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# CS 224W Project Report: Financial Data Analysis

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## Abstract

In this report, we propose to apply network theory tools to the analysis of the New-York Stock Exchange stock prices. Our goal is to appraise the risk associated to a portfolio by analyzing the joint evolution of the correlations between return prices of 2,355 stocks over a period of 3 years. Our approach relies on an analysis of the dynamics of the inferred graph structure that aims to capture strong meaningful interdependencies between stocks.

## 1 Introduction

The 2007 financial turmoil has come as a tough reminder of the exposure of financial markets to systemic risk, as well as the necessity of finding better indicators of dependencies between financial agents. In particular, one could be interested in trying to infer a particular investment's risk from the observation of the joint evolution of the stock prices over time. However, the very noisy nature of these time series renders this an extremely delicate task, thus begging the question: how can we extract meaningful information from the data?

Recent literature in financial engineering has underlined the potential of networks to provide- at least partially- an answer to this question: each stock can be modeled as a node, and the complex interactions between stocks can be represented by weighted edges, indicating the strength of the interaction between agents. One can then use the tools from Social Network Analysis to analyze the joint risk of the system: for instance, we could be interested in achieving a classification of nodes in terms of "core" (highly connected stocks, thus more likely to be exposed to systemic risk) vs "peripheries". This new perspective can also provide us with some further insight into the mechanisms underlying the formation and evolution of the financial system, and could thus potentially be employed to devise optimal investment strategies.

In this project, we use the adjusted closing prices for 2,355 companies from 2012 to 2015. We use this dataset to infer a meaningful network that acts as the basis of our analysis. We then study different properties of this graph: we compute several centrality measures for this graph and cluster the graph prior to analyzing the results.

## 2 Related Work and Proposed Approach

Over the past few years, several papers have tackled the problem of inferring -and analyzing- networks from time series data. For instance, in [3], T. Di Matteo et al. use correlations between each pair of *transformed* time series to measure *the strength* of each interaction. In another paper [13], D. Kennett et al. use partial correlations of these time series as a basis for their graph. However, the challenges of using the empirical correlation matrix for network analysis are two-fold. First of all, this creates a complete graph (every node being connected to all the others): most of the techniques

that are currently available in network theory work for unweighted sparse graphs and hence, they can not be directly applied to our case. The previous authors propose to filter this information by first constructing a complete weighted graph and then, finding the minimum spanning tree (MST) or planar maximally filtered graph (PMFG). In other words, in order to get a sparse graph, they restrict their domain of search to a pre-specified set of graphs. But this raises the following question: why should we expect the network of interdependency between stock prices to be best represented by a tree or planar? Not only might this assumption be too restrictive, the inferred topology that they span is difficultly interpretable both from a statistical view point and from a financial engineering perspective, and we thus propose two alternative methods to overcome this problem. In contrast to the mentioned methods which are fully unsupervised, our methods are *supervised*.

Moreover, in our setting, where the number of stocks  $N$  is at least three times greater than the length of the time interval  $\Delta_T$  (i.e. the number of days in the time series), we expect a large proportion of these correlations to be due to randomness, and we thus have to devise a method that would allow us to filter signal from noise. The challenge of properly distinguishing signal from noise was already raised in 1999 by Bouchaud et al [11]. However, their analysis only focuses on the spectral properties of  $\hat{\Sigma}$ , and thus does not provide us with any insight as to the strength of the interaction between any two nodes or the topology of the network.

Another challenge in this network-based approach consists in the assessment of the relevance of the information extracted from the graph. Indeed, checking the existence of co-dependencies between any pair of companies would require hours of scraping the web or going over companies' (private) financial records. The only labels that we have straightforwardly at our disposal are the labels of companies in terms of the 12 standardly defined sectors (Finance, Consumer Durables, etc. ), or of the 138 industries (Pharmaceuticals, Banks, etc.). Noting that we expect companies from identical sectors to exhibit high correlations, we propose an analysis based on the cross-comparison of the inferred structure of our network with this benchmark clustering. Further noting that we expect meaningful information to be persistent over time, following the approach of Di Matteo et al [6], we also suggest an evaluation of the results based on their resilience over time.

### 3 Exploring the dataset

As mentioned earlier, our dataset consists of the stock prices for 2,355 companies over a period of three years. Let us denote the stock price of the  $i^{th}$  company at day  $t$  by  $P_{it}$ . For each company, we also have access to a few covariates such as the industry and the sector that it belongs to: for instance, Apple belongs to the *Computer Manufacturing* industry, this industry being itself a subset of the *Technology* sector. This defines a partition of the stock market into sectors, each of which broken down into several industries, and each company is associated with exactly one industry (and hence, one sector).

The main idea in this report and the aforementioned papers is to exploit correlations to discover latent relationships. For given time series  $X_{it}$  and  $X_{jt}$  defined over a period of  $\Delta_t$  days, the empirical correlation is defined as:

$$\rho_{ij} = \frac{\sum_{t=1}^{\Delta_T} (X_{it} - \bar{X}_i)(X_{jt} - \bar{X}_j)}{\sqrt{\sum_{t=1}^{\Delta_T} (X_{it} - \bar{X}_i)^2} \sqrt{\sum_{t=1}^{\Delta_T} (X_{jt} - \bar{X}_j)^2}} \quad \text{where } \bar{X}_i = \frac{\sum_{t=1}^{\Delta_T} X_{it}}{\Delta_T}$$

Due to inflation, most of these time series exhibit an increasing trend. Hence, the computation of the correlations for raw prices  $P_{it}$  yields very high values – about 0.42 on *average* over 3 years. This shared overall trend is the dominating factor contributing to these high correlations, and makes it difficult to distinguish any real inter-dependencies from randomness and this shared market effect. Figure 1a depicts the histogram of the correlations for the raw data.

The idea to overcome this problem is to use the *log-return* of a stock. The log-return of a stock  $i$  is defined as the log ratio of the adjusted closing prices  $P_{it}$  for two consecutive business days, i.e:  $X_{it} = \log(P_{i(t+1)}/P_{it})$ . Throughout our analysis, we operate under the standard assumption in

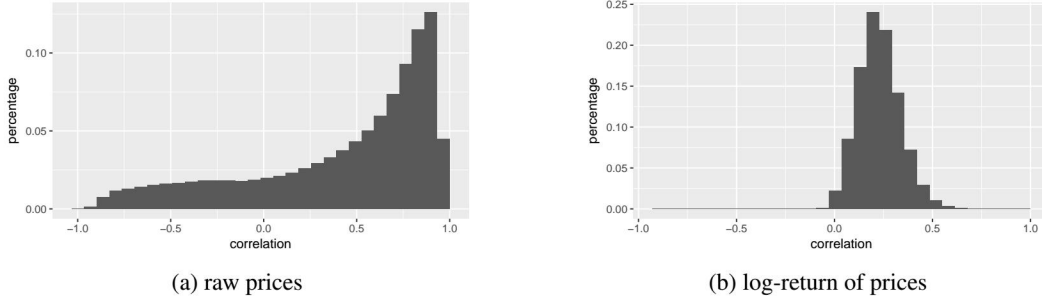
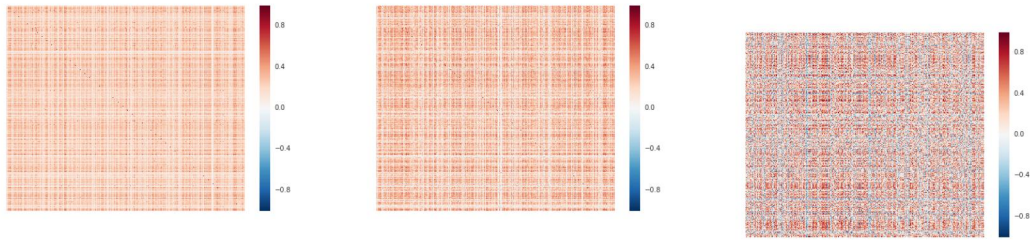


Figure 1: Histogram of correlations of raw and transformed.

financial engineering that these log-returns are independent and normally distributed. Figure 1b shows the histogram of the transformed data. whereas figure 2 depicts the heat map generated for the intervals of 10 days, 100 days and three years. We note in particular that the correlations depend on the value of the time frame  $\Delta_T$  selected, and we will thus be interested in analyzing the evolution of this empirical correlation matrix over  $T$  time frames.



(a) Heatmap for the correlations between stocks over the 3-year time span (b) Heatmap of the correlation for the first 100 days of our time series (c) Heatmap of the correlation for the first 10 days of our time series

Figure 2: Plots for the correlation over different time spans: we see more and more contrasts as we decrease the length of the time window. In particular, correlations can become negative for a shorter time frame, which might induce a better differentiation of the data, and make our graph inference task easier

Not only do the log-returns exhibit attractive distributional properties, they also allow us to provide a more solid statistical foundation to our approach, as explained in section 4. Hence, from section 5 onwards, we will work with the “transformed” correlation matrix  $\hat{\Sigma} = (\rho_{ij})$  spanned by the log-returns.

## 4 Significance of Correlations

In the previous section, we pointed out that the raw prices are highly correlated, but the correlations can be explained by inflation and randomness. Now, we claim that it is *not* the case about log-return prices. This in turn raises the question: how can we correctly define a null hypothesis to assess the significance of the interdependencies that we wish to exhibit?

In [12], Atse et al. studied the significance of correlations by performing a permutation test to the transformed time series. They shuffle transformed time series to simulate randomly generated data (acting as the null distribution). They then plot the histogram of these “random null” correlations to conclude that the actual observed correlations are significantly high [12]. Even though this is a fascinating experiment, it fails to provide insight into the real interdependency between stock, as shown by the following possibility: fix two time series. Assume that in each time series, the first half of the data are mostly positive log-returns and the second part of the data are mostly negative – this might occur if, for instance, the prices were increasing in the first half of the data, but decreasing in

the second due to some political/economical crisis. We expect the correlation for these time series to be high, but the correlation of the shuffled data to be very low. Therefore, Atse's suggested method would imply that our sets of observed correlations are significantly different from the null. However, the stocks might still be unrelated to each other, and Atse's method has only achieved in detecting a statistically significant "overall market effect". A more accurate way of assessing interdependencies would be achieved by looking at the coincidence of negative values in the first half of the data and positive values in the second: the intuition is that the fact that two companies show negative log-returns at day 10 (when the overall market exhibit a positive trend) might indicate a stronger and more relevant interdependency between these companies. One way of formalizing this idea is to apply *local shuffling* to the time series. We split each time series into segments of fixed length  $l$  (say 3, for instance) and we then locally shuffle each segment, leaving the overall block ordering unchanged. Thus, if the correlations are high, just because of some global conditions changing nationwide, the local increases and decreases should still be random and thus independent of each other, and we should not observe any significant difference between the histogram of the locally shuffled data and the actual data. On the other hand, if we do observe a significant difference between the histograms, this would imply that even local changes of some companies are synchronized, which is a strong evidence that our correlations are meaningful. We have performed this experiment for the raw data and the transformed data and the results are displayed in Figure 3.

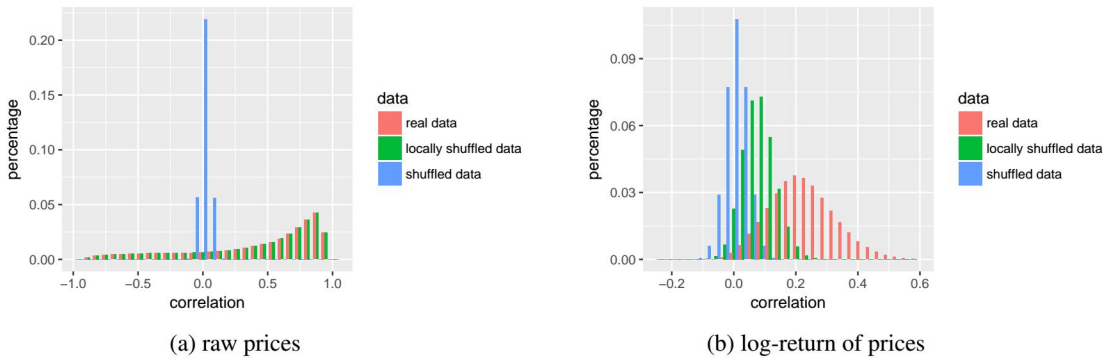


Figure 3: Histogram of correlations of raw and transformed.

As expected, Figure 3a suggests that, even though correlations of the *raw* prices are meaningful with respect to the complete shuffling, our approach shows that this can be attributed to a global trend, since this significance vanishes on local level. However, Figure 3b implies that even if we shuffle windows of length 3 in each log-transformed time series, then the distribution of the locally shuffled correlations is significantly different from the observed one, thus acting as strong evidence in favor of the existence of inter-dependencies among stocks: in other words, there is signal to be extracted from these correlations. Hence, this experiment succeeds in both justifying using the log-returns prices as data, and the relevance of our approach.

## 5 Thresholding Correlations

In this section, we propose to infer a financial network structure by thresholding the empirical correlation matrix  $\hat{\Sigma}$ . We model our correlation as  $\rho_{ij} = C_{ij} + \alpha + \epsilon_{ij}$  where  $C_{ij}$  is a sparse matrix corresponding to the "excess correlation", i.e, the signal encoding the existence of a strong tie between stocks  $i$  and  $j$ ,  $\alpha$  is the overall trend of the market, and  $\epsilon_{ij}$  corresponds to background noise, and is thus centered at 0.

This yields a first natural threshold-based approach for inferring a network topology, while making as little distributional assumptions as possible: we propose to add edges between stocks whose correlation exceeds a certain predefined threshold. Here, we propose two thresholding approaches, one based on the rank of the correlations, and the other relying only on their magnitude.

In the first approach, we suggest sorting the correlations in a decreasing order

$$\rho'_1 \geq \rho'_2 \geq \dots \geq \rho'_{n(n-1)/2}.$$

For each  $k \in [1, n(n-1)/2]$ , let  $x_k$  be the indicator of whether or not the companies corresponding to  $\rho'_k$  belongs to the same industry. Now, define

$$f_k = \frac{1}{k} \sum_{i=1}^k x_i.$$

Namely,  $f_k$  denotes the proportion of correlations among  $\rho'_1, \dots, \rho'_k$  for which the corresponding companies belong to the same industry. Intuitively, we expect the companies within each industry to have higher correlations on average. Therefore, we expect this number to be decreasing. Figure 4 plots the values of  $f_k$ 's and confirms this intuition.

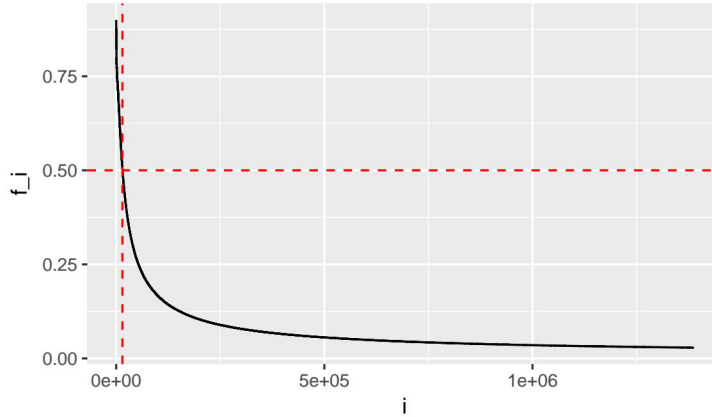


Figure 4: The proportion of same-industry companies among  $i$  highest correlations.

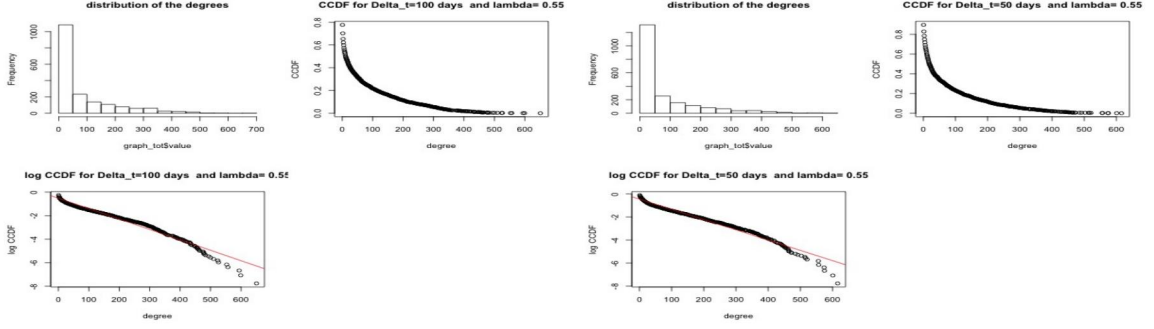
The above figure shows that if we construct a graph with 15,000 edges corresponding to companies with highest correlations, then about half of the edges belong to companies within the same *industry*. Therefore, our inferred network structure is strongly consistent with the traditional partitioning of the market, and, as explained in section 2, this upvotes the validity of our approach.

Our second method is a generalization of the previous one, and simply consists in thresholding correlations at a predefined threshold  $\lambda$ , for different lengths of time series  $\Delta_T$ . The intuition is that we also want to use the information contained in the dynamics of the graph (i.e, its actual evolution over time) that springs from this analysis as further information about the structure of the market. This approach thus requires us to select two parameters: the correlation threshold  $\lambda$  as well as the value of the time window  $\Delta_T$ .

## 5.1 Thresholded Graph Properties

In this subsection, we study the properties of the second approach proposed above. One can thus begin the analysis by investigating the effect of these parameters on the distribution of the inferred graph features. Figure 5 shows the impact of the threshold level on the degree distribution and the Complementary Cumulative Distribution function (CCDF) of the degree distribution. In particular, this analysis shows that the inferred graph follows closely a power law distribution, with parameter  $\alpha \approx 1.0$  (with an  $R^2$  value for the fit close to 0.97). This thus corresponds to a very "small world" system: the graph that we have obtain is thus very sparse and "filtered".

As mentioned earlier, we now propose to investigate the relevance of our graph by assessing two quantities:



(a) Distribution of the edges of  $G_1$  (first graph snapshot) with  $\Delta = 100$ ,  $\lambda = 0.55$  (b) Degree distribution of  $G_1$  (first graph snapshot) for  $\Delta_T = 50$ ,  $\lambda = 0.55$

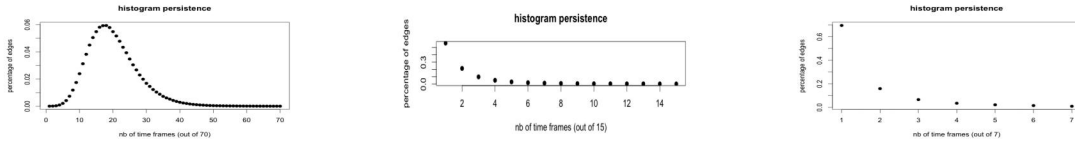
Figure 5: CCDF and degree distribution of the graph inferred by correlation-thresholding

- "statistical significance": we compute the p-values associated to each observed correlation. Here, we suggest a data-driven estimation of the p-values : assuming that the excess-correlation matrix  $C$  is very sparse, we consider the centered correlations (so that we have removed the market effect), and consider these to be our null distribution. We can then compute empirical p-values for any of our observed correlations by comparing them to the quantiles of these centered correlations.
- "time-significance": the underlying idea is that meaningful correlations (ie, when  $C_{ij}$  is non null) have higher probability of persisting over time, whereas spurious effects will vanish. To be more rigorous, using the notations defined in the previous section, and after correcting for the market effect:  $\mathbb{P}(\rho_{ij} > \lambda) = \mathbb{P}(\epsilon_{ij} > \lambda - C_{ij}) \geq \mathbb{P}(\epsilon_{ij} > \lambda)$ , and hence a first significance indicator would be to consider an edge  $(i, j)$  as significant if:

$$\mathbb{P}(\rho_{ij} > \lambda) \geq 1 - \epsilon \stackrel{\text{estimated by}}{\implies} \frac{1}{D} \sum_{i=1}^D \mathbb{1}((i, j) \in G_i) \geq 1 - \epsilon$$

where  $G_i$  denotes the  $i^{\text{th}}$  snapshot of our network.

Studies were led for time frames of 10, 50 and a 100 day. The persistence of the edges is shown in the graph below:



(a) Persistence of the edges for 70 time shots ( $T \sim 10$  days), correlation thresholded at 0.5 (b) Persistence of the edges for 15 time shots ( $T \sim 50$  days), correlation thresholded at 0.39 (c) Persistence of the edges for 7 time shots ( $T \sim 100$  days), correlation thresholded at 0.6

Figure 6: Plots of the persistence of the edges over time as a function of the threshold level and the time frame  $\Delta$

The previous plot shows the resilience of the edges over time. In particular, we observe a high proportion of "vanishing edges": when considering  $\Delta_T = 100$  and  $\lambda = 0.6$ , 70% of nodes only appear in one time shot. This value decreases to 65% when thresholding at level  $\lambda = 0.39$ . In parallel, we also note that when analyzing from 10-days long time series, the resilience of the edges almost follows a binomial distribution with parameters  $n = 70$  and  $p = 0.28$ , which is the distribution that we would expect if the edges were drawn purely at random with a market effect of  $\alpha \approx 0.15$  and conditioning on the correlations to be high enough to be selected. This shows that our thresholding

technique produces many edges hold no particular meaning in terms of interdependencies, which in turn, pushes us to try to restrict the graph to the edges that exhibit high resilience.

Another approach would have been to consider the graph inferred by retaining only the statistically portion of the graph (i.e, retaining only edges whose p-values are statistically at the Bonferroni-adjusted level). Applying this technique, we note that less then 100 edges are deemed as statistically significant for the 100-days time frame, and between 200 to 2000 for the 50-day time frame: the variability from snapshot to snapshot in the number of persistent edges is thus very high. Moreover, this approach yields quickly vanishing edges: for a time frame of 11 days for instance, conditionally on the edge being deemed as significant a time  $t$ , 86% of the edges will only appear once, and only 1.7% actually appear more than 3 time over the 70 time frames. This is due to the fact that the variability of the correlation is important, and the Bonferroni-based approach is perhaps too conservative, detecting edges that change often. This in turn justifies considering the time based approach as a more pertinent marker of significance, since it enables us to select a larger number of more stable edges.

## 6 Analysis of the core graph

Building up on the observations made in the previous section, we propose to analyze in depth the core graph that was obtained by looking at time-persistent edges. We begin by noting the strong dependency of this graph structure with the threshold level that was selected.

In particular, let us begin by evaluating the results for the latent graph corresponding to a time window of  $\Delta_T = 100$  consecutive days.

In this approach, we use a correlation of 0.55, so as to have a graph sparsity varying from snapshot to snapshot between 12% and 20%. In particular, to assess the properties of the graph, we suggest cross-checking our results against the hard labeling provided by the standard sector classification. Figure 9b shows the flow  $f_{ij}$  between sector  $i$  and  $j$ , defined as a normalized cut  $f_{ij} = \frac{\sum_{n_i \in S_i} \sum_{n_j \in S_j} \mathbb{1}[(i,j) \in G]}{\sqrt{\text{vol}(S_i)\text{vol}(S_j)}}$ :

dark blue edges indicate a high flow, whereas thin yellow lines indicate a weak interaction between sectors. In particular, we note the predominance of the financial sector as well as the capital goods: the sectors seem to act as "two suns" around which the entire financial universe revolves, and, as shown the weighted flows displayed by figure 9, these sectors are themselves tightly connected.

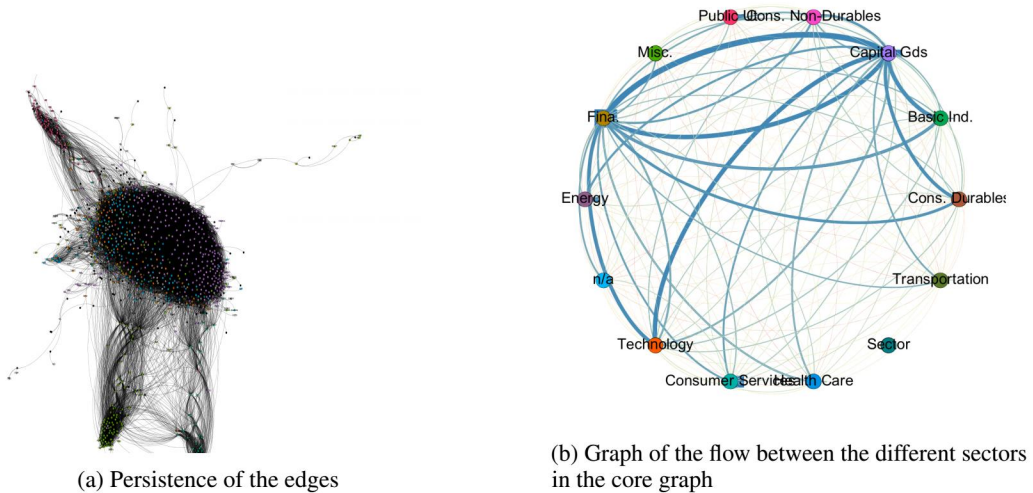
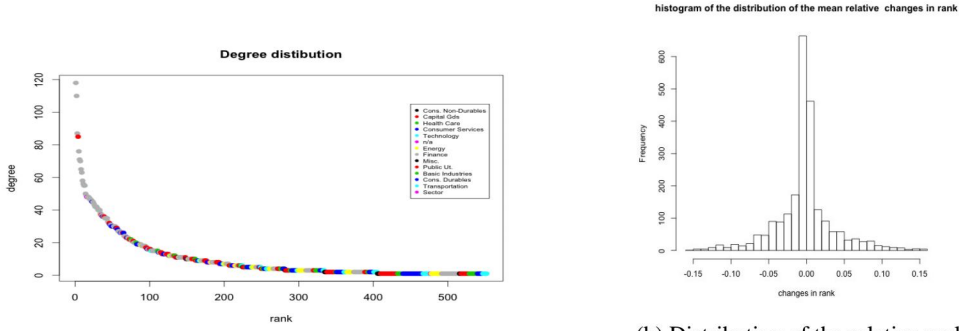


Figure 7: Visualization of the "core graph"

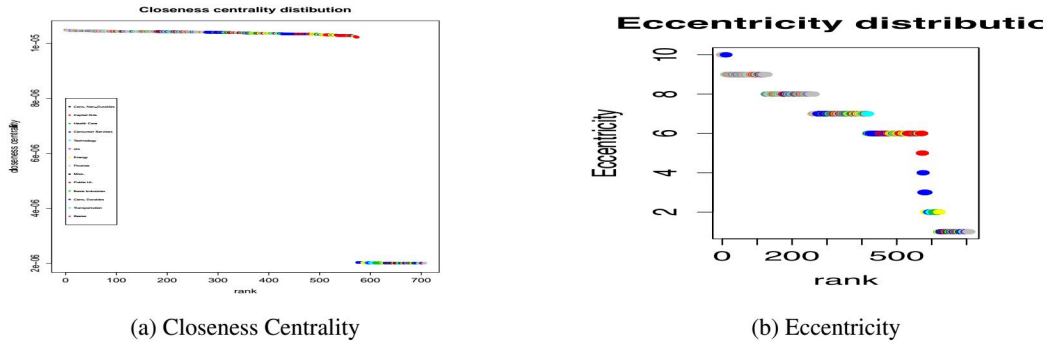
The then propose to focus on analyzing the features of our graph, looking in particular at the degree distribution and betweenness centrality measures.



(a) Degree distribution of the most persistent edges (persisting in at least 6 of the 7 time frames). The colors indicate the nodes's sector

(b) Distribution of the relative rank variation: we compute for each node  $i$  the difference in rank  $r_i^t$  between two consecutive time series:  $\Delta r_i = (r_i^{t+1} - r_i^t)/r_i^t$

Figure 8: Degree distribution and rank variability for  $\Delta_T = 100$  and  $\lambda = 0.55$



(a) Closeness Centrality

(b) Eccentricity

Figure 9: Visualization of properties of the graph for  $\Delta_T = 100$ ,  $\lambda = 0.55$ : the different colors correspond to the node's sector

First of all, we begin by noting the predominance of the financial and capital good sectors in our graph: regardless of the time frame selected, these sectors consistently rank first in terms of highest degree, eccentricity and closeness centrality. The highly connected components of the graph contains information about the stocks that act as reference for a large set of other stocks. Hence, our method highlights the financial and capital goods sector to be "key" sector: they are the most connected, and hence, the more likely to be exposed to contagion. Interestingly, the fact that these sectors are so well connected also guarantees a certain stability, as they can benefit from so many stabilizing influences, and this mitigated effect is actually an ongoing-research topic. On the other hand, sectors such as transportations and Public Utilities, are less well connected to the graph, and hence, can be considered at safer distance from systemic risk. We also note that the companies that score the highest in terms of "market influence" (as defined by their degree) are financial institutions that are unknown to the general public: "IBOC" (a financial institution) is the most connected component, participating in 118 edges, followed by "CBU", with a total of 110 connections. We also note that these features also exhibit some variability, as depicted in figure 8b, which shows the relative change in degree rank from time window to time window (in the thresholded graph).

## 7 Spectral Clustering

The intuition that we started with is that there are some *latent* communities in which there are some shared economical, political, ... factors that contribute to the stock prices. We apply spectral clustering algorithm to our graph to detect these communities. Since companies within the same industry are likely to share more contributing factors, we expect the resulting clustering to have high overlap with the given labels. Hence, we hope to end up with high purity when the industries are



used as the class labels. Afterwards, for each cluster, we compute the proportion of each industry and output the five most represented of these industries. We note that this analysis is more relevant for larger cluster size.

We applied the abovementioned analysis to the graph obtained by considering correlations above 0.55 over three years. This graph contains 15,000 edges. About half of the nodes have degree zero, and we thus choose to remove them. Prior to applying any clustering algorithm, we compute the clustering coefficient of this graph. The clustering coefficient for this graph is equal to about 0.57. Note that this graphs contains about 2.3% of the edges that the complete graph has. Hence, our graph has high potential to be clustered.

We then apply spectral clustering, which requires to pre-specify the number of clusters. Since there are 138 industries, we use 100 clusters. For this number of clusters, the purity that we get is equal to 0.48. Moreover, in the table below, we have listed the five most contributing industries to each of the three largest clusters.

Cluster 1[353]	Cluster 2[109]	Cluster 3[98]
Power Generation(0.94)	Fluid Controls(0.5)	Banks(0.38)
Electric Utilities: Central(0.79)	Paints/Coatings(0.5)	Major Banks(0.30)
Oil/Gas Transmission(0.71)	Railroads(0.4)	Office Equipment/Supplies/Services(0.30)
Investment Managers(0.56)	Consumer Electronics/Appliances(0.38)	Life Insurance(0.23)
Accident &Health Insurance(0.50)	Industrial Machinery/Components(0.25)	Savings Institutions(0.19)

Table 1: Most contributing industries to three largest clusters. Number in the brackets indicate the size of the cluster and the numbers in the paranthesis indicate the proportion of companies in that industry that lie in the specified cluster.

As shown in this table, the industries in each column are tightly related to each other – for instance, the first cluster regroups industries from the Energy sector. The coherence of the industry labels with the detected clusters upvotes the validity of our results. We also note that this clustering gives insight into less obvious inter-dependencies: for instance, in cluster one, this clustering has exhibited the existence of strong links between some energy companies and some firms from the Accident and Health Insurance industry. Hence, the new clustering can be used as a basis for more "risk-proof" investment: we have provided new "pools" of stocks, and, a reasonable investment strategy would consist in sharing the risk across these pools.

## 8 Closing remarks

Our analysis builds upon work from prior papers to extend their conclusions to a more interpretable and flexible network structure. We have in particular achieved in extracting a core subgraph of highly interdependent – and thus susceptible to risk– stocks from more independent peripheries, as well as a means of evaluating this classification’s significance. Moreover, the clusters that have been obtained by running Spectral Clustering on our inferred network yields new pools of interdependent stocks, thus giving more insight into the improvement of one’s risk diversification strategy.

## References

- [1] Ulrike von Luxburg *A Tutorial on Spectral Clustering*. Statistics and Computing, 17 (4), 2007.
- [2] M Soltanolkotabi, EJ Candès *A geometric analysis of subspace clustering with outliers* The Annals of Statistics, 2195-2238
- [3] T. Di Matteo, F. Pozzi, and T. Aste, *The use of dynamical networks to detect the hierarchical organization of market sector*, European Physics Journal B, 73,3-11 (2010).
- [4] E.B. Fox, M.I. Jordan, *Mixed Membership Models for Time Series*, arXiv preprint arXiv:1309.3533 (2013).

- [5] E.P. Xing, W. Fu, L. Song, A State-Space Mixed Membership Blockmodel For Dynamic Network Tomography, *The Annals of Applied Statistics*, 2010, Vol4, No.2 , 535-566.
- [6] Fox, Emily, et al. *Sharing features among dynamical systems with beta processes*. Advances in Neural Information Processing Systems. 2009
- [7] Tola, V., Lillo, F., Gallegati, M., Mantegna, R. N. (2008). *Cluster analysis for portfolio optimization* . *Journal of Economic Dynamics and Control*, 32(1), 235-258
- [8] Mantegna, R.N, *Hierarchical structure in financial markets*. *The European Physical Journal Matter and Complex Systems*, 11(1), 193-197.
- [9] T. Di Matteo, T. Aste, *Spread of risk across financial markets: Better to invest in the peripheries*, *Scientific Reports* · April 2013, DOI: 10.1038/srep01665
- [10] Jukka-Pekka Onnela, *Complex Networks in the study of financial and social systems* , PhD dissertation, HELSINKI UNIVERSITY OF TECHNOLOGY, 2006
- [11] Laurent Laloux, Pierre Cizeau, Jean-Philippe Bouchaud, and Marc Potters, *Noise Dressing of Financial Correlation Matrices*, *Physical Review Letters*, Volume 83 number 7, 16 August 1999
- [12] Aste, Tomaso, W. Shaw, and Tiziana Di Matteo. *Correlation structure and dynamics in volatile markets*. *New Journal of Physics* 12.8 (2010): 085009.
- [13] Kenett, Dror Y., et al. *Dominating clasp of the financial sector revealed by partial correlation analysis of the stock market*. *PloS one* 5.12 (2010): e15032.

## 9 Teammate Contribution

- Claire: Section 5 and 6 (dynamic approach implementation)-text and code
- Nima: Section 4 and 7 (significance testing and Spectral Clustering approach)-text and code
- Interpretation and writing up of the poster and other sections was done together.