Outbreak Detection in Networks

CS224W: Social and Information Network Analysis
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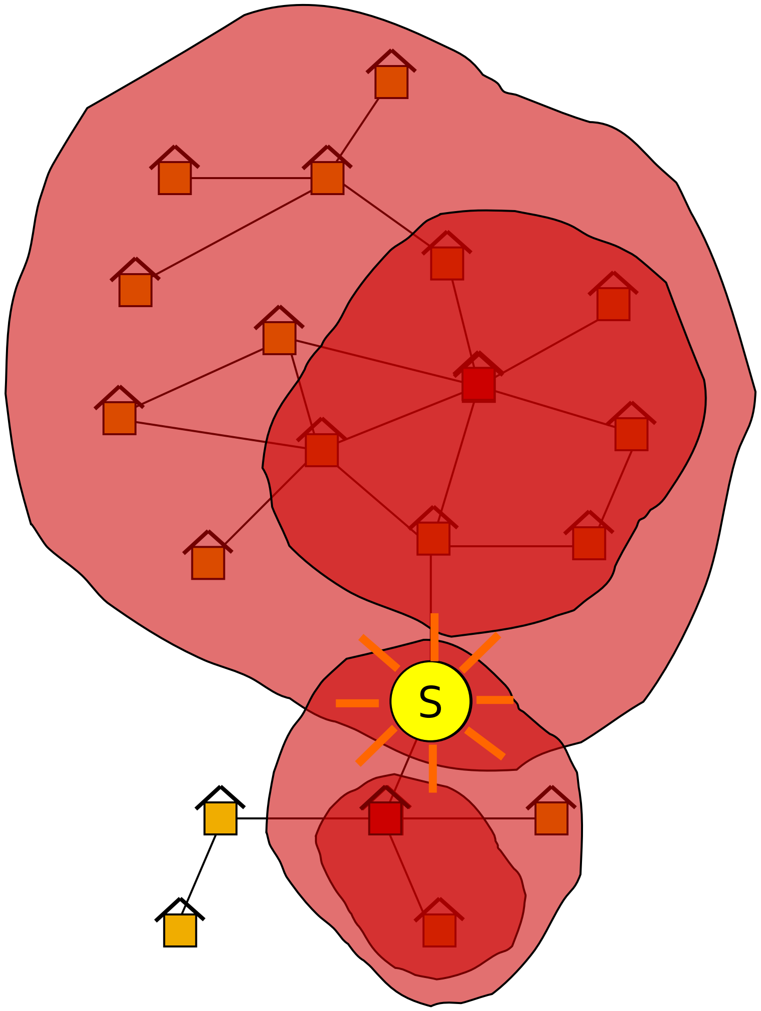
http://cs224w.stanford.edu

Plan for Today

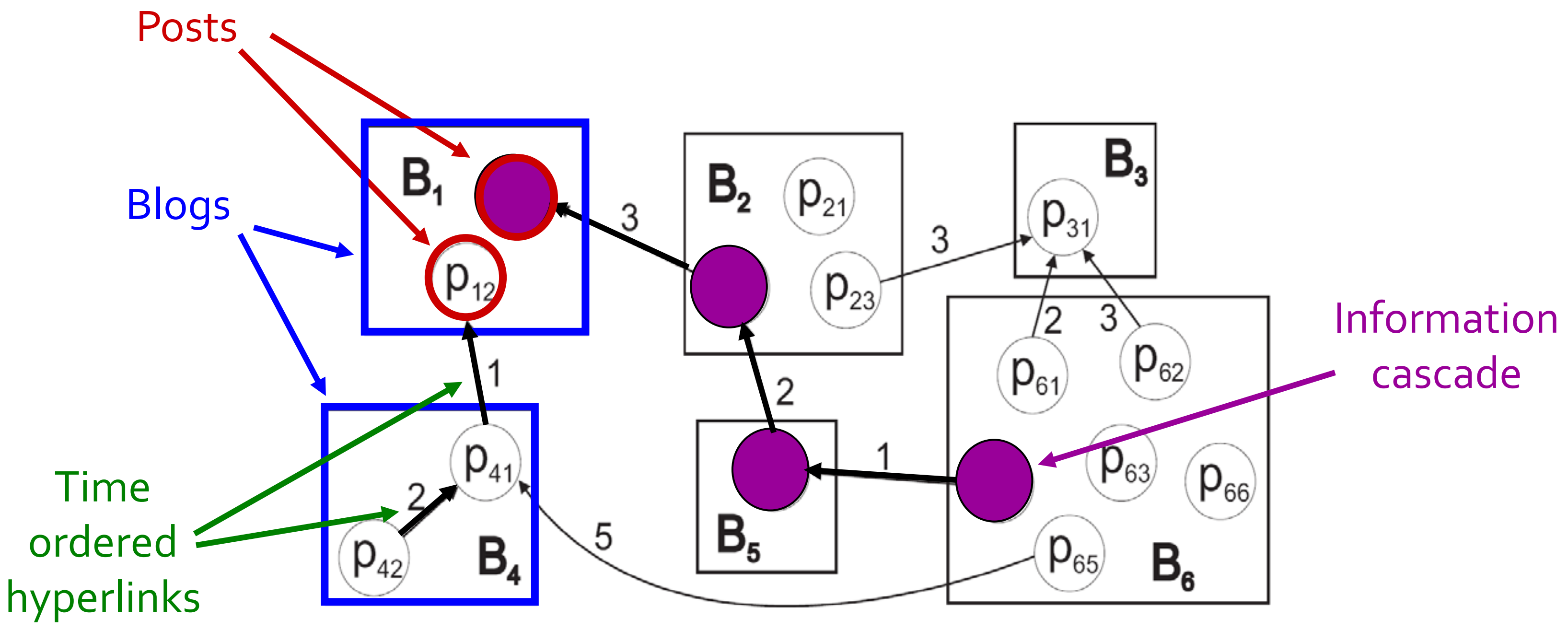
- **(1) New problem: Outbreak detection**
- **(2) Develop an approximation algorithm**
  - It is a submodular opt. problem!
- **(3) Speed-up greedy hill-climbing**
  - Valid for optimizing general submodular functions (i.e., also works for influence maximization)
- **(4) Prove a new "data dependent" bound on the solution quality**
  - Valid for optimizing any submodular function (i.e., also works for influence maximization)

Detecting Contamination Outbreaks

- Given a real city water distribution network
- And data on how contaminants spread in the network
- Detect the contaminant as quickly as possible
- Problem posed by the *US Environmental Protection Agency*

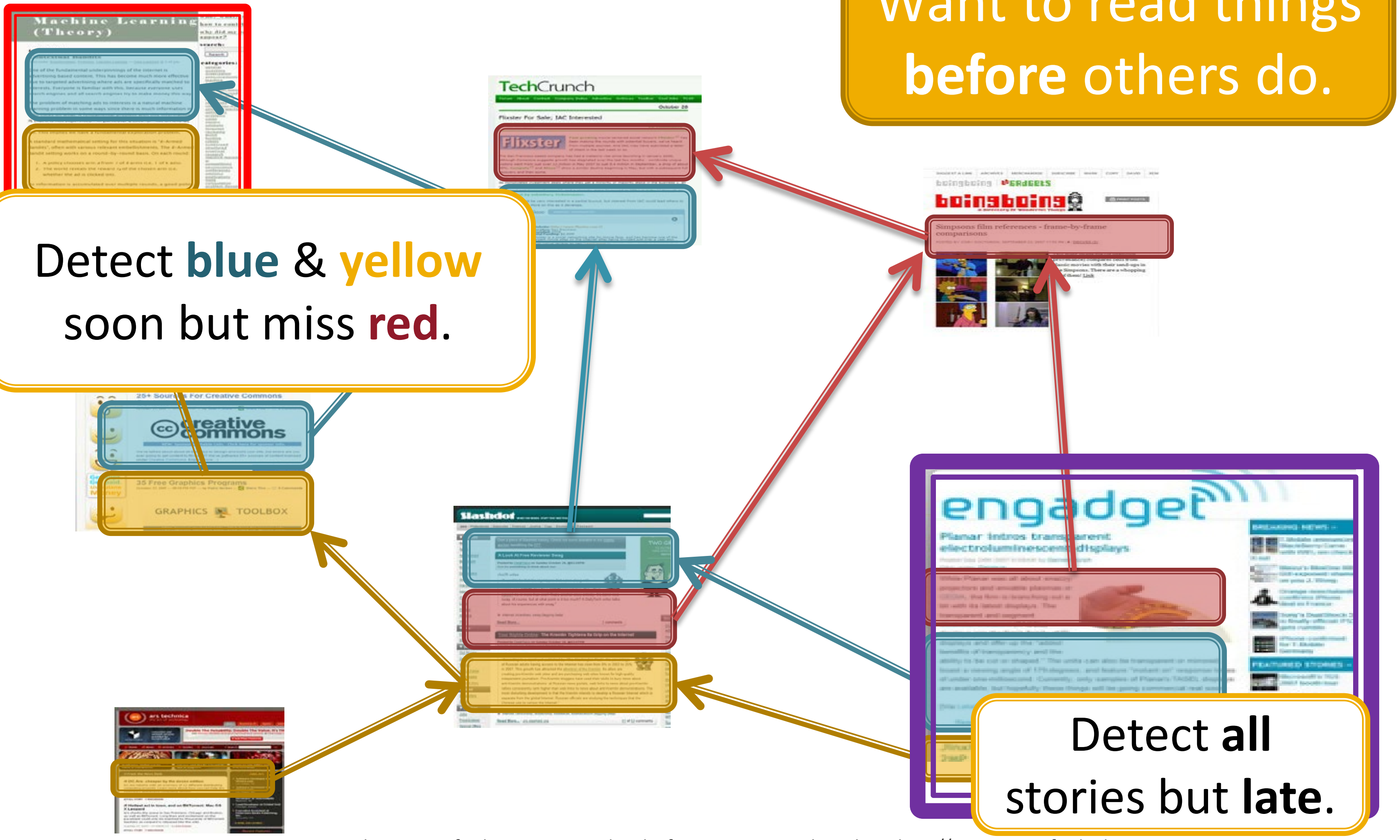


Detecting Information Outbreaks



Which blogs should one read to **detect cascades** as **effectively** as possible?

Detecting Information Outbreaks



Want to read things **before** others do.

Detect **blue** & **yellow** soon but miss **red**.

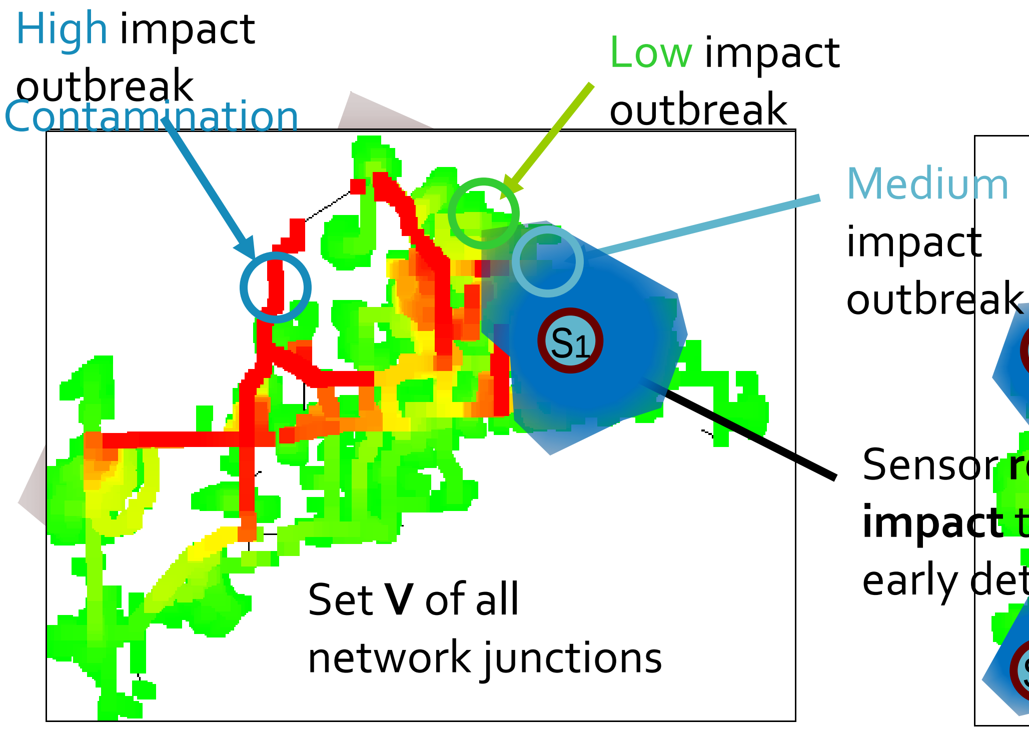
Detect **all** stories but **late**.

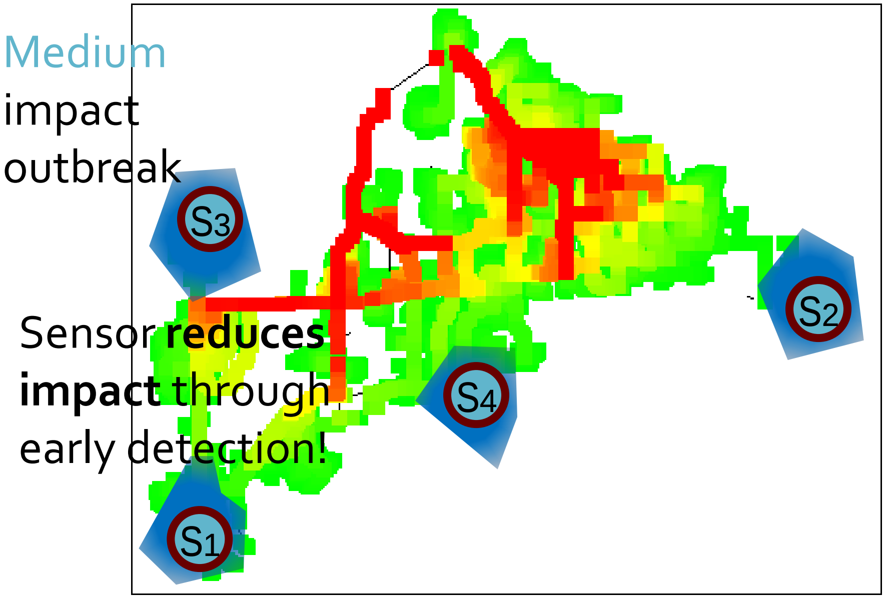
General Problem

- **Both of these two are an instance of the same underlying problem!**
- **Given a dynamic process spreading over a network we want to select a set of nodes to detect the process effectively**
- **Many other applications:**
  - Epidemics
  - Influence propagation
  - Network security

Water Network: Utility

- **Utility of placing sensors:**
  - Water flow dynamics, demands of households, …
- **For each subset S ⊆ V compute utility *f(S)***





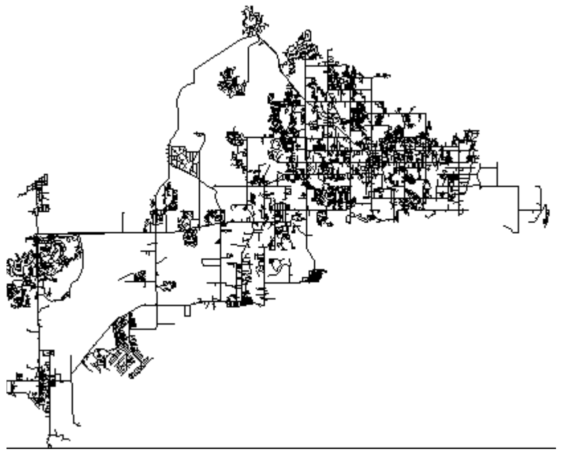
High sensing quality f(S) = 0.9

Low sensing quality f(S)=0.01

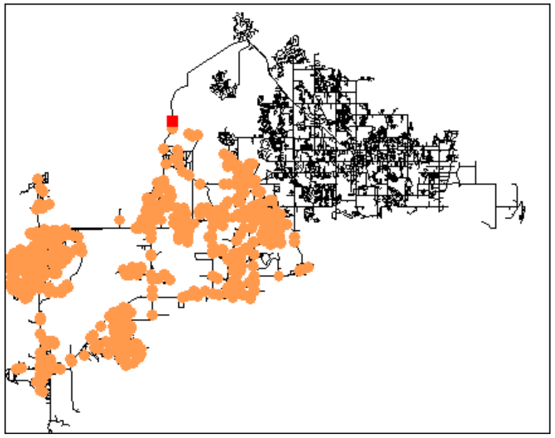
Problem Setting: Contamination

Given:

- Graph $G(V, E)$
- Data on **how outbreaks spread over the G:**
  - For each outbreak i we know the time $T(i, u)$ when outbreak i contaminates node u



Water distribution network
(physical pipes and junctions)

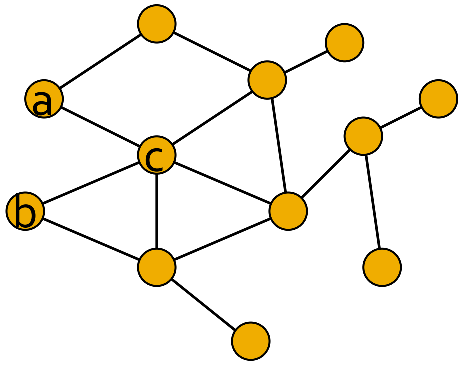


Simulator of water consumption&flow
(built by Mech. Eng. people)
We simulate the contamination spread for every possible location.

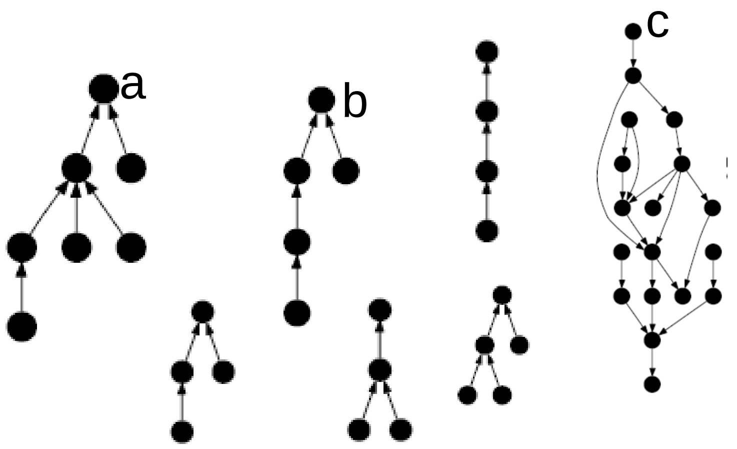
Problem Setting: Blogospehere

Given:

- Graph $G(V, E)$
- Data on **how outbreaks spread over the G**:
  - For each outbreak i we know the time $T(i, u)$ when outbreak i contaminates node u



The network of the blogosphere



Traces of the information flow

Collect lots of blogs posts and trace hyperlinks to obtain data about information flow from a given blog.

Problem Setting

Given:

- Graph $G(V,E)$
- Data on **how outbreaks spread over the G:**
  - For each outbreak i we know the time $T(i,u)$ when outbreak i contaminates node u

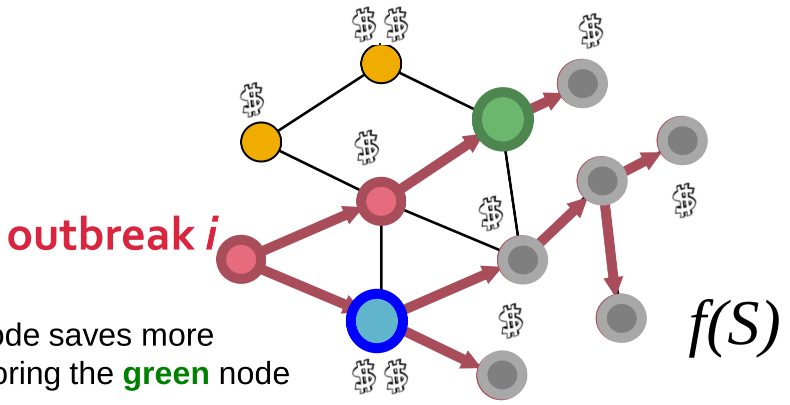
- **Goal:** Select a subset of nodes ***S*** that maximizes the expected **reward**:

$$\max_{S \subseteq V} f(S) = \sum_i \underbrace{P(i)\, f_i(S)}_{\text{Expected reward for detecting outbreak } i}$$

subject to: $cost(S) < B$

Two Parts to the Problem

- **Reward**
  - **(1)** Minimize time to detection
  - **(2)** Maximize number of detected propagations
  - **(3)** Minimize number of infected people
- **Cost** (context dependent):
  - Reading big blogs is more time consuming
  - Placing a sensor in a remote location is expensive



Monitoring **blue** node saves more people than monitoring the **green** node

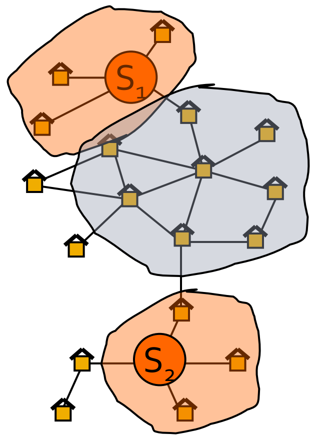
Objective functions are Submodular

$f_i(S)$ is penalty reduction:
$f_i(S) = \pi_i(\emptyset) - \pi_i(S)$

- **Objective functions:**
  - 1) **Time to detection** (**DT**)
    - How long does it take to detect a contamination?
    - **Penalty for detecting at time t:** $\pi_i(t) = \min\{t, T_{max}\}$
  - 2) **Detection likelihood** (**DL**)
    - How many contaminations do we detect?
    - **Penalty for detecting at time t:** $\pi_i(t) = 0, \pi_i(\infty) = 1$
      - Note, this is binary outcome: we either detect or not
  - 3) **Population affected** (**PA**)
    - How many people drank contaminated water?
    - **Penalty for detecting at time t:** $\pi_i(t) =$ {# of infected nodes in outbreak i by time t}.
- **Observation:**
  In all cases detecting sooner does not hurt!

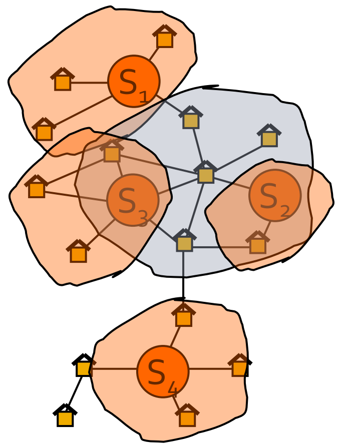
Structure of the Problem

- **Observation: Diminishing returns**



Placement $S=\{s_1, s_2\}$

New sensor:



Placement $S'=\{s_1, s_2, s_3, s_4\}$

Adding s' helps a lot

Adding s' helps very little

Objective functions are Submodular

- **Claim:** For all $\boldsymbol{A} \subseteq \boldsymbol{B} \subseteq \boldsymbol{V}$ and sensors $\boldsymbol{s} \in \boldsymbol{V} \backslash \boldsymbol{B}$
  $$\boldsymbol{f}(\boldsymbol{A} \cup \{\boldsymbol{s}\}) - \boldsymbol{f}(\boldsymbol{A}) \geq \boldsymbol{f}(\boldsymbol{B} \cup \{\boldsymbol{s}\}) - \boldsymbol{f}(\boldsymbol{B})$$
- **Proof: All our objectives are submodular**
  - Fix cascade/outbreak $\boldsymbol{i}$
  - **Show** $\boldsymbol{f_i(A) = \pi_i(\infty) - \pi_i(T(A, i))}$ is submodular
  - **Consider** $\boldsymbol{A} \subseteq \boldsymbol{B} \subseteq \boldsymbol{V}$ and sensor $\boldsymbol{s} \in \boldsymbol{V} \backslash \boldsymbol{B}$
  - **When does node $\boldsymbol{s}$ detect cascade $\boldsymbol{i}$?**
    - **We analyze 3 cases based on when $\boldsymbol{s}$ detects outbreak *i***
    - **(1)** $\boldsymbol{T(s, i) \geq T(A, i)}$: $\boldsymbol{s}$ detects late, nobody benefits:
      $f_i(A \cup \{s\}) = f_i(A)$, also $f_i(B \cup \{s\}) = f_i(B)$ and so
      $f_i(A \cup \{s\}) - f_i(A) = 0 = f_i(B \cup \{s\}) - f_i(B)$

Objective functions are Submodular

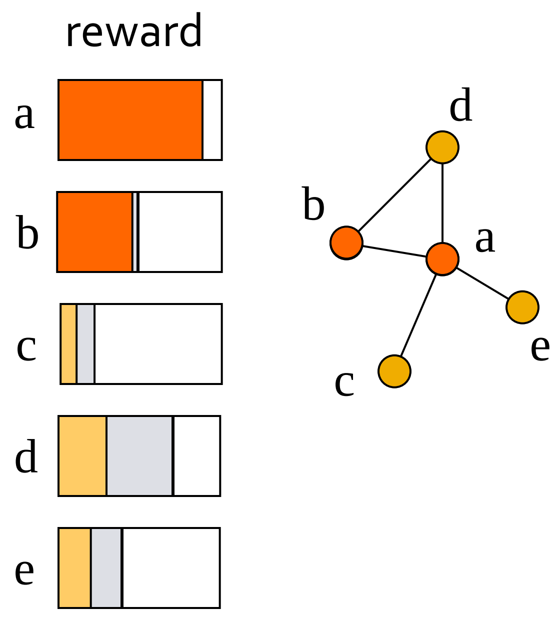
Remember $A \subseteq B$

- **Proof (contd.):**
  - **(2)$T(B,i) \leq T(s,i) < T(A,i)$**: s detects after **B** but before **A**
    s detects sooner than any node in A but after all in B.
    So s only helps improve the solution A (but not B)
    $f_i(A \cup \{s\}) - f_i(A) \geq 0 = f_i(B \cup \{s\}) - f_i(B)$
  - **(3) $T(s,i) < T(B,i)$:** s detects early
    $f_i(A \cup \{s\}) - f_i(A) = [\pi_i(\infty) - \pi_i(T(s,i))] - f_i(A) \geq$
    $[\pi_i(\infty) - \pi_i(T(s,i))] - f_i(B) = f_i(B \cup \{s\}) - f_i(B)$
    - Ineqaulity is due to non-decreasingness of $f_i(\cdot)$, i.e., $f_i(A) \leq f_i(B)$
  - **So, $f_i(\cdot)$ is submodular!**
- **So, $f(\cdot)$ is also submodular**

$$f(S) = \sum_i P(i)\, f_i(S)$$

Background: Submodular functions

Hill-climbing



Add sensor with highest marginal gain

- **What do we know about optimizing submodular functions?**
  - A hill-climbing (i.e., greedy) is near optimal: $\left(1 - \frac{1}{e}\right) \cdot OPT$
- **But:**
  - **(1)** This only works for **unit cost case!** (each sensor costs the same)
    - **For us each sensor s has cost $c(s)$**
  - **(2)** Hill-climbing algorithm is slow
    - At each iteration we need to re-evaluate marginal gains of all nodes
    - Runtime $O(|V| \cdot K)$ for placing K sensors

CELF: Algorithm for optimizing submodular functions under cost constraints

Towards a New Algorithm

- **Consider the following algorithm to solve the outbreak detection problem: Hill-climbing that ignores cost**
  - Ignore sensor cost
  - Repeatedly select sensor with highest marginal gain
  - Do this until the budget is exhausted
- **Q: How well does this work?**
- **A: It can fail arbitrarily badly! ☹**
  - Next we come up with an example where Hill-climbing solution is arbitrarily away from OPT

Problem 1: Ignoring Cost

- **Bad example when we ignore cost:**
  - $\boldsymbol{n}$ sensors, budget $\boldsymbol{B}$
  - $\boldsymbol{s_1}$: reward $\boldsymbol{r}$, cost $\boldsymbol{B}$
  - $\boldsymbol{s_2} \dots \boldsymbol{s_n}$: reward $\boldsymbol{r-\varepsilon}$, cost $\mathbf{1}$
  - Hill-climbing always prefers more expensive sensor $\boldsymbol{s_1}$ with reward $\boldsymbol{r}$ (and exhausts the budget).
    It never selects cheaper sensors with reward $\boldsymbol{r-\varepsilon}$
    $\rightarrow$ **For variable cost it can fail arbitrarily badly!**
- Idea: What if we optimize **benefit-cost ratio**?

$$s_i = \arg\max_{s \in V} \frac{f(A_{i-1} \cup \{s\}) - f(A_{i-1})}{\boldsymbol{c(s)}}$$

Greedily pick sensor $\boldsymbol{s_i}$ that maximizes benefit to cost ratio.

Problem 2: Benefit-Cost

- **Benefit-cost ratio can also fail arbitrarily badly!**
- **Consider**: budget B:
  - **2 sensors s_1 and s_2:**
    - **Costs:** $c(s_1) = \varepsilon,\ c(s_2) = B$
    - **Only 1 cascade:** $f(s_1) = 2\varepsilon,\ f(s_2) = B$
  - **Then benefit-cost ratio is:**
    - $B/c(s_1) = 2$ and $B/c(s_2) = 1$
  - So, we first select s_1 and then can not afford s_2

$\rightarrow$ We get reward 2ε instead of B! Now send $\varepsilon \to 0$ and we get **arbitrarily bad solution!**

This algorithm incentivizes choosing nodes with very low cost, even when slightly more expensive ones can lead to much better global results.

Solution: CELF Algorithm

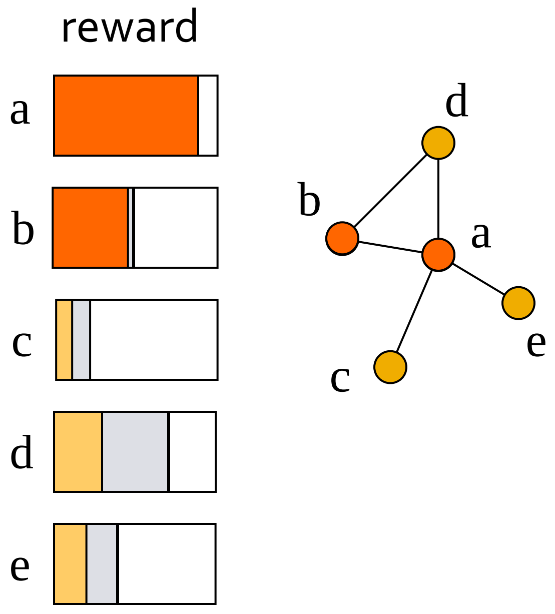
- **CELF** (**Cost-Effective Lazy Forward-selection**)
  A two pass greedy algorithm:
    - **Set (solution) S'**: Use **benefit-cost greedy**
    - **Set (solution) S''**: Use **unit-cost greedy**
  - **Final solution: $S = arg\,max(f(S'), f(S''))$**
- **How far is CELF from (unknown) optimal solution?**
- **Theorem**: **CELF is near optimal** [Krause&Guestrin, '05]
  - CELF achieves ***½(1-1/e)*** factor approximation!

This is surprising: We have two clearly suboptimal solutions, but taking the best of them always gives us a near-optimal solution.

Speeding-up Hill-Climbing: Lazy Evaluations

Background: Submodular functions

Hill-climbing



Add sensor with highest marginal gain

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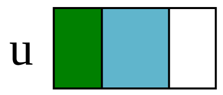
  - A hill-climbing (i.e., greedy) is near optimal $(1 - \frac{1}{e} \cdot OPT)$
- **But:**
  - **(2)** Hill-climbing algorithm is **slow!**
    - At each iteration we need to re-evaluate marginal gains of all nodes
    - Runtime $O(|V| \cdot K)$ for placing K sensors

Speeding up Hill-Climbing

- **In round $i+1$:** So far we picked $S_i = \{s_1, \ldots, s_i\}$
  - Now pick $\mathbf{s}_{i+1} = \arg\max_{u} f(S_i \cup \{u\}) - f(S_i)$
    - This our old friend – greedy hill-climbing algorithm. **It maximizes the "marginal benefit"**
      $$\delta_i(u) = f(S_i \cup \{u\}) - f(S_i)$$
- **By submodularity property:**
  $$f(S_i \cup \{u\}) - f(S_i) \geq f(S_j \cup \{u\}) - f(S_j) \text{ for } i < j$$
- **Observation: By submodularity:**
  For every u
  $\delta_i(u) \geq \delta_j(u)$ for $i < j$ since $S_i \subseteq S_j$

  Marginal benefits δ_i(u) only shrink!
  (as i grows)

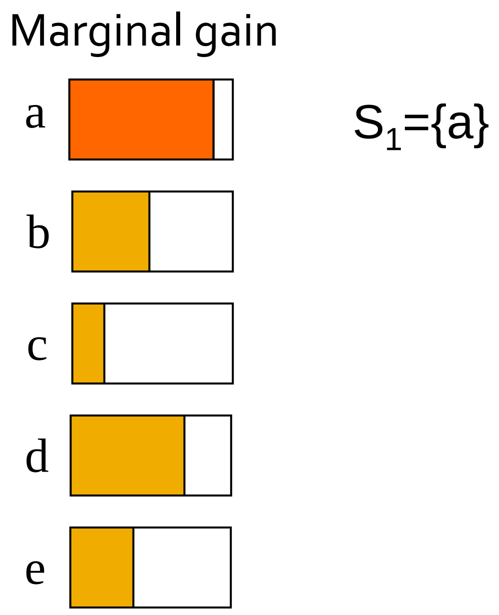
$\delta_i(u) \geq \delta_j(u)$



Activating node *u* in step *i* helps more than activating it at step *j* (j>i)

Lazy Hill Climbing

- **Idea:**
  - Use δ_i as upper-bound on δ_j $(j > i)$
- **Lazy hill-climbing:**
  - Keep an ordered list of marginal benefits δ_i from previous iteration
  - Re-evaluate δ_i only for top node
  - Re-sort and prune



$$f(S \cup \{u\}) - f(S) \geq f(T \cup \{u\}) - f(T) \qquad S \subseteq T$$

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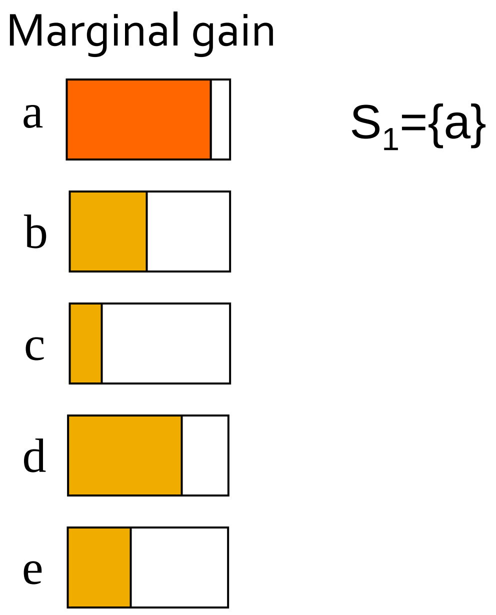

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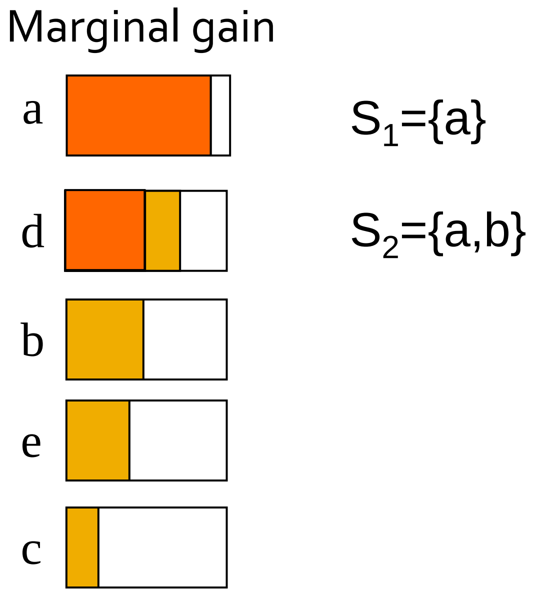

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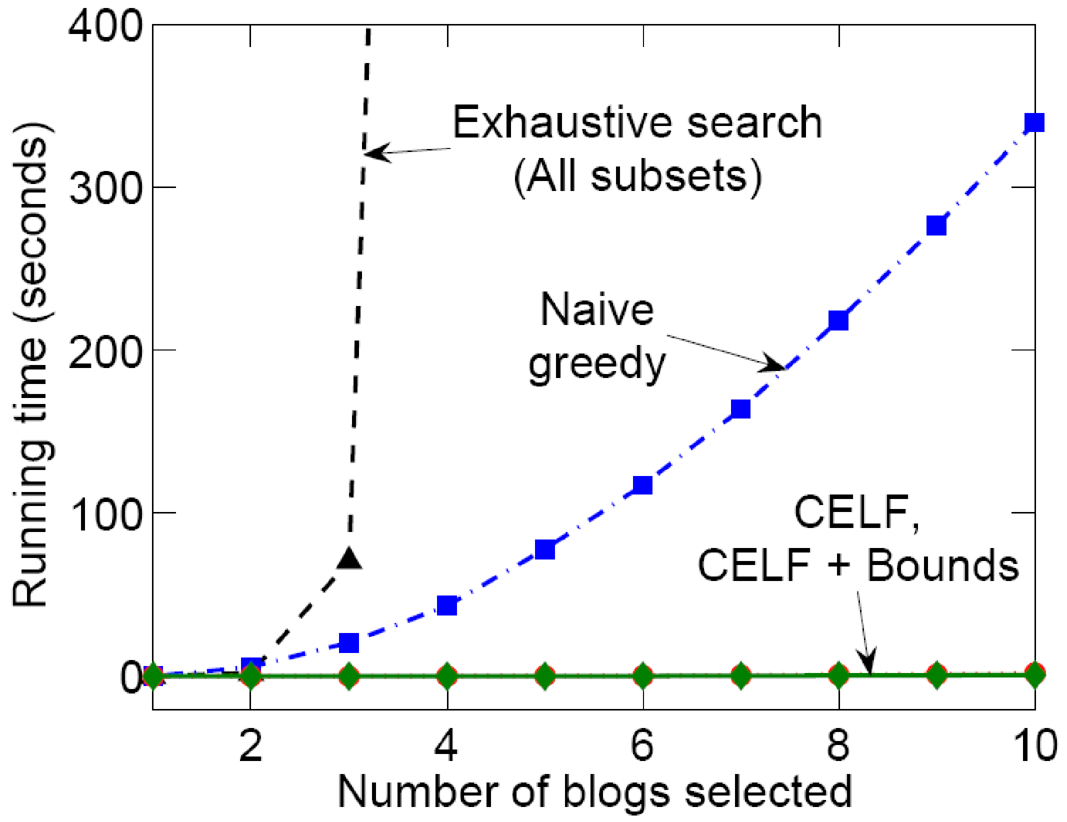
CELF: Scalability



- CELF (using Lazy evaluation) runs **700** times faster than greedy hill-climbing algorithm

Data Dependent Bound on the Solution Quality

Solution Quality

- **Back to the solution quality!**
- **The (1-1/e) bound for submodular functions is the worst case bound (worst over all possible inputs)**
- **Data dependent bound:**
  - Value of the bound depends on the input data
    - On "easy" data, hill climbing may do better than 63%
  - **Can we say something about the solution quality when we know the input data?**

Data Dependent Bound

- Suppose $\boldsymbol{S}$ **is some solution** to $\boldsymbol{f}(\boldsymbol{S})$ s.t. $|\boldsymbol{S}| \leq \boldsymbol{k}$
  - $\boldsymbol{f}(\boldsymbol{S})$ is monotone & submodular
- Let $\boldsymbol{OPT} = \{\boldsymbol{t}_1, \ldots, \boldsymbol{t}_k\}$ be the OPT solution
- For each $\boldsymbol{u}$ let $\boldsymbol{\delta}(\boldsymbol{u}) = \boldsymbol{f}(\boldsymbol{S} \cup \{\boldsymbol{u}\}) - \boldsymbol{f}(\boldsymbol{S})$
- Order $\boldsymbol{\delta}(\boldsymbol{u})$ so that $\boldsymbol{\delta}(\mathbf{1}) \geq \boldsymbol{\delta}(\mathbf{2}) \geq \ldots$
- **Then:** $\boldsymbol{f}(\boldsymbol{OPT}) \leq \boldsymbol{f}(\boldsymbol{S}) + \sum_{i=1}^{k} \boldsymbol{\delta}(\boldsymbol{i})$
  - **Note:**
    - This is a data dependent bound ($\delta(u)$ depends on input data)
    - Bound holds for **any** algorithm
      - Makes no assumption about how $\boldsymbol{S}$ was computed
    - For some inputs it can be very “loose” (worse than 63%)

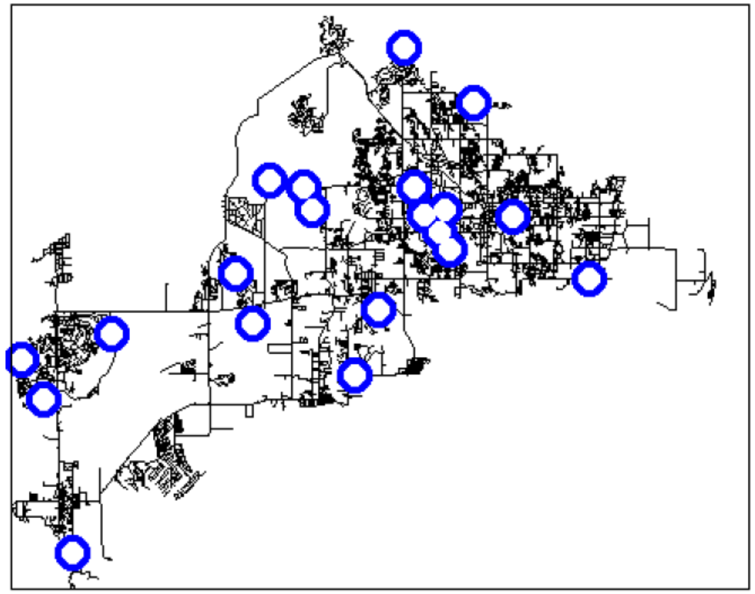
Data Dependent Bound

- **Claim:**
  - For each u let $\boldsymbol{\delta(u) = f(S \cup \{u\}) - f(S)}$
  - Order $\boldsymbol{\delta(u)}$ so that $\boldsymbol{\delta(1) \geq \delta(2) \geq \ldots}$
  - **Then:** $\boldsymbol{f(OPT) \leq f(S) + \sum_{i=1}^{k} \delta(i)}$
- **Proof:**
  - $f(OPT) \leq f(OPT \cup S) =$
    $f(S) + \sum_{i=1}^{k} [f(S \cup \{t_1 \ldots t_i\}) - f(S \cup \{t_1 \ldots t_{i-1}\})]$
  - $\leq f(S) + \sum_{i=1}^{k} [f(S \cup \{t_i\}) - f(S)]$ (we proved this last time)
  - $= f(S) + \sum_{i=1}^{k} \delta(t_i)$ Instead of taking $t_i \in OPT$ (of benefit $\delta(t_i)$), we take the best possible element ($\delta(i)$)
  - $\leq f(S) + \sum_{i=1}^{k} \delta(i) \Rightarrow \boldsymbol{f(T) \leq f(S) + \sum_{i=1}^{k} \delta(i)}$

Case Study: Water distribution network & blogs

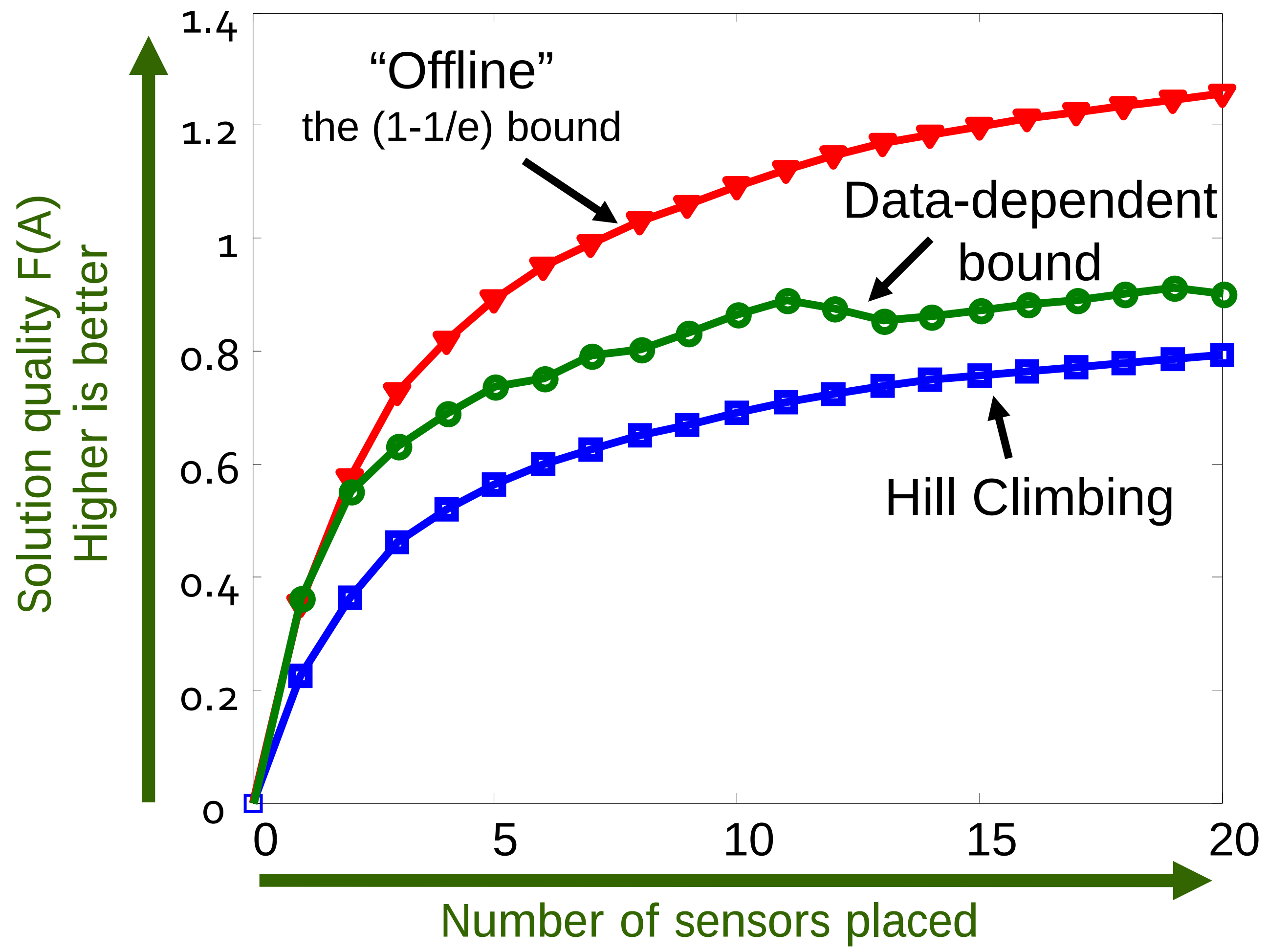
Case Study: Water Network

- **Real metropolitan area water network**
  - V = 21,000 nodes
  - E = 25,000 pipes



- Use a cluster of 50 machines for a month
- Simulate 3.6 million epidemic scenarios (random locations, random days, random time of the day)

Bounds on the Optimal Solution

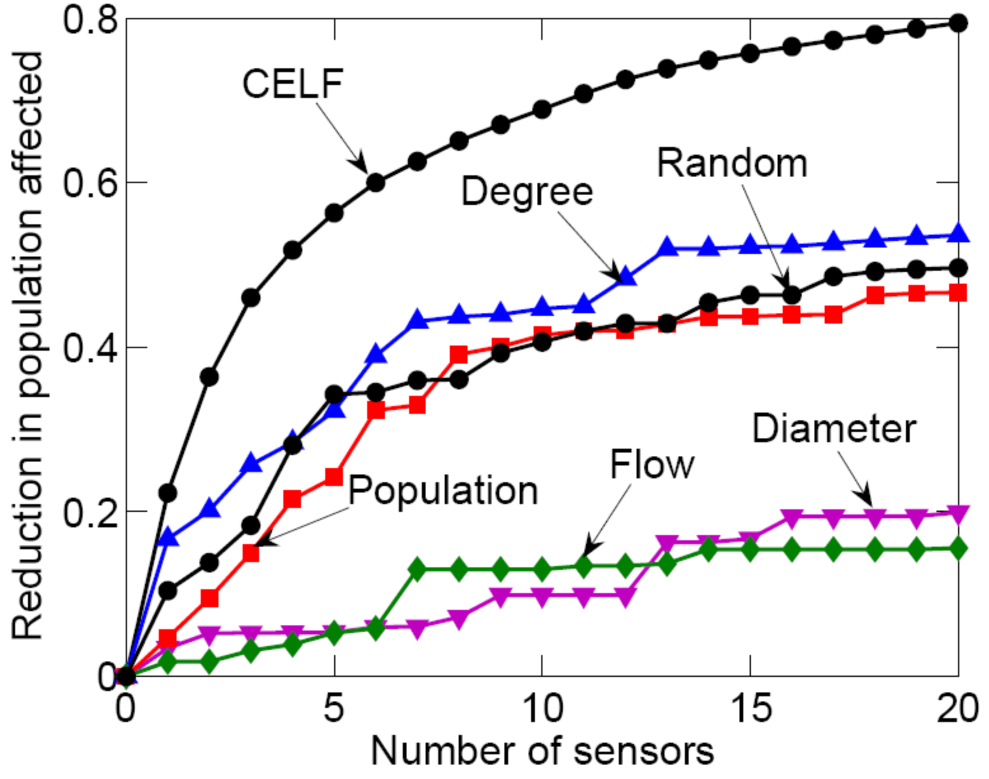


Data-dependent bound is much tighter

(gives more accurate estimate of alg. performance)

[w/ Ostfeld et al., J. of Water Resource Planning]

Water: Heuristic Placement



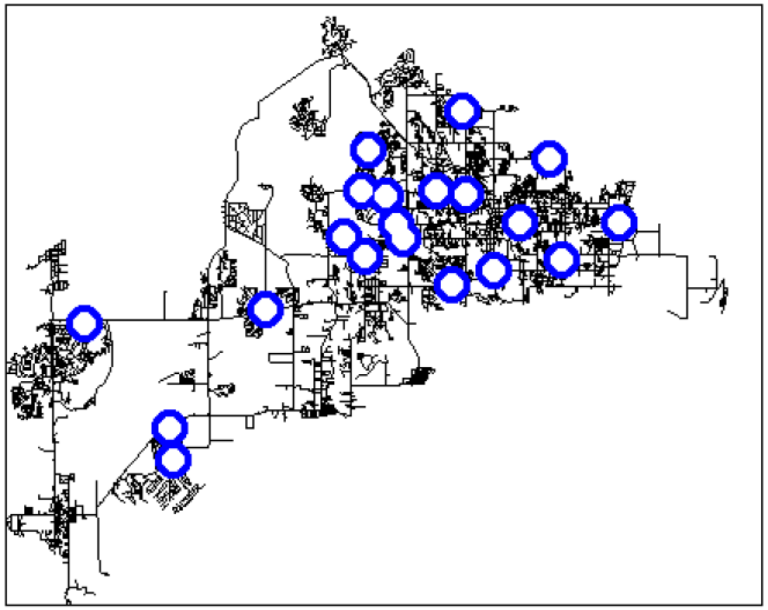
- **Placement heuristics perform much worse**

Author	Score
CELF	**26**
Sandia	21
U Exter	20
Bentley systems	19
Technion (1)	14
Bordeaux	12
U Cyprus	11
U Guelph	7
U Michigan	4
Michigan Tech U	3
Malcolm	2
Proteo	2
Technion (2)	1

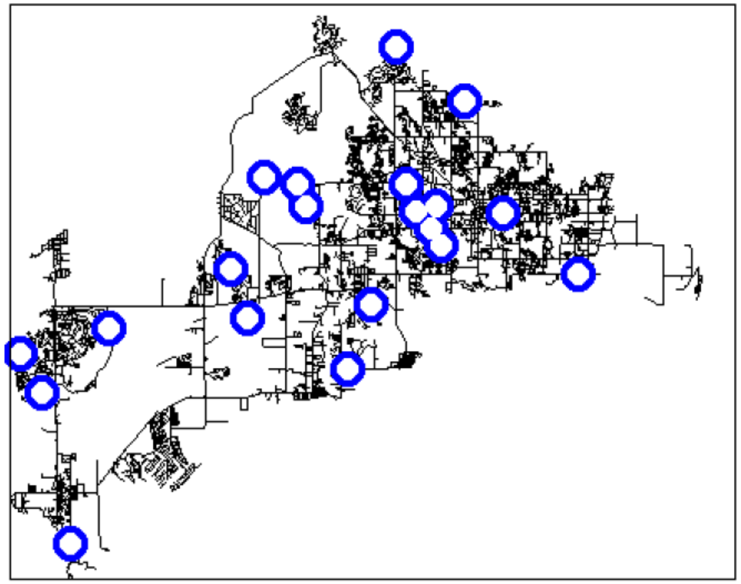
Battle of Water Sensor Networks competition

Water: Placement visualization

- Different objective functions give different sensor placements

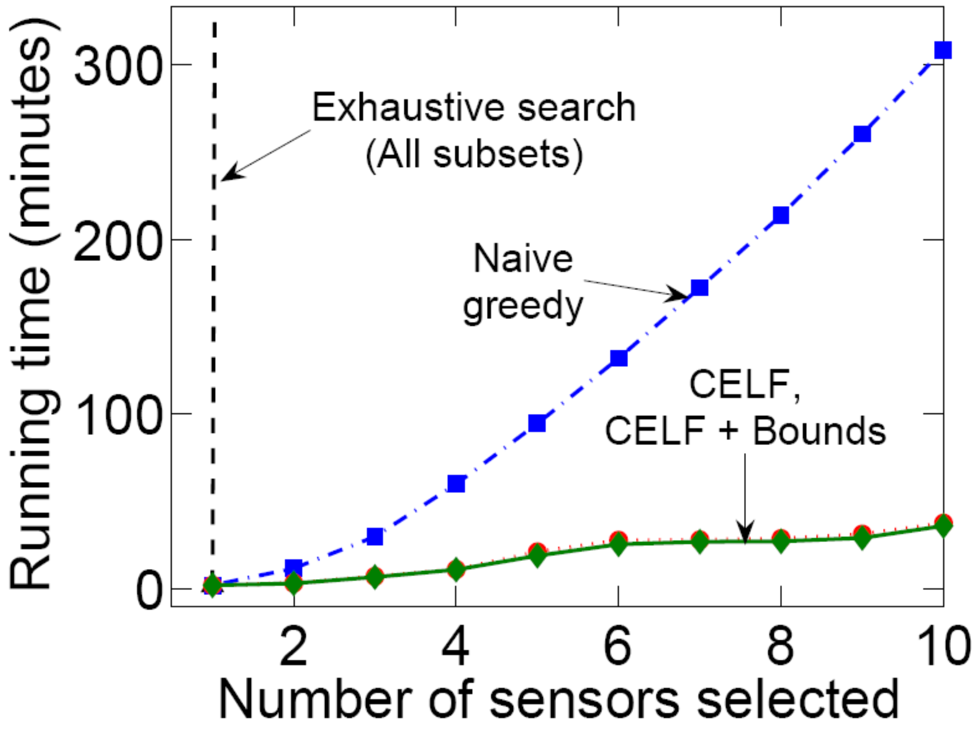


Population affected



Detection likelihood

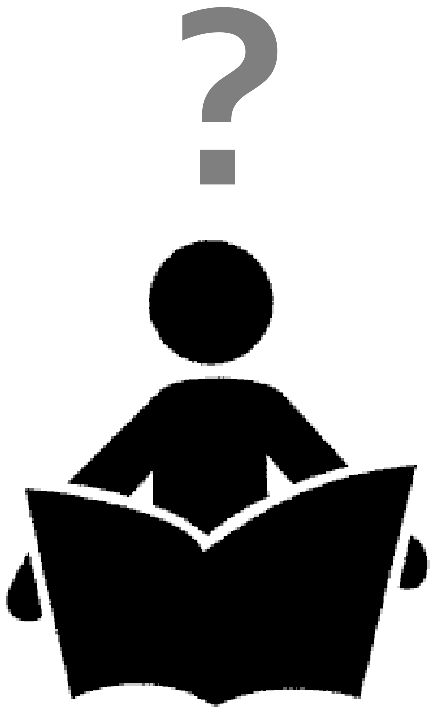
Water: Scalability



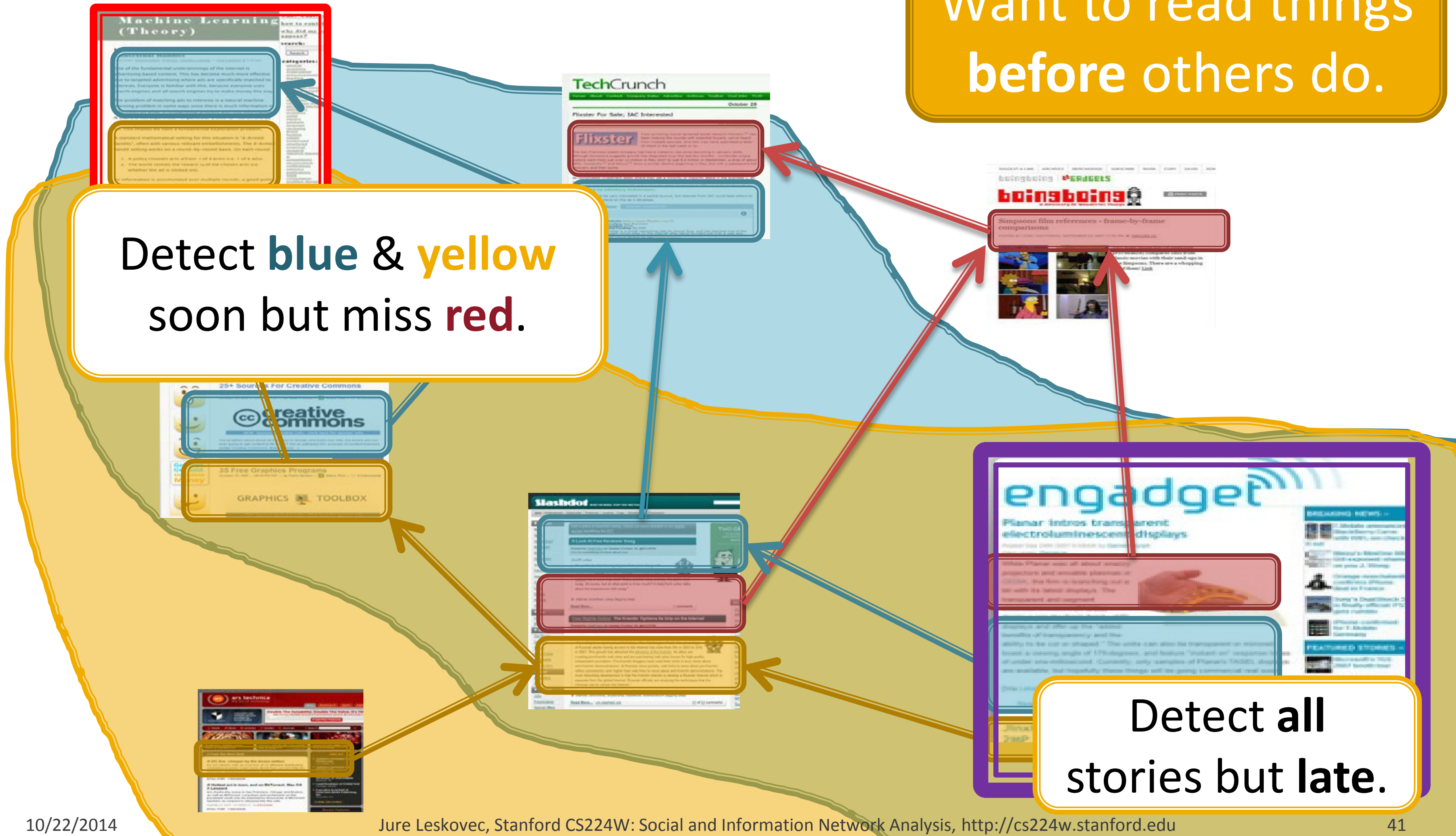
- CELF is **10** times faster than greedy hill-climbing!

Question…

- I have 10 minutes. Which blogs should I read to be most up to date?
- Who are the most influential bloggers?

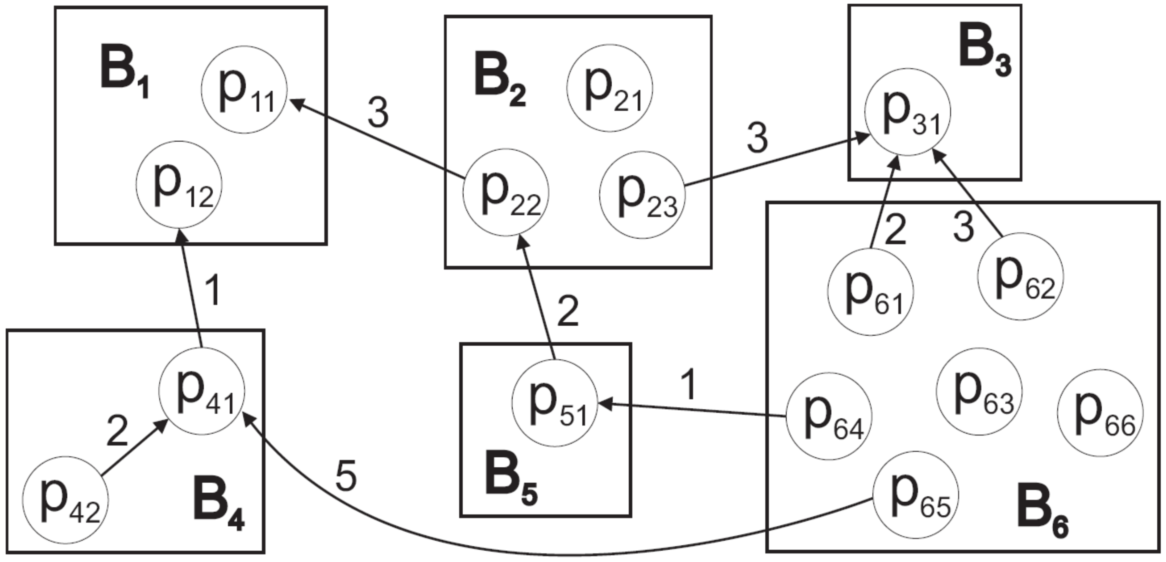


Detecting information outbreaks


Want to read things **before** others do.
Detect **blue** & **yellow** soon but miss **red**.
Detect **all** stories but **late**.
Machine Learning (Theory)
TechCrunch
Flixster
boingboing
Simpsons film references - frame-by-frame comparisons
creative commons
25+ Sources For Creative Commons
35 Free Graphics Programs
GRAPHICS TOOLBOX
Slashdot
ars technica
engadget
Planar intros transparent electroluminescent displays

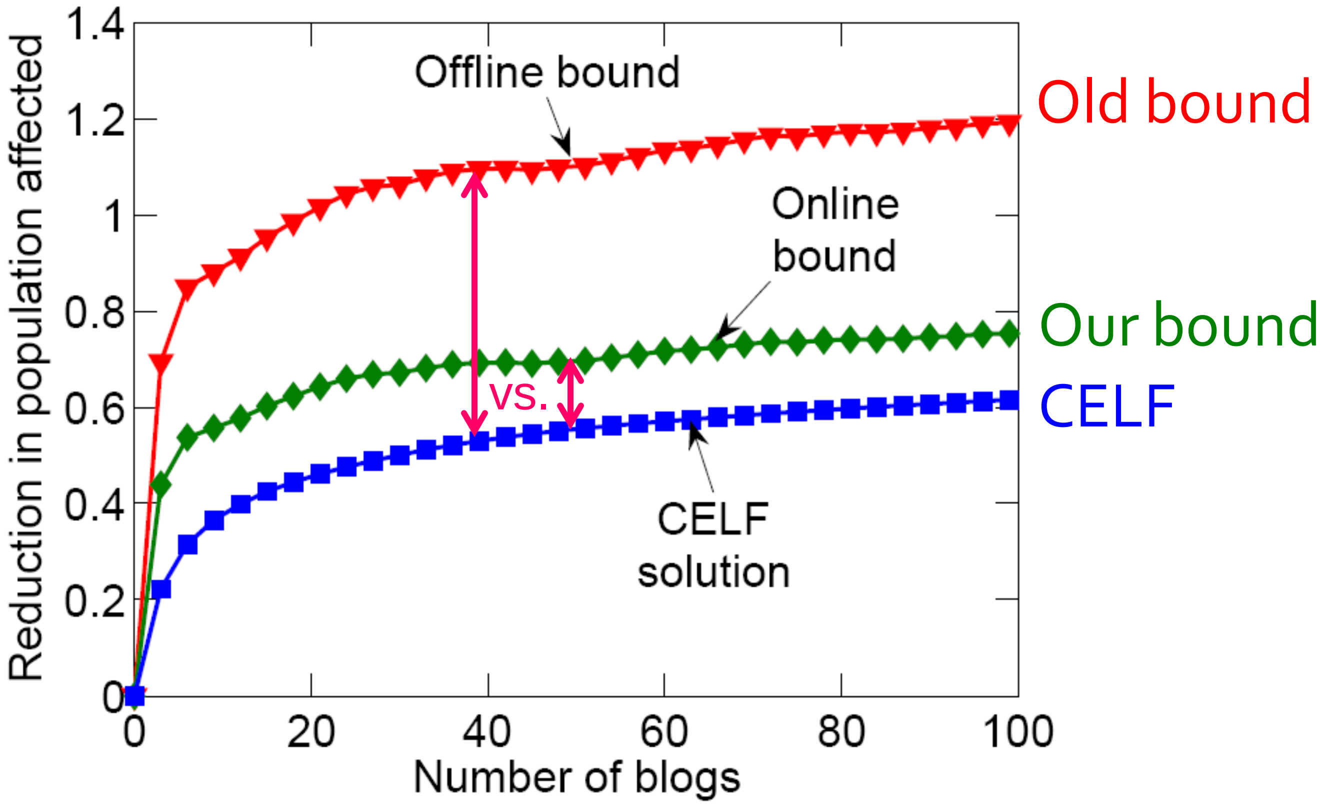
Case study 2: Cascades in blogs

- Crawled 45,000 blogs for 1 year
- Obtained 10 million posts
- And identified 350,000 cascades
- Cost of a blog is the number of posts it has

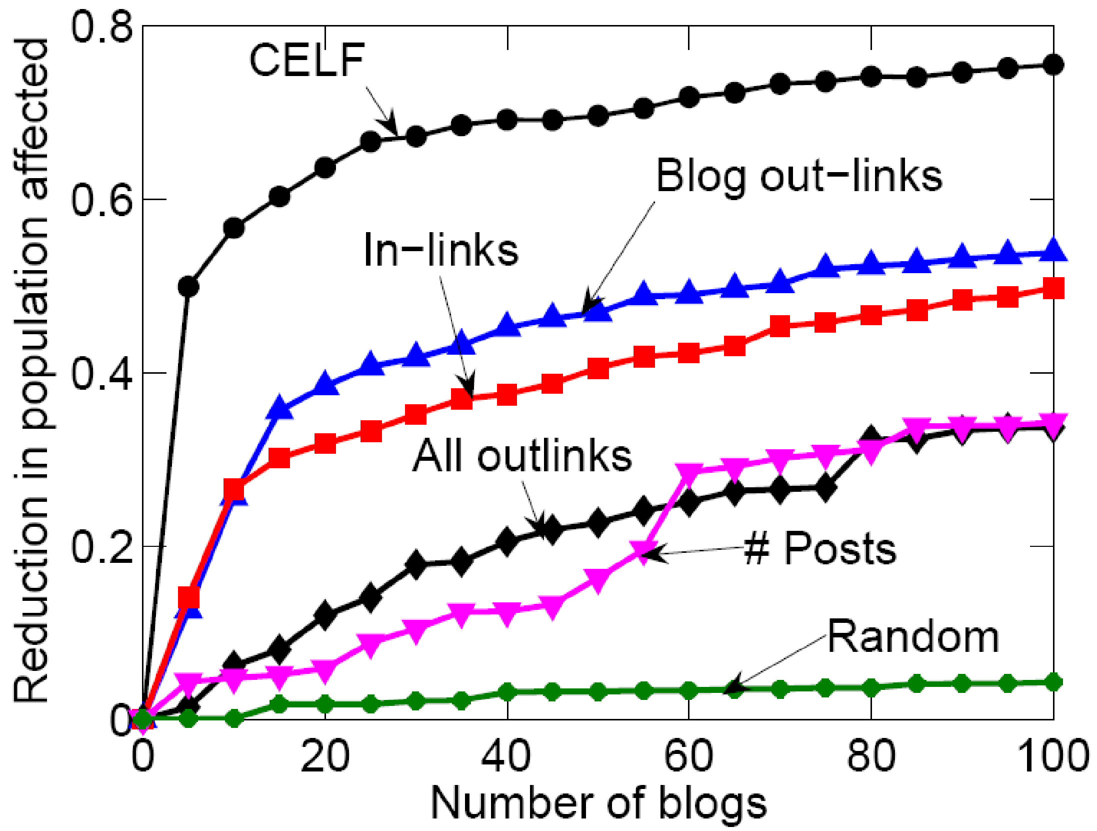


Blogs: Solution Quality

- **Online bound turns out to be much tighter!**
  - Based on the plot below: 87% instead of 63%



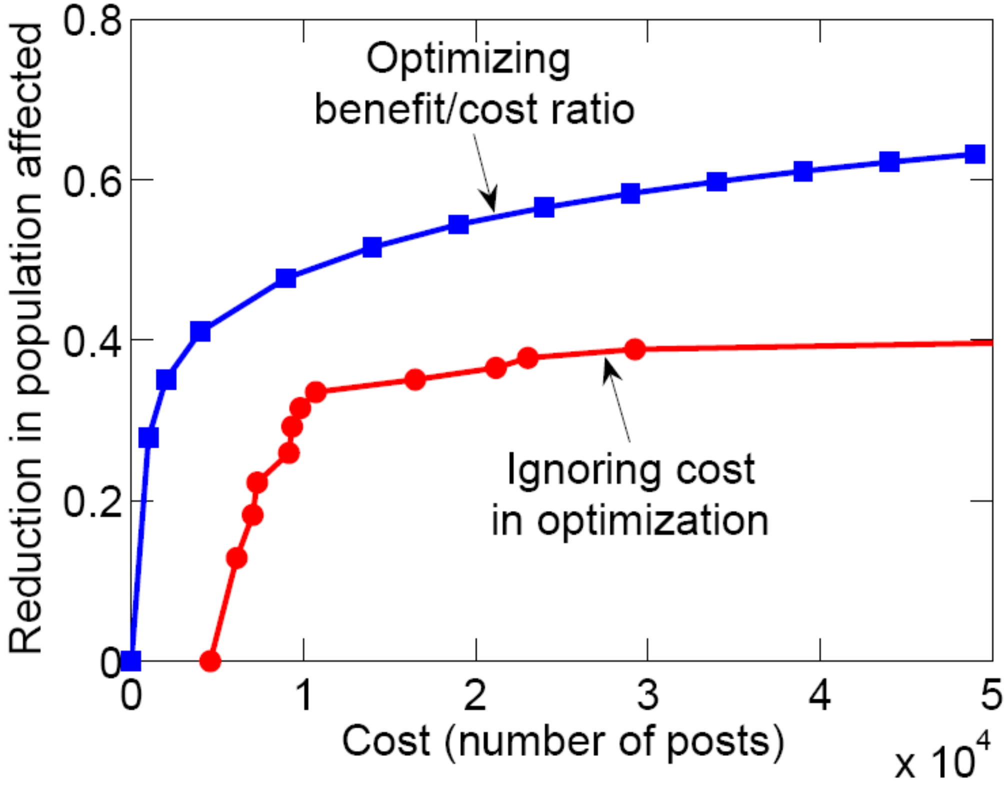
Blogs: Heuristic Selection



- **Heuristics perform much worse!**
- **One really needs to perform the optimization**

Blogs: Cost of a Blog

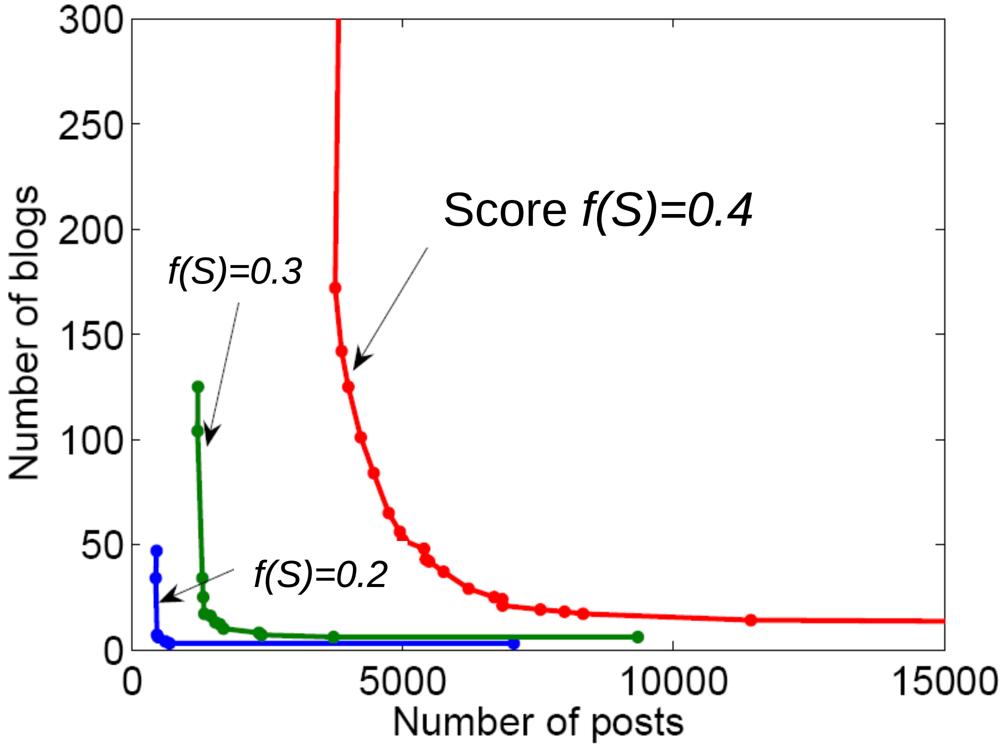
- **CELF has 2 sub-algorithms. Which wins?**
- **Unit cost:**
  - CELF picks large popular blogs
- **Cost-benefit:**
  - Cost proportional to the number of posts



- **We can do much better when considering costs**

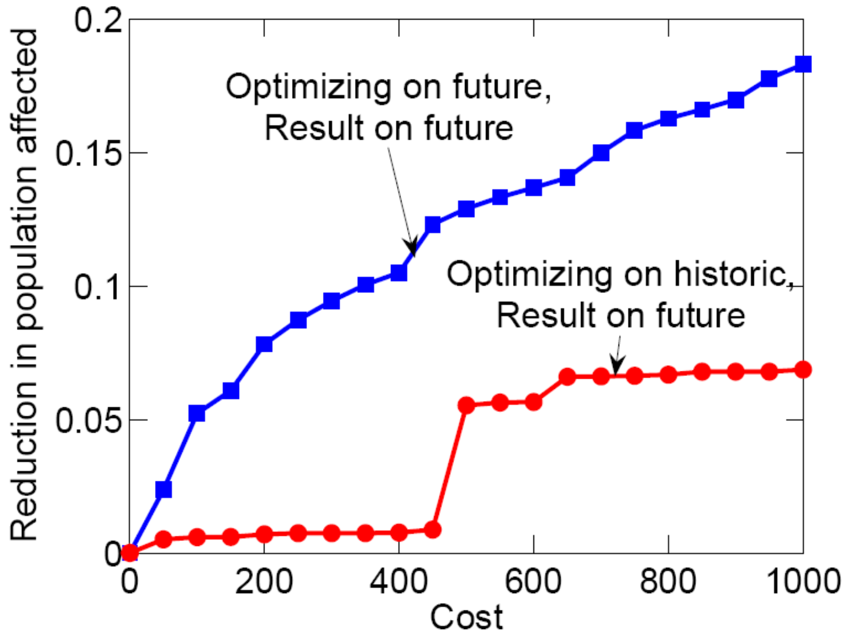
Blogs: Cost of a Blog

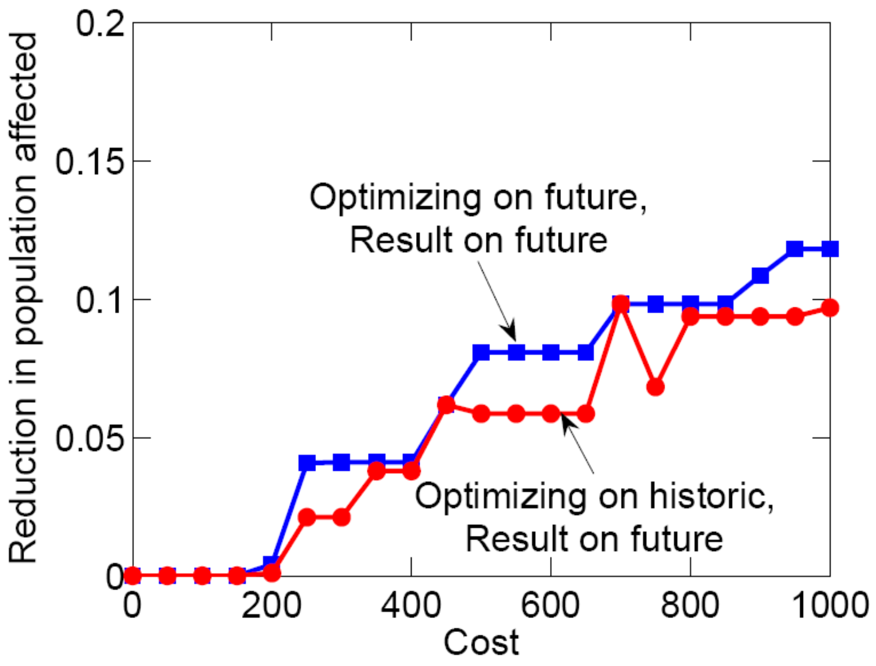
- **Problem:** Then CELF picks **lots of small blogs** that participate in few cascades
- We pick best solution that interpolates between the costs
- We can get good solutions with **few blogs and few posts**



Each curve represents a set of solutions *S* with the same final reward *f(S)*

Blogs: Generalization to Future

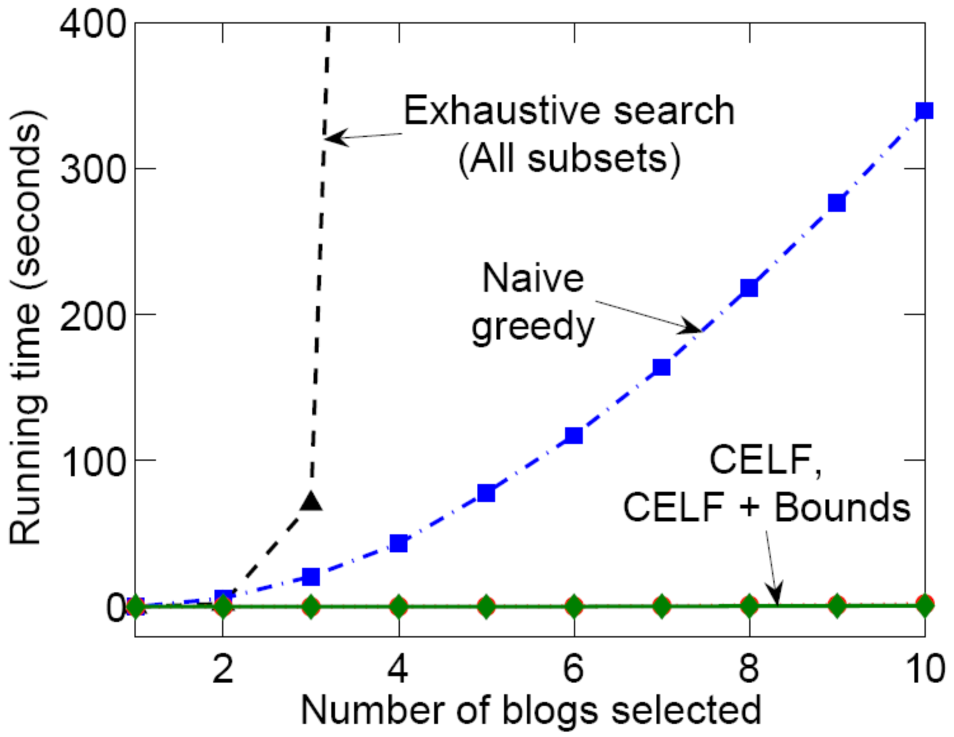




- We want to generalize well to future (unknown) cascades
- Limiting selection to bigger blogs improves generalization!

[Leskovec et al., KDD '07]

Blogs: Scalability



- CELF runs **700** times faster than simple hill-climbing algorithm

Next Time: New Topics

Observations	Models	Algorithms
Small diameter, Edge clustering	Erdös-Renyi model, Small-world model	Decentralized search
Patterns of signed edge creation	Structural balance, Theory of status	Models for predicting edge signs
Viral Marketing, Blogosphere, Memetracking	Independent cascade model, Game theoretic model	Influence maximization, Outbreak detection, LIM
Scale-Free	Preferential attachment, Copying model	PageRank, Hubs and authorities
Densification power law, Shrinking diameters	Microscopic model of evolving networks	Link prediction, Supervised random walks
Strength of weak ties, Core-periphery	Kronecker Graphs	Community detection: Girvan-Newman, Modularity