes of the financial network is susceptible to potential risks and would potentially go banckrupt.

In our study, these conventional networks will be calculated. However, these centrality notions may fail to capture the importance of nodes in very specific circumstances. As an effort to understand the spread of crisis over network *in-situ*, we will do an outbreak study to find out the best nodes to place monitors for early detection and compare these essential nodes to the critical nodes obtained through conventional calculations.

4 Dataset Description

The Mexican financial system is composed (among other institutions) of 47 commercial banks, 6 development financial institutions, and 36 brokerage firms operating up to date. The loaning activities between all the financial institutions are on the record on a daily basis from the year of 2000. However, given that recently the Central Bank of Mexico has been intervening in the financial system via an increase in the reference interest rate, we are studying the period where these interventions happened (from July 2015 to October 2017). Over this period there have been 10 interventions from the Central Bank as shown in Table 1.

Table 1: Monetary Policy from the Central Bank of Mexico

Date	From	To
2015-12-17	3.00	3.25
2016-02-17	3.25	3.75
2016-06-30	3.75	4.25
2016-09-29	4.25	4.75
2016-11-17	4.75	5.25
2016-12-15	5.25	5.75
2017-02-09	5.75	6.25
2017-03-30	6.25	6.50
2017-05-18	6.50	6.75
2017-06-22	6.75	7.00

One of the advantages that presents the Mexican financial system is the comprehensive information that it reports to the authorities. We analyzed the banking funding from two different dimensions to understand it's structure and how a monetary policy (i.e. a raise on the reference rate) impacts these markets, and further understand what are the strategies taken by the banks.

1. The repo market consists of two parties where one acquires the ownership of some assets on exchange of money, and commits to resell them on a conveyed period for the same price plus a prime. In Mexico, this market is regulated, and only commercial and development banks, and brokerage firms are allowed to demand liquidity under this market; on the side of liquidity providers, the counter-party can be diverse (from a natural person to a financial institution). The operation is allowed to last up to 3 days (there may be some exceptions), but there exist roll-over repos where the counter-parties pact to re-do the repo once the term has passed.

    - One of the main risks of the repo market happens when the interest rate increases: not only funding is more expensive, but also, the price of securities diminish which may evolve on a liquidity or credit risk.

2. The inter-banking market, the only participants are commercial and development banks, and brokerage firms. They lend money between each other and pact an interest rate and a term. Based on the institution's situation it may or may not find funding on this market for an acceptable interest rate.

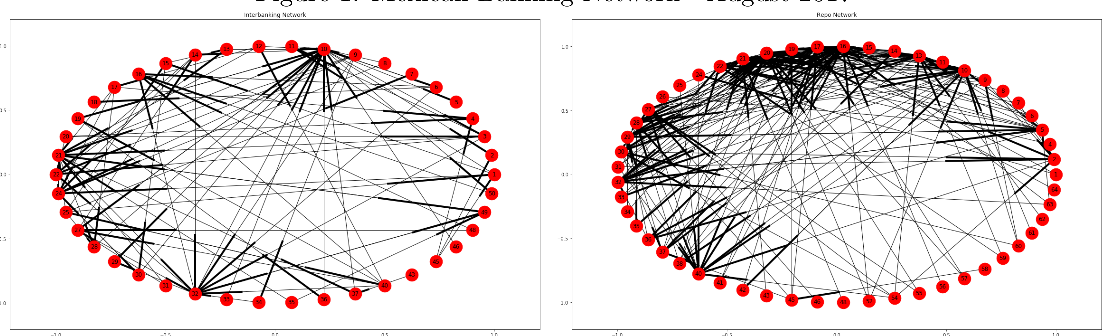
    - If an institution starts to be perceived as facing liquidity risk, some counter-parties may cut the funding or increase the interest rate to compensate the risk.

5 Static Network: Basics

5.1 Static Network Construction and Characteristics

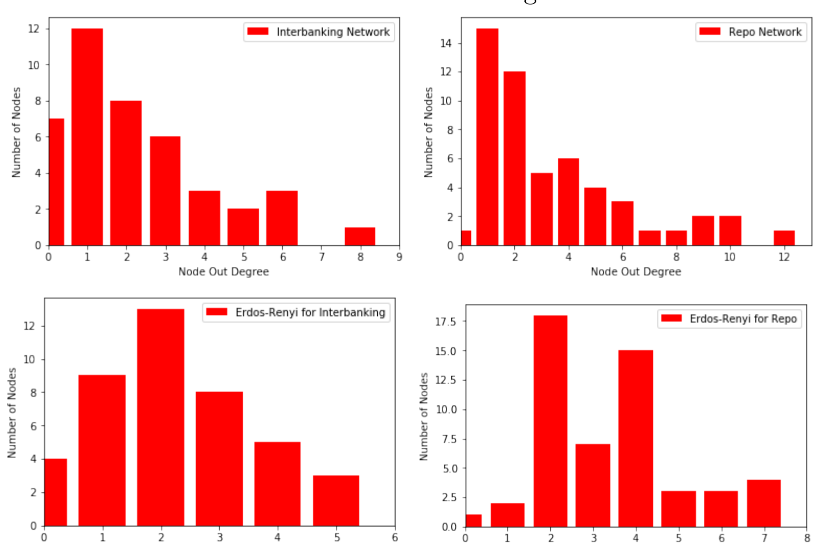
The funding transactions between Mexican banks are recorded in database by rows. Each row of the dataset contains the information of the lender, borrower and the amount of the transaction. The transaction starting date, ending date as well as the interest rates are also recorded. To have a basic understanding of the bank network structure, for a moment, we set aside the information of loan amount, interest rate as well as the transaction times. A simple static directed network is built from data by taking each bank as a node. A directed edge is pointed from node A to node B if A has a record lending money to B. The static network for the transactions that were active on August, 2017 is visualized in Figure 1.

Figure 1: Mexican Banking Network - August 2017



In total, the Mexican inter-banking network has 42 nodes and 95 directed edges, the repo network has 53 nodes and 182 edges. The histogram of the out degree is shown in Figure 4.
Given the network basic parameters, we compared it to a null model which is a randomly generated Erdos-Reyni graph with the same total number of nodes and directed edges. The out degree histogram is also shown in Figure 4. We observe a significant difference in the out degree distributions as the Mexican banking network (for both markets) has a node with the highest degree, which it seems to be a main/common source of funding. Also, it seems that most of the nodes on the empirical graph have an out-degree around 2, while Erdos-Reyni graph has a more belled-formed histogram.

Figure 2: Comparison of out degree for the Mexican bank network and a randomly generated Erdos-Reyni network with same number of nodes and directed edges.



We also calculate the clustering coefficient of the interbanking network (.2497) and it's corresponding random graph (0.0847), and the repo network (.3268) and it's corresponding random graph (.1211). The difference in clustering coefficients shows that the borrowing activities do not take place randomly but often happen within small groups of banks.

5.2 Network Topology

We analyze the network topology by first finding the largest WCC and SCC of the network. We find that for the interbanking network the largest SCC contains 17 nodes and the largest WCC contains all 42 nodes. For repo network the largest SCC contains 24 nodes and the largest WCC contains all 53 nodes. For all the nodes we then run a forward BFS search to find out the size of the forward BFS tree. As a summary, we present Table 2.

Table 2: Number of Nodes and BFS Size

Interbanking Network	
BFS Size	Number of Nodes
1	7
2	1
23	17
24	9
25	4
26	1
27	1
31	2

Repo Network	
BFS Size	Number of Nodes
1	7
2	1
25	17
26	9
27	4

These calculations indicate that for the interbanking network, 17 banks belong to the IN component of the network which mean they only lend money to other banks. Eight banks are in the OUT component of the network which only borrow money from other banks, while the rest 17 banks form the SCC where it is an interchange between borrowing and lending from each other. Whereas in the repo network, 6 banks belong to the IN component, 8 banks belong to the OUT component, and 24 to the SCC. Given the sensibility of the information, we are not able to give the characteristics of the nodes/banks that are part of each of the components, however, this omission won't affect the interpretation of our final results.

5.3 Critical Nodes

Nodes are of different importance to the bank network. With simple heuristics, we pay close attention to the nodes which are either central to the graph or in the special components of the graph. The following nodes are picked out as examples to study the contagious behaviors we are interested in.

5.3.1 Closeness

The Closesness centrality of a node n_i is defined to be c_{n_i} in Equation 3, where w_{n_i,n_j} is the length of the shortest path between node n_i and n_j and k_{n_i} is the out degree of the node n_i. One hypothesis we make is that closeness makes node susceptible to contagious outbreaks. Nodes with highest closeness are listed in Table. 3.

$$c_{n_i} = \frac{1}{k_{n_i} \sum_{n_j} w_{n_i,n_j}}, \tag{3}$$

Table 3: Top three nodes with largest closeness

Interbanking Network		**Repo Network**	
Node ID	Closeness	Node ID	Closeness
40	0.327	10	0.291
24	0.322	14	0.276
2	0.295	16	0.270

5.3.2 Eccentricity

Eccentricity is defined in [3] as the largest path length of the shortest path between the source node n_i and the destination node n_j. One hypothesis we make is that nodes with larger eccentricity are on the edges of the network and may adopt the contagious outbreaks the last. Nodes with highest eccentricity are listed in Table 4.

Table 4: Top three nodes with largest eccentricity

Interbanking Network		**Repo Network**	
Node ID	Eccentricity	Node ID	Eccentricity
45	6	24	6
48	5	2	6
10	5	64	6

6 Static Network: Potential Risk and Outbreak Detection

Transactions within the financial networks can spread financial risks. Earlier detection of misconducts and risk of default of banks is crucial to prevent systematic failure of the network. Monitoring the bank activities has associated legal, person power and other costs which scale as the number of banks the authorities in practice monitor. In this section, we explore the optimal number of monitors and optimal placement of the monitors in the network, such that the authorities can quickly detect outbreak of crisis at fixed budget.

6.1 Problem Formulation

Given the static interbanking network we have in Section 5, without loss of generality, we assume the crisis outbreak could happen at any node with uniform probability. The crisis would spread out through the directed edges of the network. At time t, if a node n is infected, then we assume at time $t+1$ the children of node n will be infected.

Minimizing the total wait time before detecting a crisis translates to minimizing the expected penalty that comes along with a late detection. We define the penalty to be $\pi_i(t|S) = t$, which represents the penalty of an outbreak initiated from node i and detected at time t for a specific monitor placement S. In case, the outbreak will never be detected, for instance the monitor is not connected to the outbreak node, we define assign a constant which is larger than $\max(\pi_i(t))$, i.e. $\pi_i(\infty) = constant > \max(\pi_i(t))$. In practice this constant is set to be 5.

To summarize, the final objective we are interested in is $\max_S f(S) = \sum_i \pi_i(S)$, subject to $|S| = k$, where k is the number of monitors we are allowed by the budget.

As the cost of having a particular number of monitors is hard to decide, we leave it as a free parameter in the model. In our study, we scan through different numbers of monitors we install.

6.2 Algorithm

For this problem we employ the CELF (Cost-Effective Lazt Forward-selection) algorithm as covered in class [9]. Since the cost is uniform in our problem, we only need to implement the unit-cost greedy search . The full algorithm goes as follows:

Algorithm 1: `Find Best Monitors`

Input: graph, K = [0, 1, 2,...] list of the number of monitors we want to scan

SensorMap = { }

for *k in K* **do**

    // Initialize empty list of sensors

    sensors = [ ]

    while *len(sensors)*< *k* **do**

        new_monitor = UnitCostGreedySearch(graph, sensors)

        sensors += new_monitor

    SensorMap[k]=sensors

return *SensorMap*

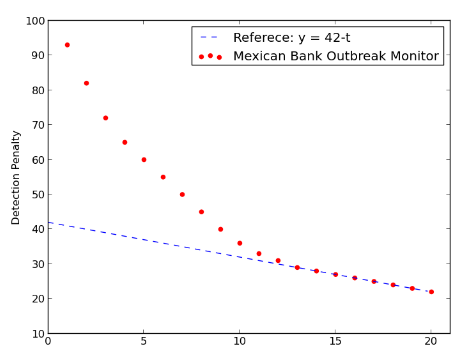
6.3 Outbreak Detection Result

The result of minimized penalty as a function of number of monitors allowed budget is shown in Fig.3. As expected, as the number of monitors we install grow the penalty drops. Once we install around 10 monitors, the total penalty becomes around 40 which is close to the total number of nodes (42) in the graph, meaning we can detect on average all outbreaks within one time stamp.

The diminishing return of monitor installation is obvious in the graph. With more than 13 monitors installed, the return for installing one more monitor is shown to be always 1 (blue line in Fig.3). This behavior indicates that the additional monitor installed only improves the detection of outbreak at the particular place where that particular monitor is installed.

Since greedy algorithm is used for this analysis, we can write out the top ten best nodes to place the monitor. The node IDs are: 10, 16, 4, 21, 2, 5, 7, 18, 29, 24 in decreasing order of influence. (We are only publishing the node IDs not the node names for restrictions of the dataset.) Comparing to Table. 3, it is worth noting that the top three critical nodes we find in the outbreak study does not coincide with the nodes of top closeness. In this particular case, the notion of closeness failes to capture the major player of the financial market.

Figure 3: Minimized Penalty Vs. Number of Monitors Installed



7 Network Dynamics: Using the daily interchange as a dynamic network

7.1 Evaluation of the network's evolution

Now that we had understood the general structure of the graph from the overview of the static network. We are going to evaluate how the graph has changed from July 2015 to November 2017. We first obtained the characteristics from the network on the starting date and compared them to the ending date. Additionally, we created an artificial temporary network and compared it with the original graph. The method we followed to construct the artificial network was as follows:

- We started by estimating the probabilities of addition and deletion of an edge in the network. For every date, we analyzed the network and compared it with the network of the day before.
- To estimate probabilities of an edge being added or removed, every deletion of an edge (compared with the previous day) was counted, and the probability of deletion was obtained by dividing it by the total number of edges on the previous graph. Every addition of an edge was also counted and divided by the number of nodes to obtain the probability of addition. We then averaged these probabilities over all dates.
- Once the average probabilities were estimated, we took the starting network as the base network and we proceeded to add and remove edges for the given probabilities over the same time period as our analysis (599 days).

Results are shown in Table 5. Where it seems that for the interbanking network the estimation of probabilities ended up with a close enough number of edges and a similar number of WCC. Whereas, in the repo network, the probabilities sub-estimated the number of edges. And, in both cases, as we have seen, there are nodes in the network that are only lenders and others that are just borrowers (IN and OUT components). This polarization is lost on the randomized network.

Table 5: Comparison between a temporal random graph and empirical graph

Interbanking Network					
Network	**Number of Nodes**	**Number of Edges**	**SCC**	**WCC**	**Clustering Coef**
Starting	43	117	5	43	0.3316
Ending	44	84	6	44	0.1385
Ending Simulated	43	81	26	41	0.0918
Repo Network					
Network	**Number of Nodes**	**Number of Edges**	**SCC**	**WCC**	**Clustering Coef**
Starting	45	168	19	45	0.4878
Ending	52	167	22	52	0.3707
Ending Simulated	45	126	39	45	0.123

To continue evaluating the evolution of the networks, in Table 6 now, instead of obtaining probabilities for appearing or disappearing edges over all the network, we estimate the probabilities over each pair of nodes and create the random network based on these probabilities. The estimation of probabilities also give us information about the consistency of the nodes' relationship. The results show an over-estimation of the number of edges for both networks, and the polarization behaviour is also lost (based on the value of the SCC).

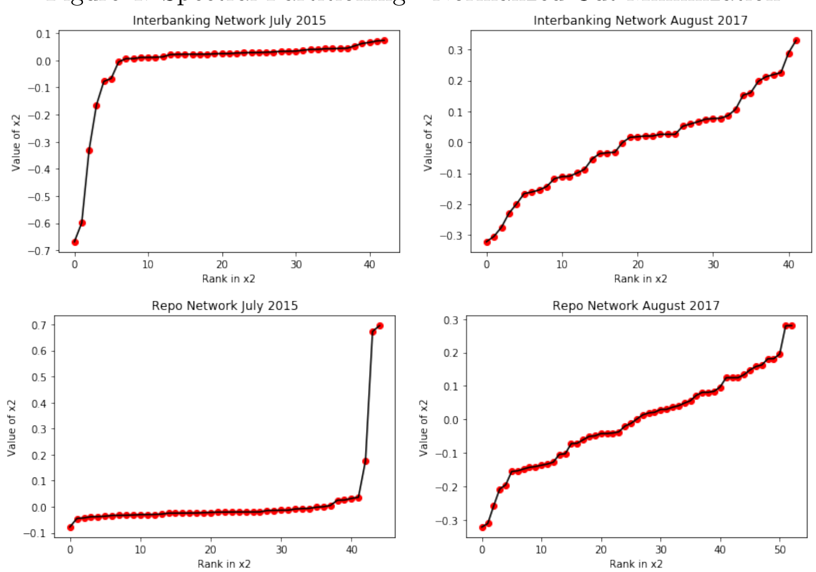
Table 6: Random graph generated with node's probabilities

Network	Number of Nodes	Number of Edges	SCC	WCC	Clustering Coef
Interbanking	52	146	37	52	0.073
Repo	53	204	39	53	0.2059

7.2 Community Formation

With our previous results we conclude, there were some characteristics of the networks that could be reached at random based on the probabilities calculated. However, the network had a polarization particularity (meaning there were banks that exclusively lend money and other that exclusively borrow money) that couldn't be modeled at random. We now proceed to analyze if the community structure of the network has changed, for this end, we implemented the *spectral algorithm for normalized cut minimization*[10].

Figure 4: Spectral Partitioning - Normalized Cut Minimization



The results were pretty interesting because although we will see on subsection 7.3 that measures such as full diameter, SCC, WCC, and clustering coefficient don't show a clear difference across time, the network evolution in terms of communities show a big change in both markets. It started with only one clear split in 2015 and ended with several splits in 2017. This is of great importance to authorities because a network with higher number of communities is less susceptible to contagion.

7.3 Analyzing the existence of cascading effect

We wanted to understand what was the effect of the an increase of the reference interest rate on the networks. By understanding the effect, we wanted to further estimate some parameters to predict the impact of future changes considering some of the ideas done in Myers et al[6], and Goyal et al[7], but adapted to the properties of our networks. For this end, we first calculated the correlations between the reference interest rate and both the weighted average interbanking interest rate and the weighted average repo interest rate, in both cases the correlation is greater than .99, this shows that the adoption of the reference interest rate is almost direct in the markets.

We then evaluated the characteristics of the graphs over time. Table 7 presents the basic statistics, which show that for both markets the variation of full diameter, SCC, WCC, and clustering coefficient is not large.

Table 7: Historical Network Information from 2015 to 2017

Measure	**Interbanking Network**				**Repo Network**			
	Full Diameter	SCC	WCC	Clustering Coef	Full Diameter	SCC	WCC	Clustering Coef
min	4	2	36	0.0460	4	9	43	0.1961
median	7	16	43	0.2263	6	19	48	0.3788
mean	7.29	15.29	42.54	0.2292	5.91	18.63	48.07	0.37
max	16	30	48	0.3979	10	27	53	0.5357
std.	1.44	5.86	1.68	0.063	0.96	2.97	2.23	0.054

We then calculated the correlations between the rate that was charged for banks on the dates when a monetary policy was implemented and the rate that those banks charged the next date (Table 8). All of the results for both markets were negative, these results are consistent with the characteristics of the graphs: the networks are quite polarized, there are some nodes that lend money and others which exclusively borrow money. This polarization implies that when processing the vectors banks that received funding had a particular interest rate, however, the rate they charge the next day (second vector) was zero because they did not lend money, and viceversa. Also, in average, interest rates are close to the reference interest rate, which explains the values around $-.5$. These results allow us to conclude that for these kind of episodes (monetary policy) there is not a cascading effect but rather an adjustment of banks to the new reference interest rate.

Table 8: Correlation between the rate charged to banks and the rate they charge

	2015-12-17	2016-06-30	2016-11-17	2017-02-09	2017-05-18
interbanking	-0.4491	-0.6070	-0.5837	-0.4291	-0.4795
repo	-0.3558	-0.2689	-0.3695	-0.4010	-0.3830

8 Conclusion

The Mexican financial network is a very well connected network with its largest WCC having size equal to the total number of nodes. The static network analysis demonstrates that the network crisis outbreak can be detected early (within one time stamp) if around 10 monitors (a quarter of the total network size) could be established in the network. The study also demonstrates that conventional notion of centrality like closeness is not equivalent to the nodes being essential for outbreak detection.

The evolution of the network for edge creation follows a pattern among nodes, however, there is a clear polarization on the network structure that can't be modeled at random. The current analysis show, that the community structure of network has evolved from 2015 to 2017, from one-split large communities to several splits small communities. This result is interesting because the increase in number of communities allow the network to be more stable when there is a contagion episode.

The current study also indicates that the adoption of interest rate among the networks is instantaneous for almost all nodes. However, there is no evidence of a cascading behaviour but rather and direct adoption of the new rates.

9 Reference

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