Network Effects and Cascading Behavior

CS224W: Social and Information Network Analysis
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http://cs224w.stanford.edu

How the Class Fits Together

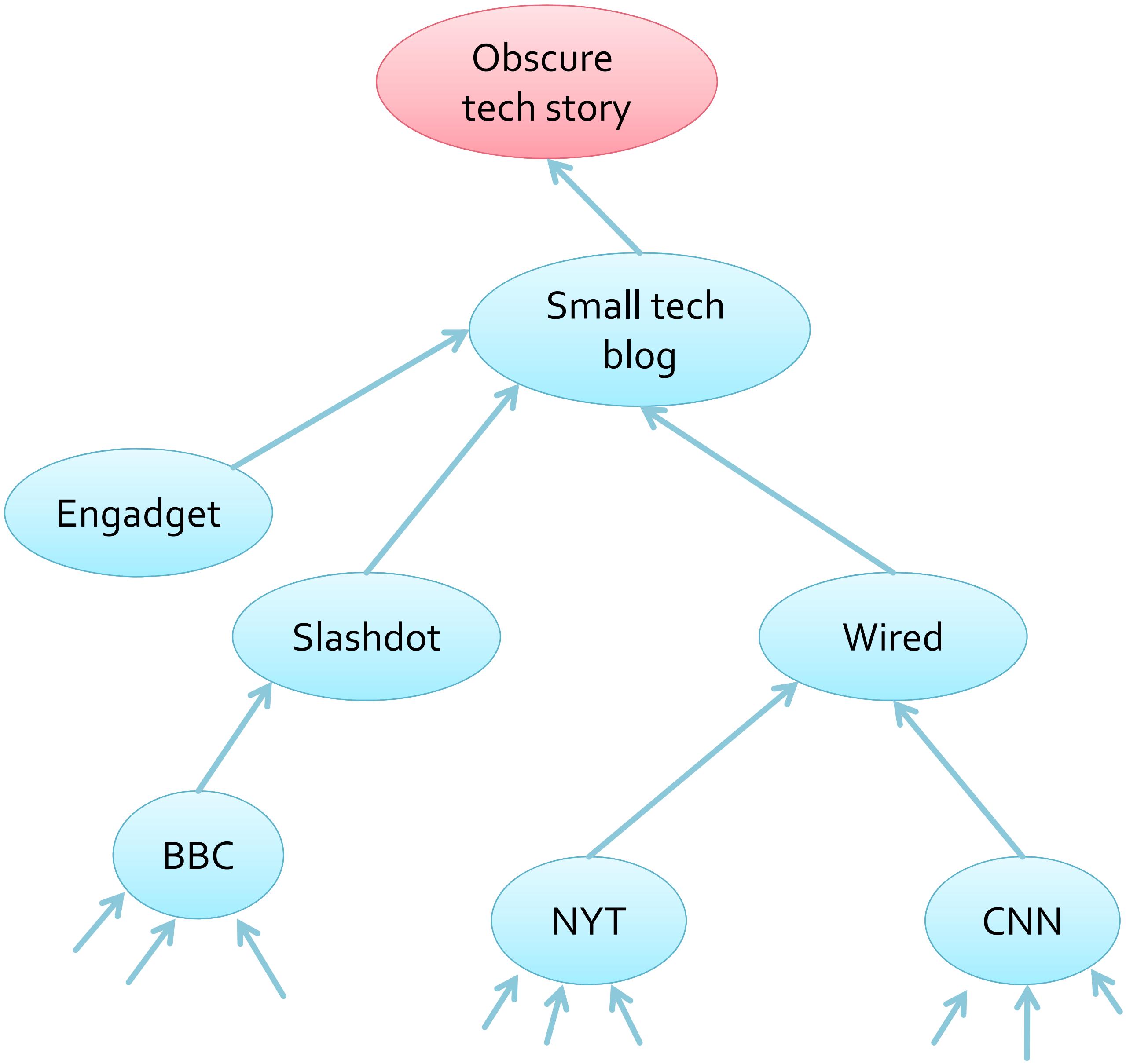
Observations	Models	Algorithms
Small diameter, Edge clustering	Erdös-Renyi model, Small-world model	Decentralized search
Patterns of signed edge creation	Structural balance, Theory of status	Models for predicting edge signs
Viral Marketing, Blogosphere, Memetracking	Independent cascade model, Game theoretic model	Influence maximization, Outbreak detection, LIM
Scale-Free	Preferential attachment, Copying model	PageRank, Hubs and authorities
Densification power law, Shrinking diameters	Microscopic model of evolving networks	Link prediction, Supervised random walks
Strength of weak ties, Core-periphery	Kronecker Graphs	Community detection: Girvan-Newman, Modularity

Spreading Through Networks

- **Spreading through networks:**
  - Cascading behavior
  - Diffusion of innovations
  - Network effects
  - Epidemics
- **Behaviors that cascade from node to node like an epidemic**

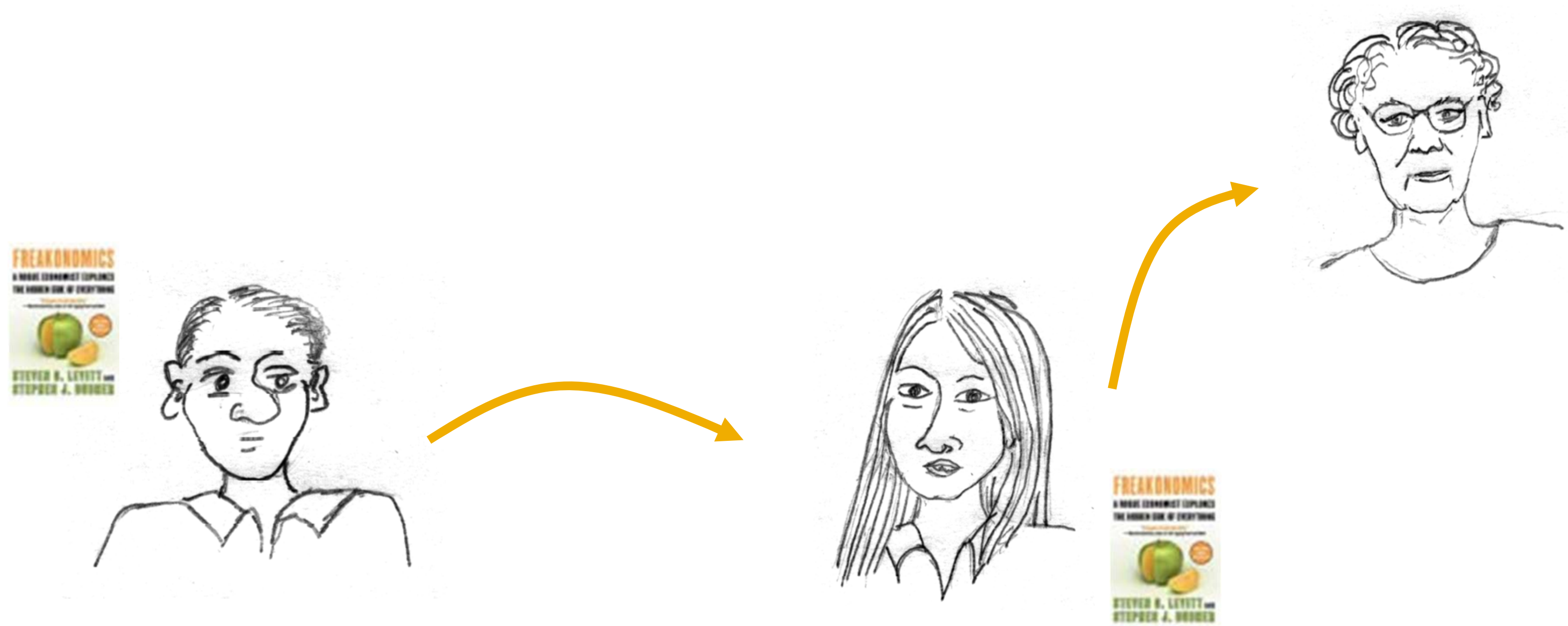
- **Examples:**
  - **Biological:**
    - Diseases via contagion
  - **Technological:**
    - Cascading failures
    - Spread of information
  - **Social:**
    - Rumors, news, new technology
    - Viral marketing

Information Diffusion

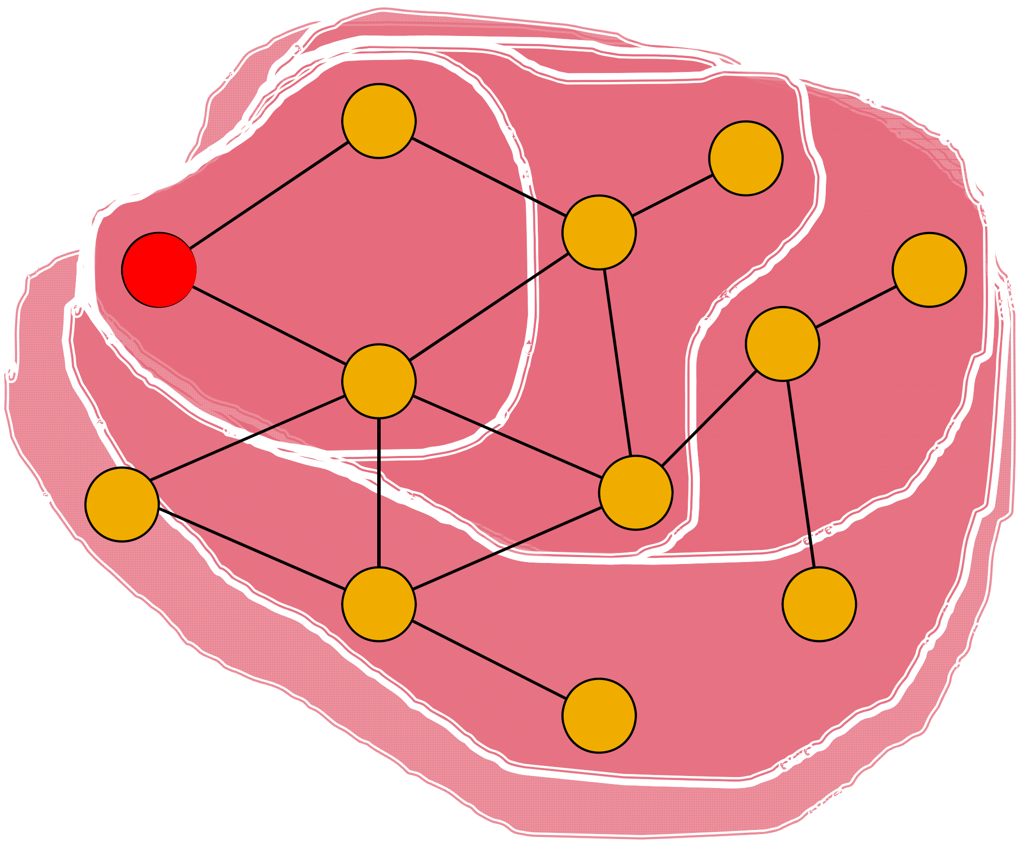


Diffusion in Viral Marketing

- **Product adoption:**
  - **Senders and followers of recommendations**

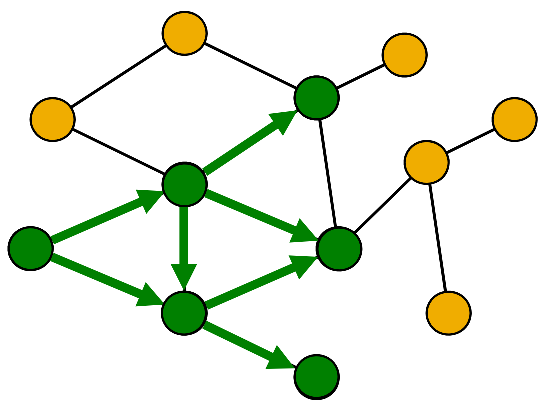


Spread of Diseases

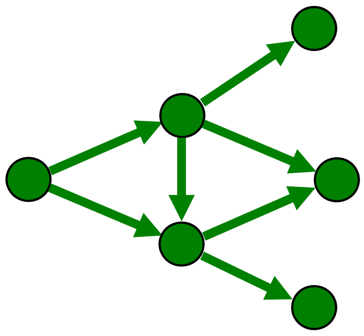


Network Cascades

- **Contagion that spreads over the edges of the network**
- It creates a propagation tree, i.e., **cascade**



Network



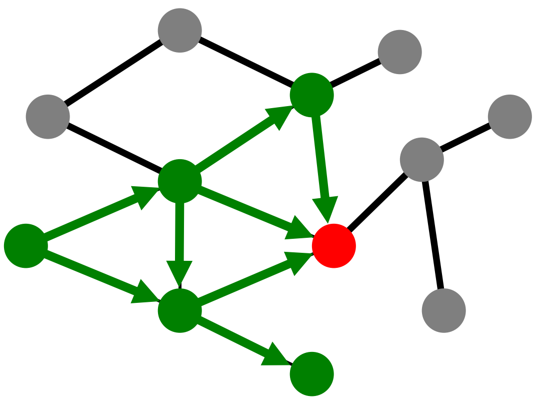
Cascade
(propagation graph)

Terminology:

- Stuff that spreads: Contagion
- "Infection" event: Adoption, infection, activation
- We have: Infected/active nodes, adoptors

How to Model Diffusion?

- **Probabilistic models:**
  - Models of influence or disease spreading
    - An infected node tries to "push" the contagion to an uninfected node
  - **Example:**
    - You "catch" a disease with some prob. from each active neighbor in the network

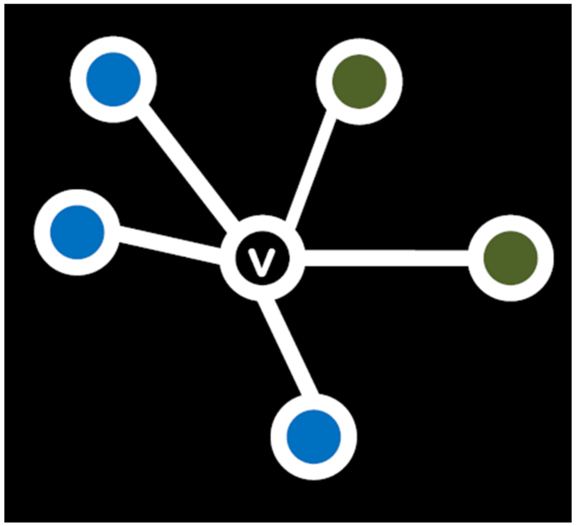


- **Decision based models (today!):**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if *k* of your friends do so too

Decision Based Model of Diffusion

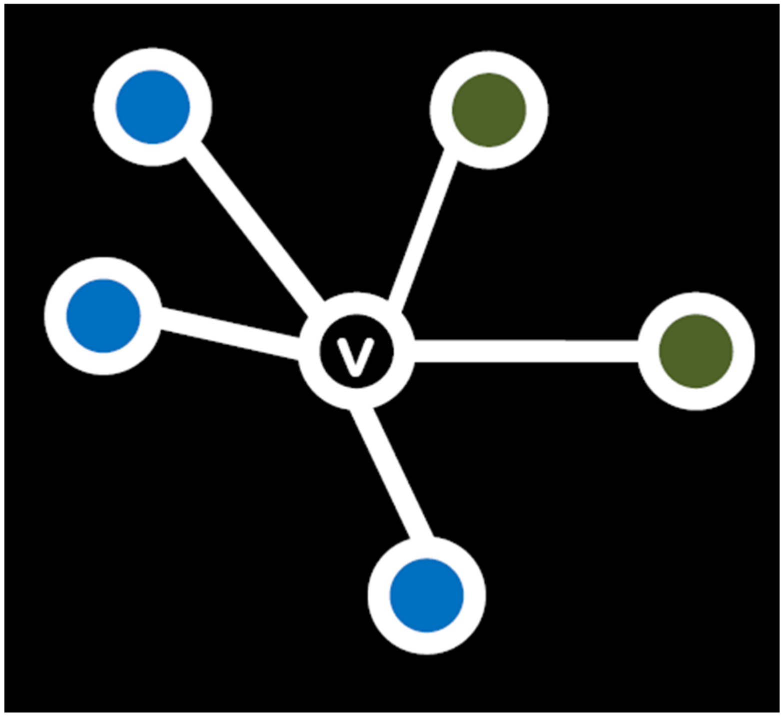
Decision Based Models

- **Two ingredients:**
  - **Payoffs:**
    - Utility of making a particular choice
  - **Signals:**
    - Public information:
      - What your network neighbors have done
    - (Sometimes also) Private information:
      - Something you know
      - Your belief
- **Now you want to make the optimal decision**



Game Theoretic Model of Cascades

- **Based on 2 player coordination game**
  - 2 players – each chooses technology A or B
  - Each person can only adopt **one** "behavior", **A** or **B**
  - You gain more payoff if your friend has adopted the **same** behavior as you

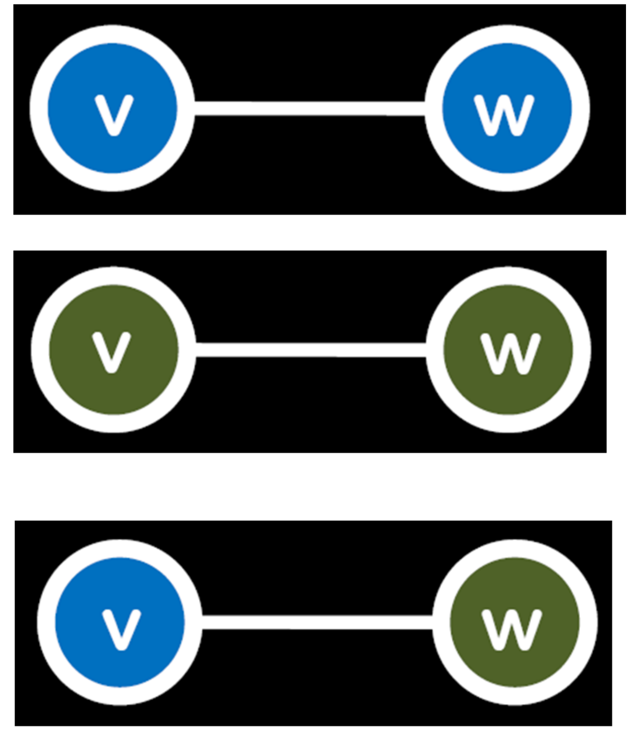


Local view of the network of node v

Example: BlueRay vs. HD DVD

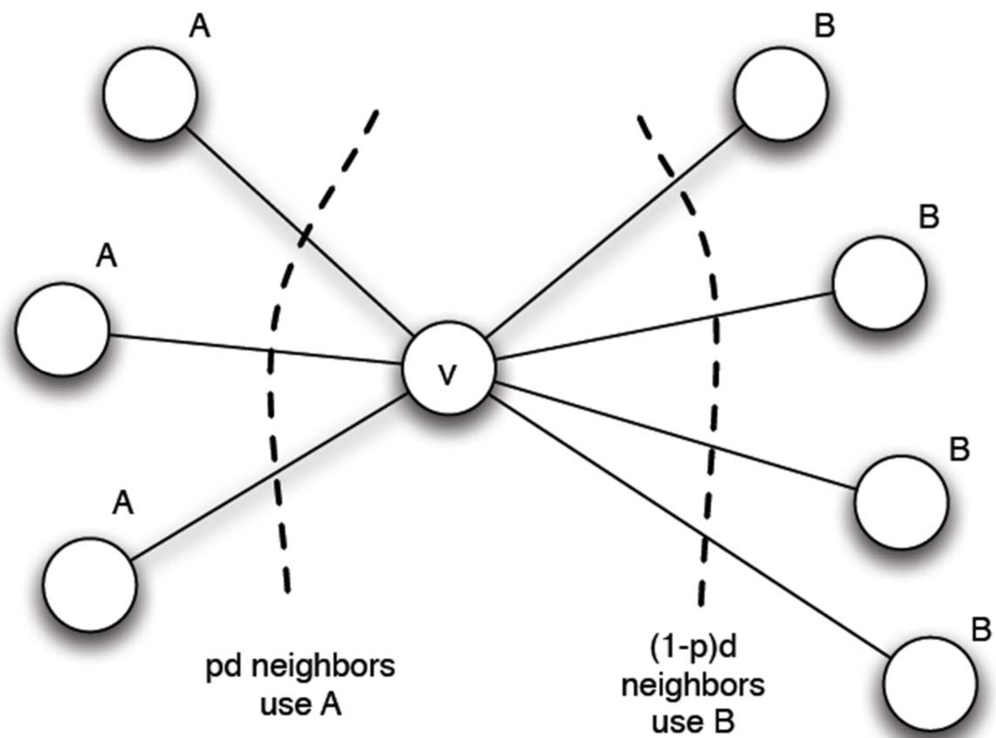
The Model for Two Nodes

- ***Payoff matrix:***
  - If both v and w adopt behavior A, they each get payoff *a > 0*
  - If *v* and *w* adopt behavior *B*, they reach get payoff *b > 0*
  - If *v* and *w* adopt the opposite behaviors, they each get *0*



- **In some large network:**
  - Each node *v* is playing a copy of the game with each of its neighbors
  - **Payoff**: sum of node payoffs per game

Calculation of Node *v*



Threshold:
v choses *A* if

$$p > q = \frac{b}{a+b}$$

- Let v have *d* neighbors
- Assume fraction *p* of *v*'s neighbors adopt *A*
  - $Payoff_v = a \cdot p \cdot d$ if *v* chooses A
    $= b \cdot (1-p) \cdot d$ if *v* chooses B
- **Thus: *v* chooses A if:** $a \cdot p \cdot d > b \cdot (1-p) \cdot d$

Example Scenario

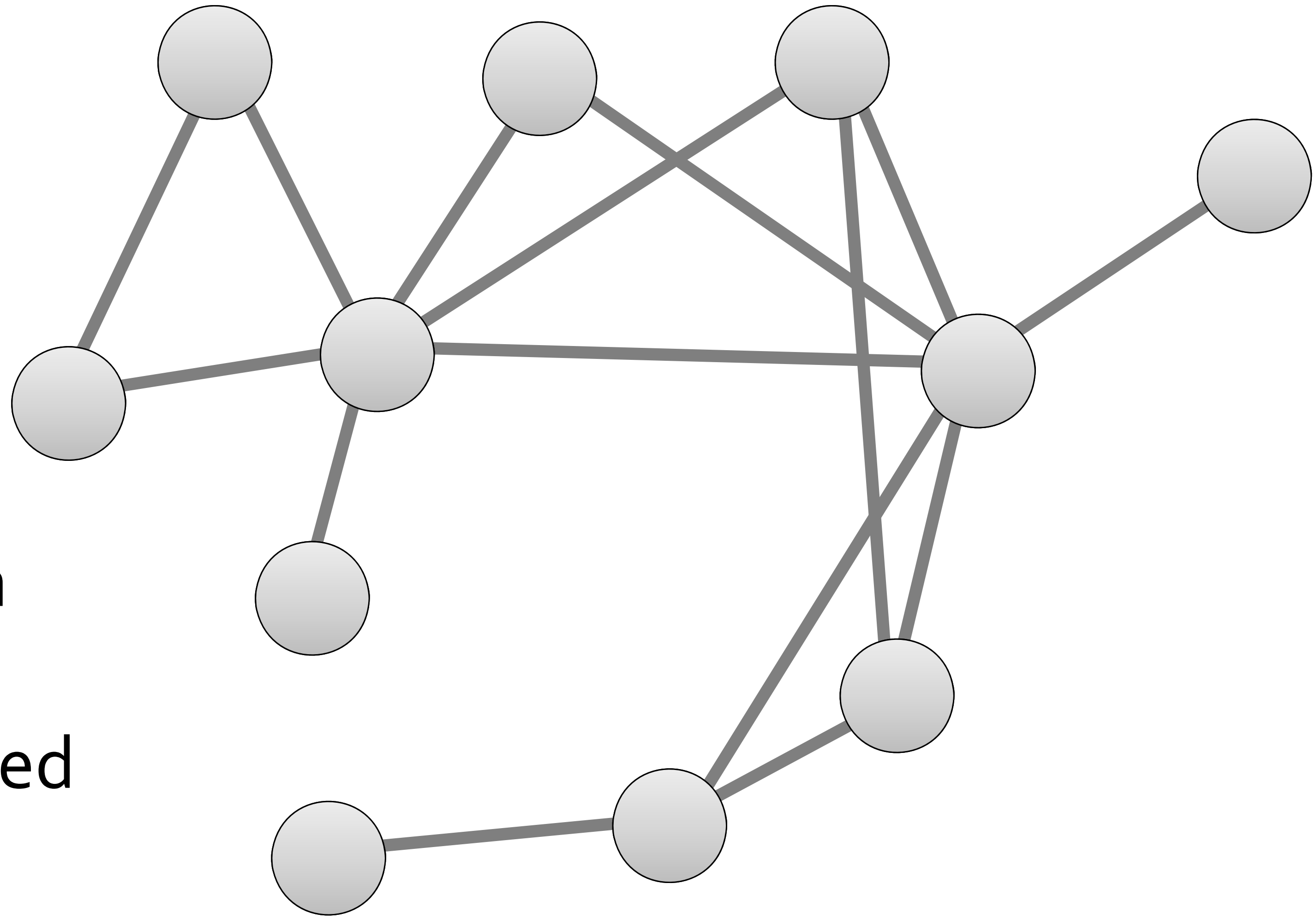
- **Scenario:**
  Graph where everyone starts with *B*.
  Small set *S* of early adopters of *A*
  - Hard-wire *S* – they keep using *A* no matter what payoffs tell them to do

- Assume payoffs are set in such a way that nodes say:
  If more than 50% of my friends take A
  I'll also take A

  (this means: $a = b - \varepsilon$ and $q > 1/2$)

Example Scenario

$S = \{u, v\}$



If more than 50% of my friends are red I'll be red

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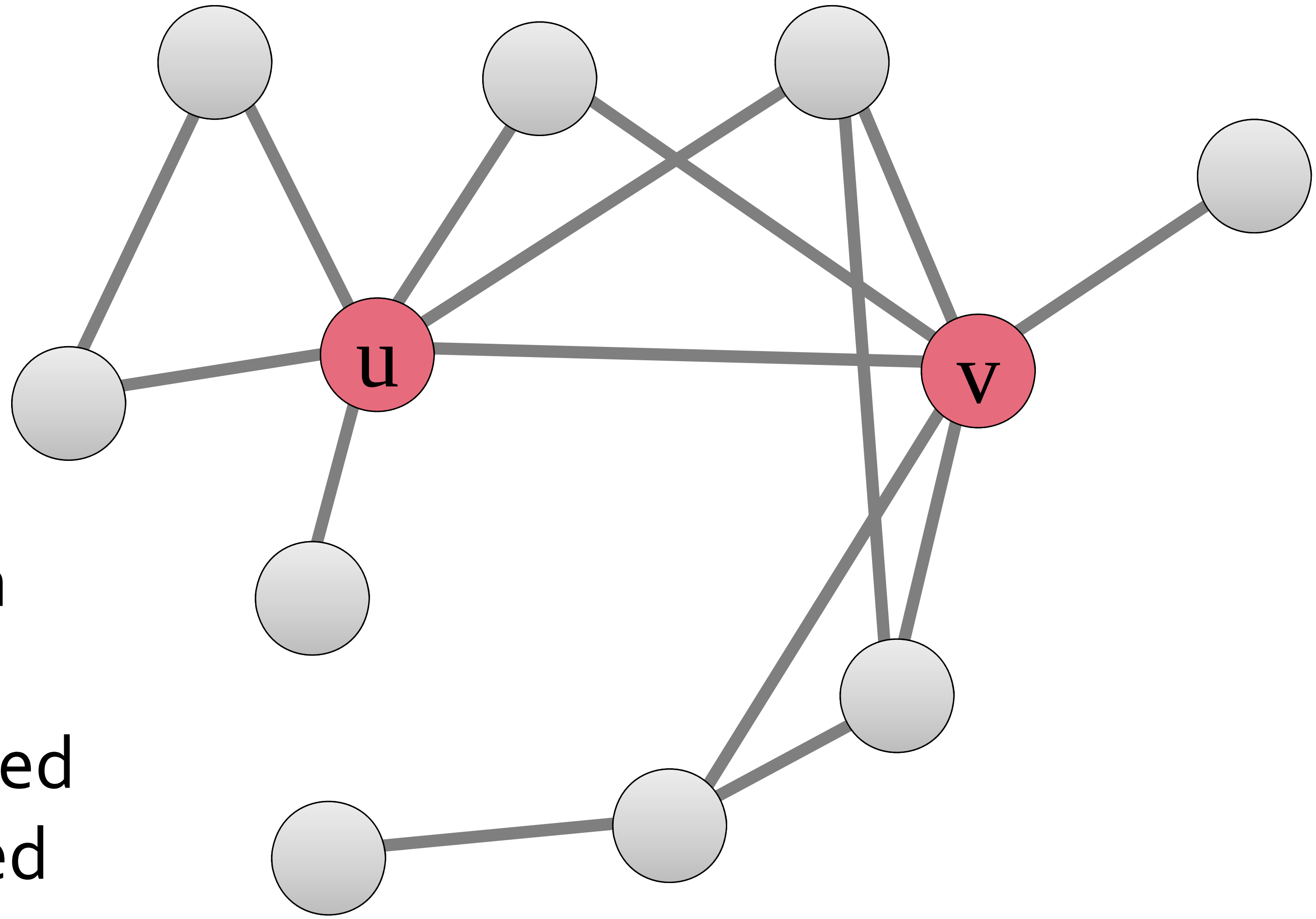

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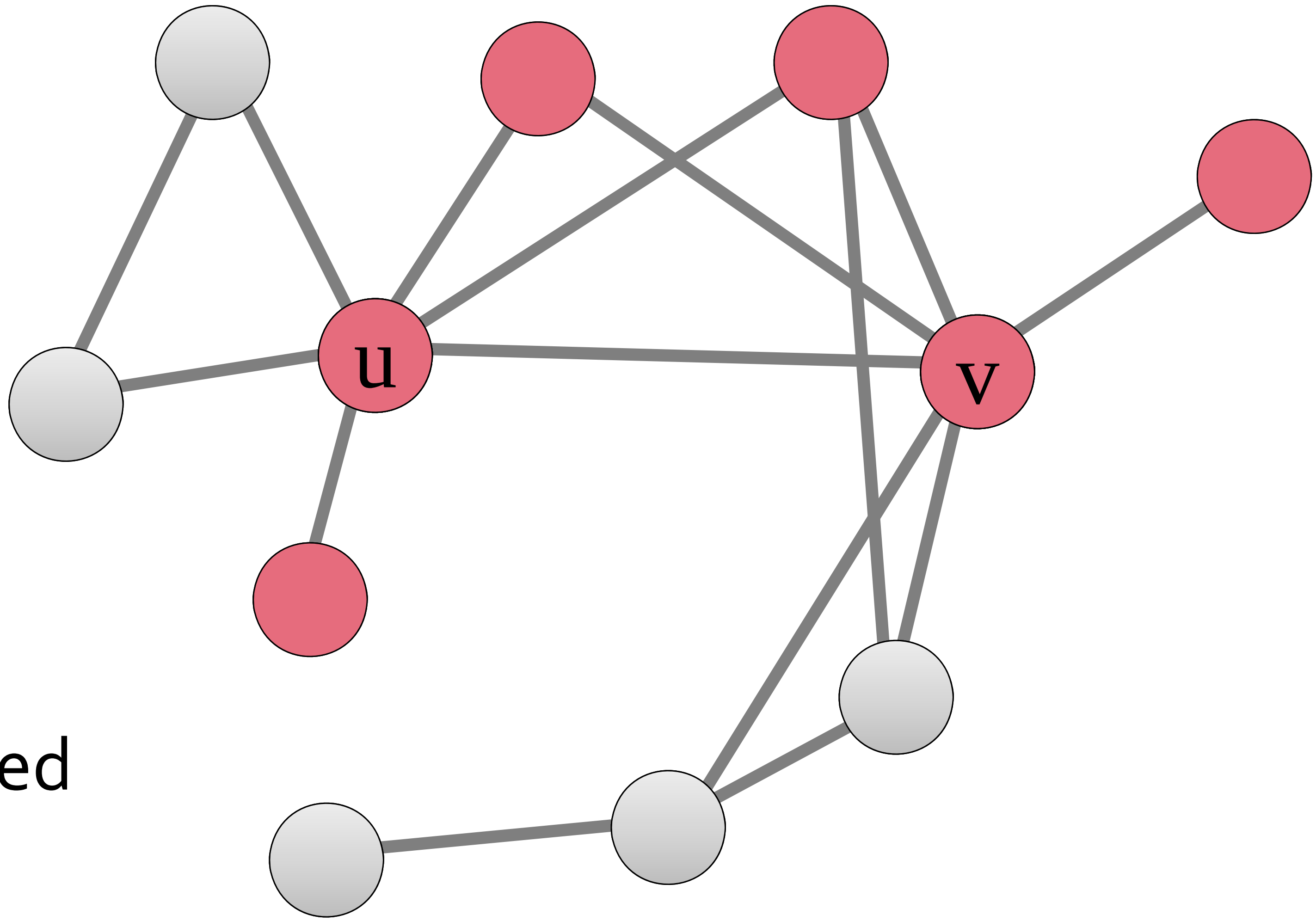

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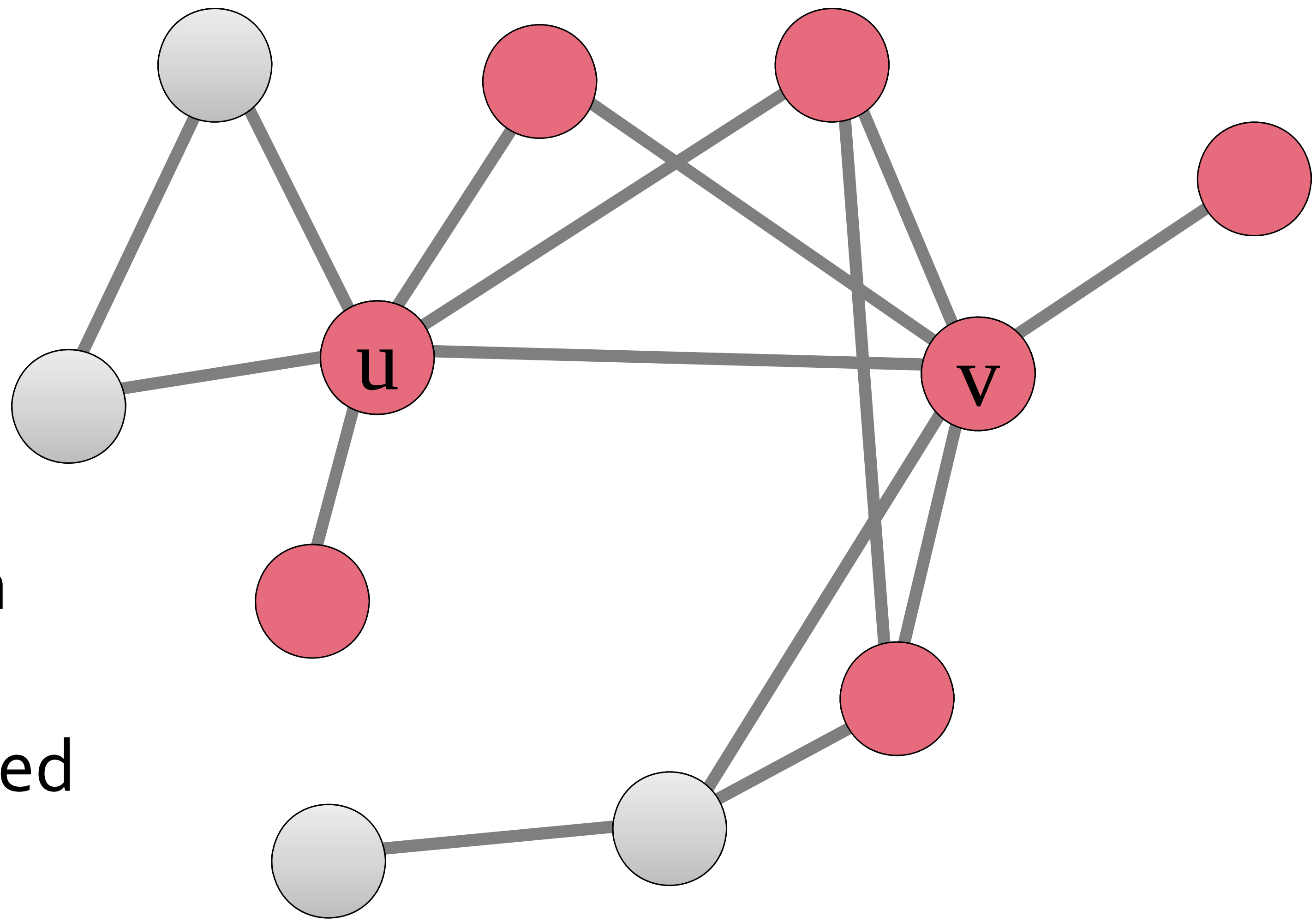

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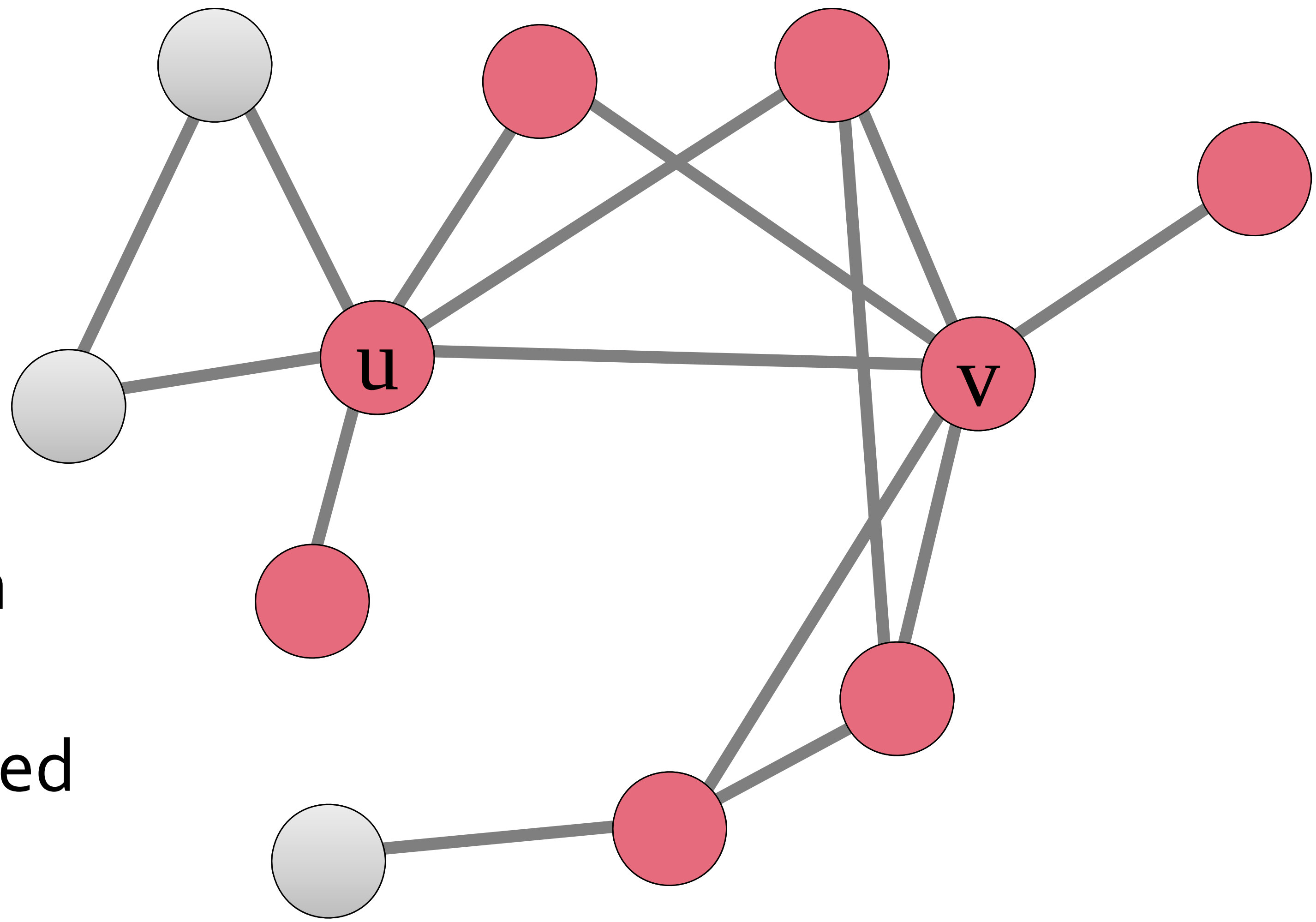

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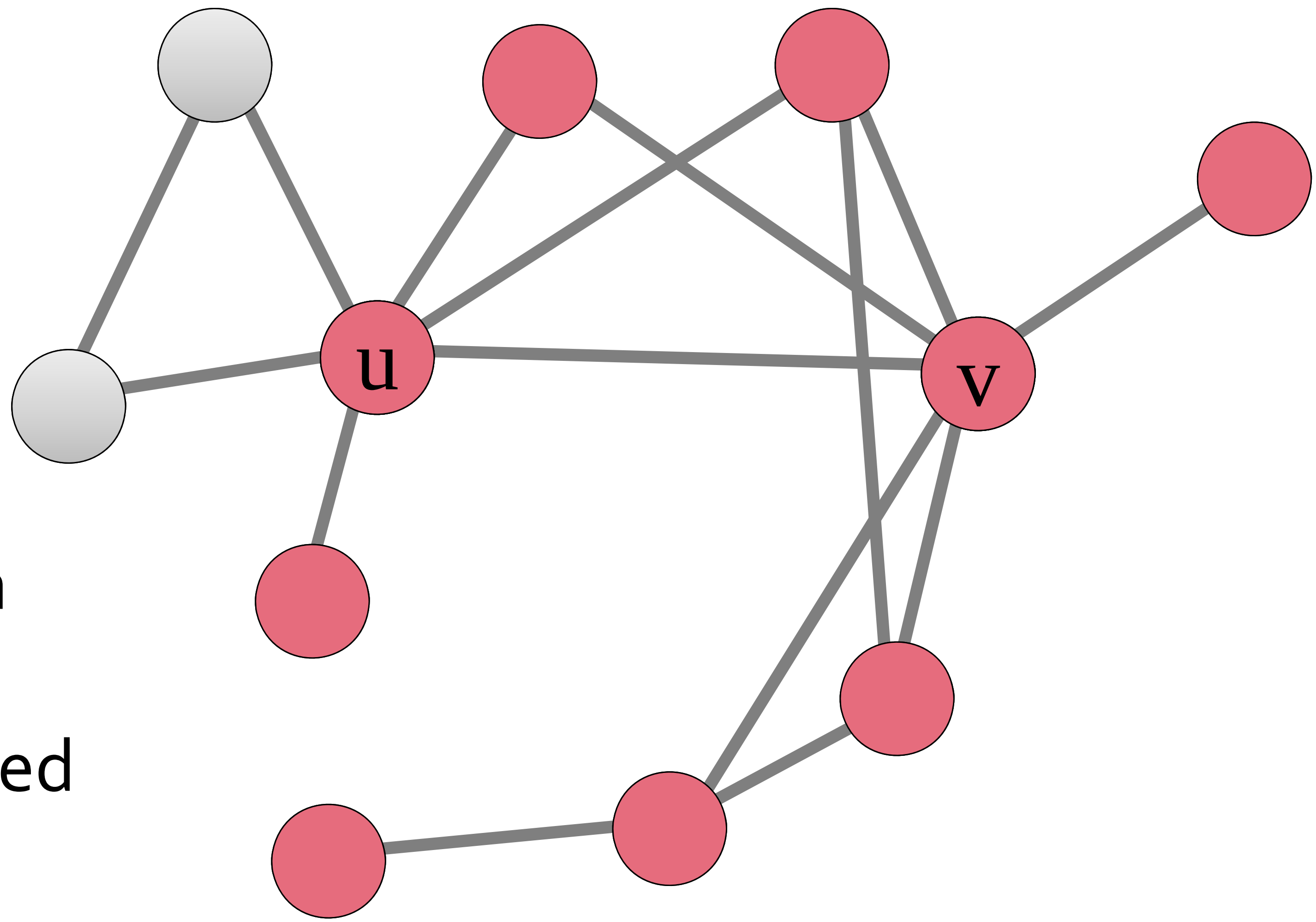

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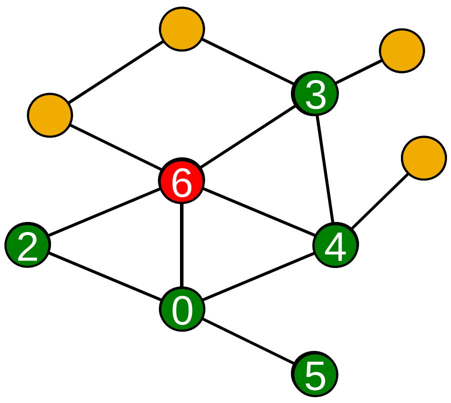
Monotonic Spreading

- **Observation: Use of A spreads monotonically**
  (Nodes only switch B→A, but never back to B)
- **Why?** Proof sketch:
  - **Nodes keep switching from B to A:** *B*→*A*
  - Now, suppose some node switched back from *A*→*B*, consider the **first** node ***u*** to do so (say at time *t*)
  - Earlier at some time *t'* (*t'*<*t*) the same node ***u*** switched *B*→*A*
  - So at time *t'* ***u*** was above threshold for *A*
  - But up to time *t* no node switched back to B, so node ***u*** could only had more neighbors who used *A* at time *t* compared to *t'*.
    There was no reason for *u* to switch.



!! Contradiction !!

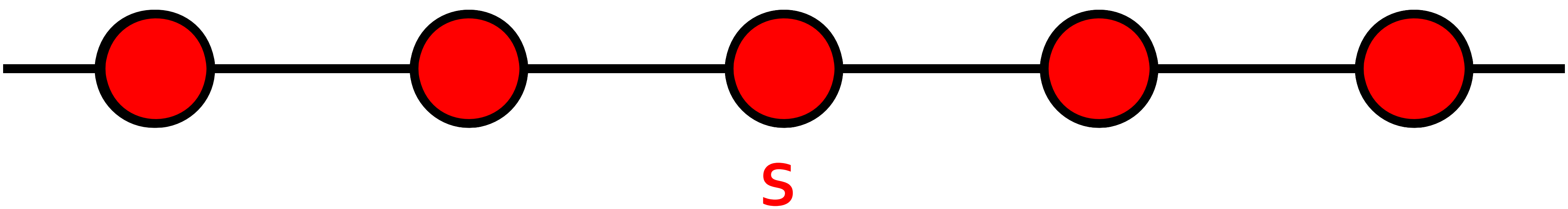
Infinite Graphs

v choses *A* if *p*>*q*

$$q = \frac{b}{a+b}$$

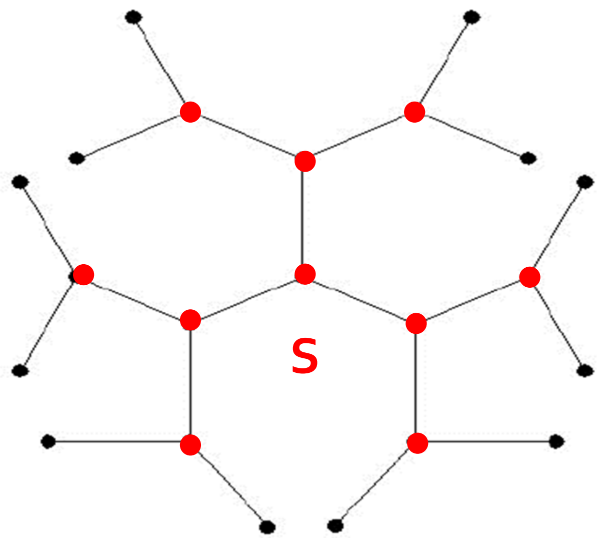
- **Consider infinite graph *G***
  - (but each node has finite number of neighbors!)
- We say that a finite set ***S*** causes a cascade in *G* with threshold *q* if, when *S* adopts *A*, eventually every node adopts *A*
- Example: **Path**

If q<1/2 then cascade occurs



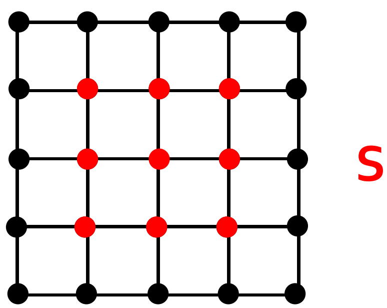
Infinite Graphs

- **Infinite Tree:**



If q<1/3 then cascade occurs

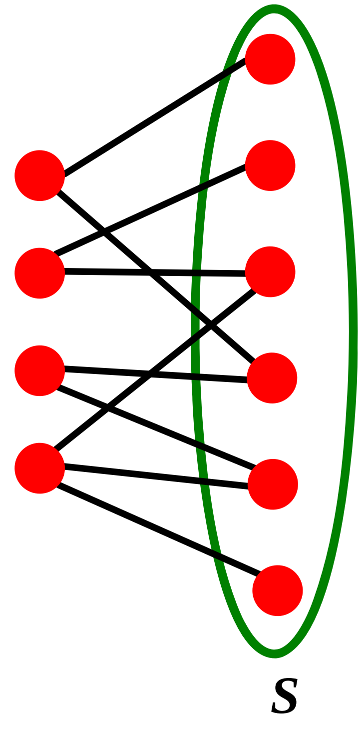
- **Infinite Grid:**



If q<1/4 then cascade occurs

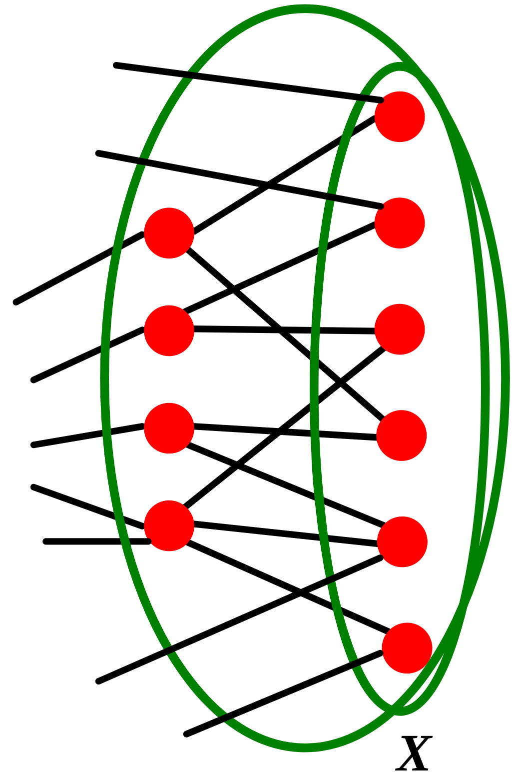
Cascade Capacity

- Def:
  - The **cascade capacity** of a graph *G* is the largest *q* for which some **finite set S** can cause a cascade
- Fact:
  - There is no *G* where cascade capacity **> ½**
- **Proof idea:**
  - Suppose such G exists: q>½, finite S causes cascade
  - **Show contradiction:** Argue that nodes stop switching after a finite # of steps



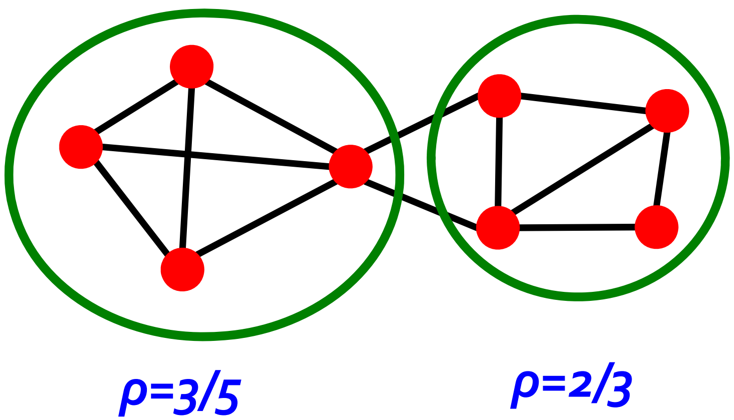
Cascade Capacity

- **Fact:** There is no *G* where cascade capacity **> ½**
- **Proof sketch:**
  - Suppose such G exists: q>½, finite S causes cascade
  - **Contradiction:** Switching stops after a finite # of steps
    - Define "potential energy"
    - Argue that it starts finite (non-negative) and strictly decreases at every step
  - "Energy": = $|d^{out}(X)|$
    - $|d^{out}(X)|$:= # of outgoing edges of active set *X*
    - The only nodes that switch have a strict majority of its neighbors in *S*
    - $|d^{out}(X)|$ strictly decreases
    - It can do so only a finite number of steps



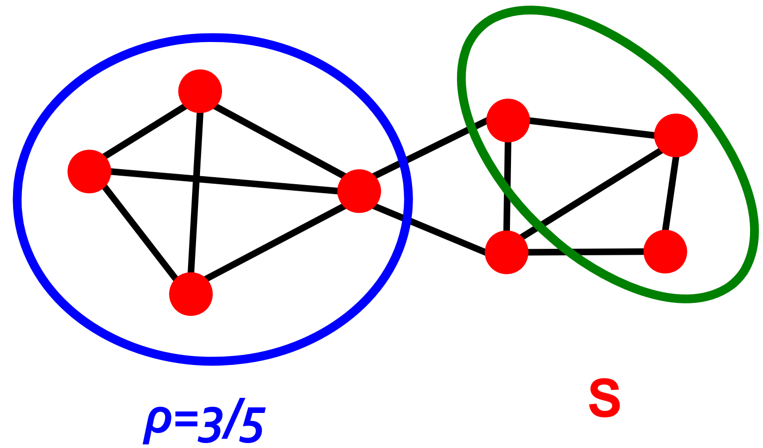
Stopping Cascades

- **What prevents cascades from spreading?**
- <u>Def:</u> **Cluster of density *ρ*** is a **set of nodes *C*** where each node in the set has at least ***ρ*** fraction of edges in *C*.



Stopping Cascades

- **Let S be an initial set of adopters of A**
- All nodes apply threshold *q* to decide whether to switch to A
- **Two facts:**
  - 1) If *G\S* contains a cluster of density *>(1-q)* then *S* can not cause a cascade
  - 2) If S fails to create a cascade, then there is a cluster of density *>(1-q)* in G\S



No cascade if q>2/5

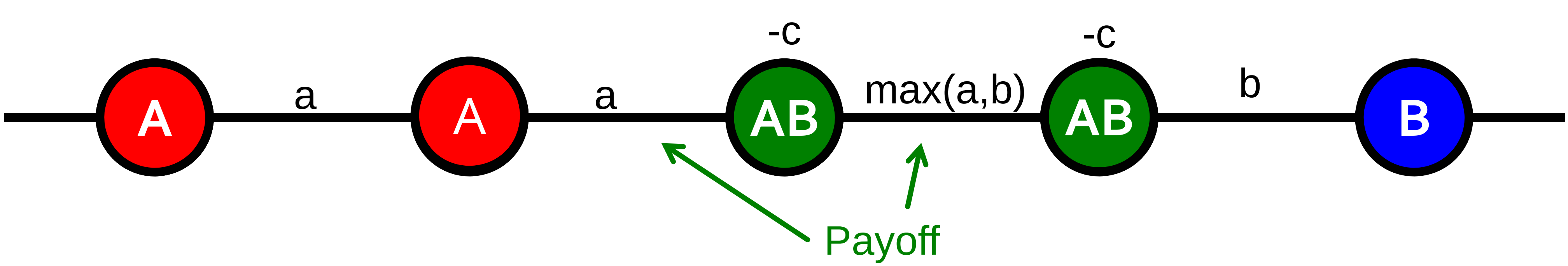
Extending the Model:
Allow People to Adopt A and B

Cascades & Compatibility

- **So far:**
  - Behaviors *A* and *B* compete
  - Can only get utility from neighbors of same behavior: *A-A* get a, *B-B* get *b*, *A-B* get *0*
- **Let's add an extra strategy "*A-B*"**
  - *AB-A*: gets *a*
  - *AB-B*: gets *b*
  - *AB-AB*: gets max(*a*, *b*)
  - Also: Some cost *c* for the effort of maintaining both strategies (summed over all interactions)

Cascades & Compatibility: Model

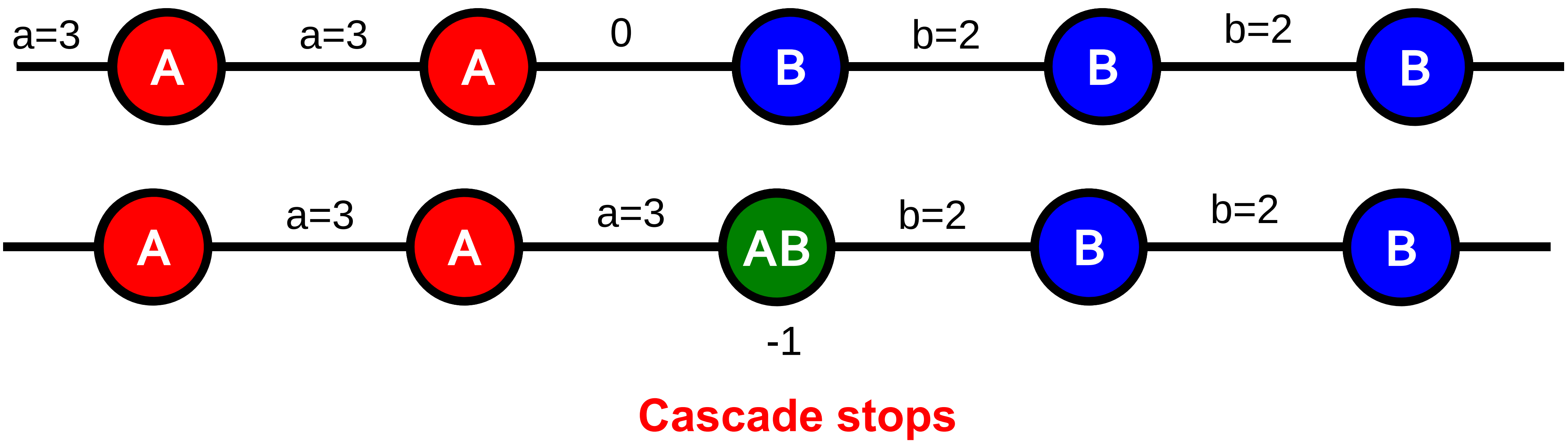
- Every node in an infinite network starts with *B*
- Then a finite set *S* initially adopts *A*
- Run the model for *t=1,2,3,…*
  - Each node selects behavior that will optimize payoff (given what its neighbors did in at time *t-1*)



- **How will nodes switch from *B* to *A* or *AB*?**

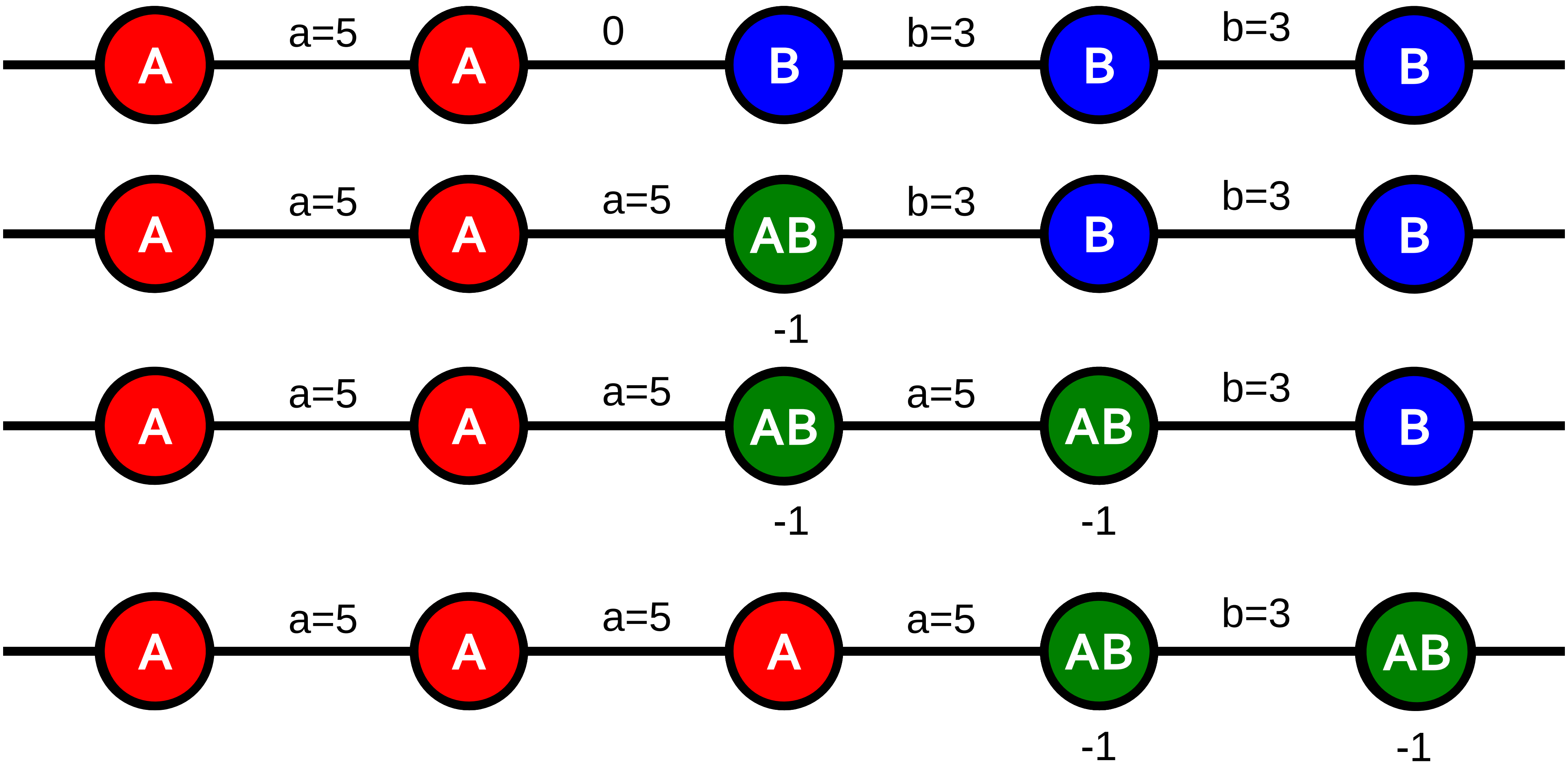
Example: Path Graph

- **Path graph:** Start with all Bs, **a > b** (A is better)
- **One node switches to A – what happens?**
  - **With just A, B:** A spreads if *a > b*
  - **With A, B, AB:** **Does A spread?**
- **Assume a=3, b=2, c=1:**



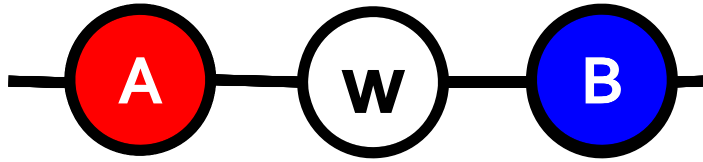
Example

- Let a=5, b=3, c=1

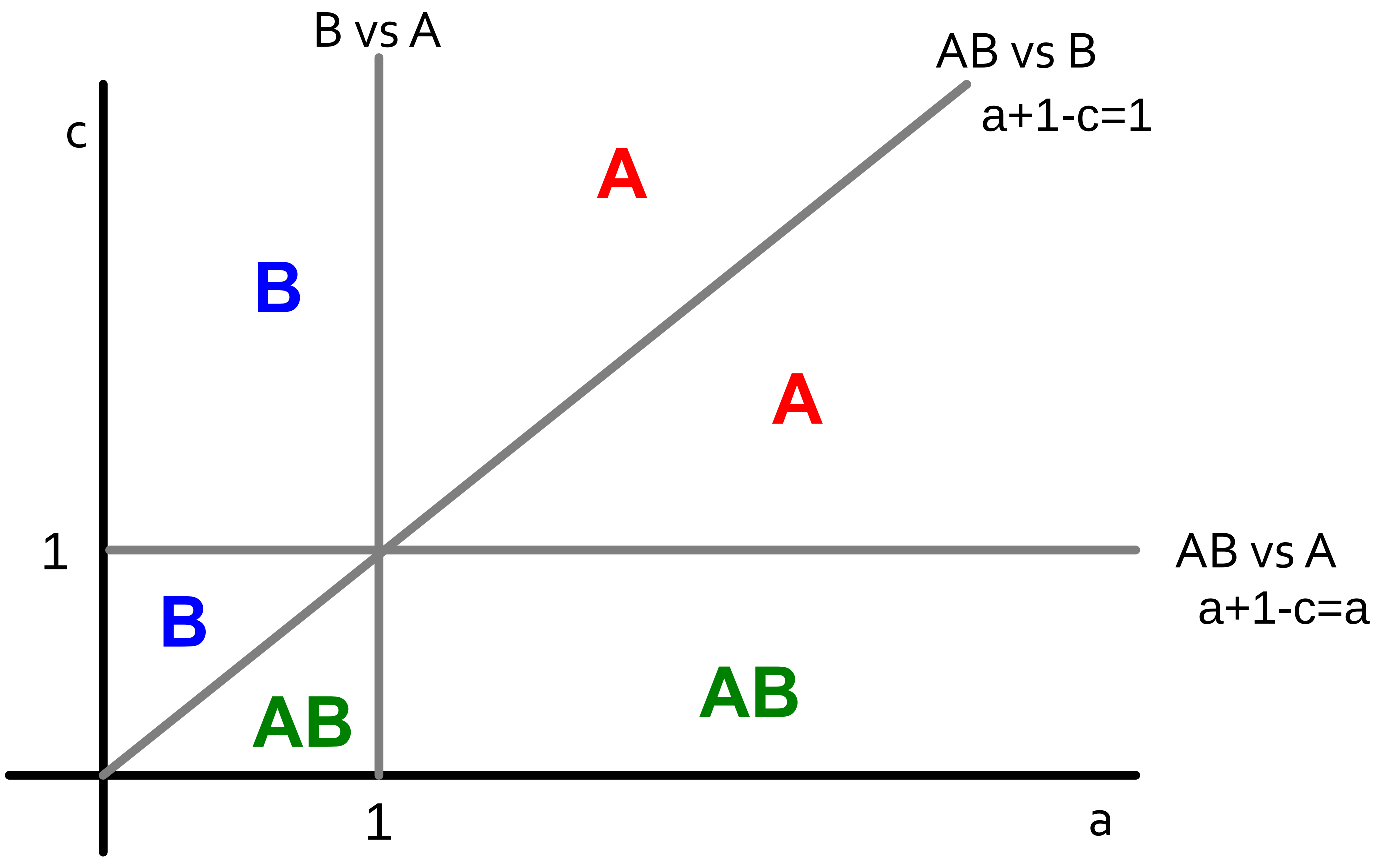


For what pairs (c,a) does A spread?

- **Infinite path, start with all Bs**



- **Payoffs for *w*:** A:a, B:1, AB:a+1-c
- What does node w in **A-w-B** do?



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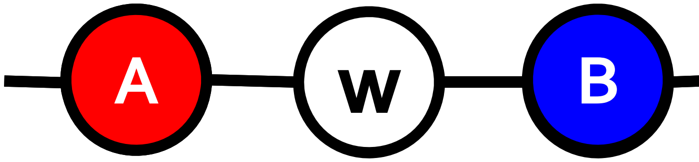

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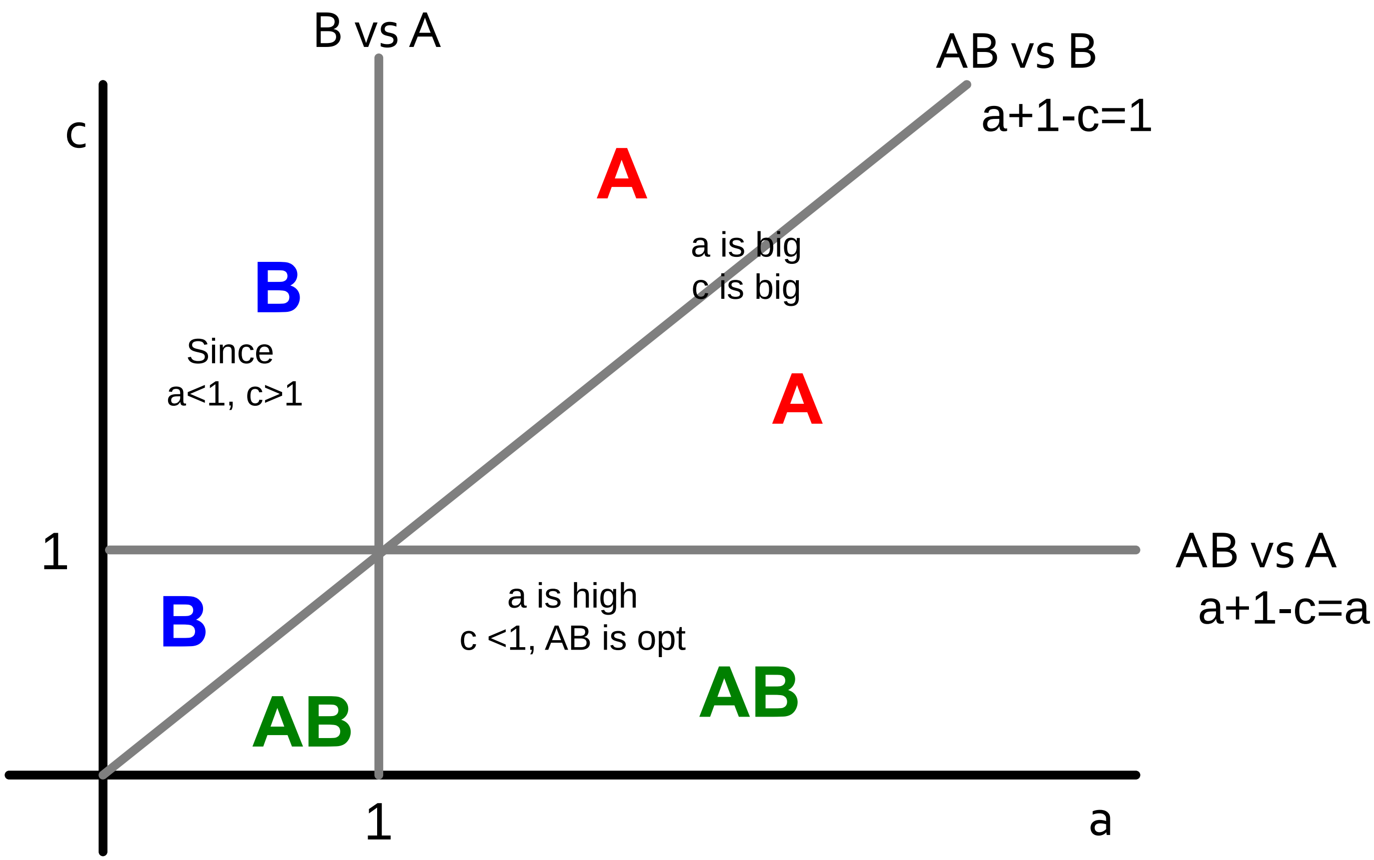
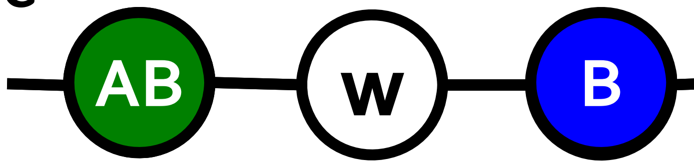
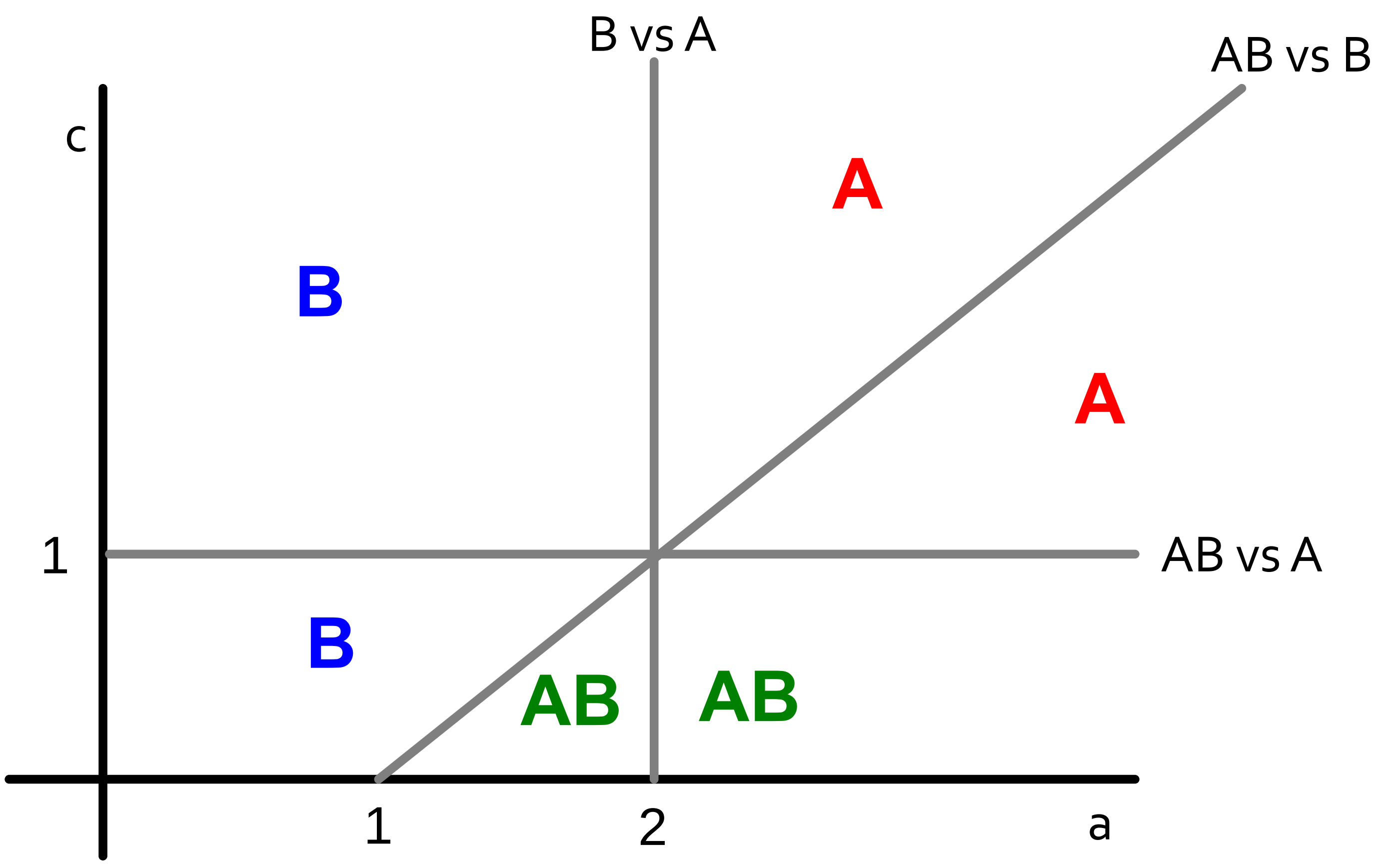


For what pairs (c,a) does A spread?

- **Same reward structure as before but now payoffs for *w* change:** A:a, B:1+1, AB:a+1-c
- **Notice: Now also AB spreads**
- What does node w in **AB-w-B** do?





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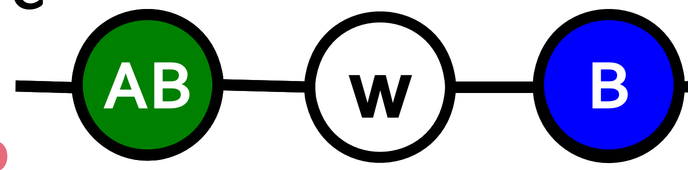
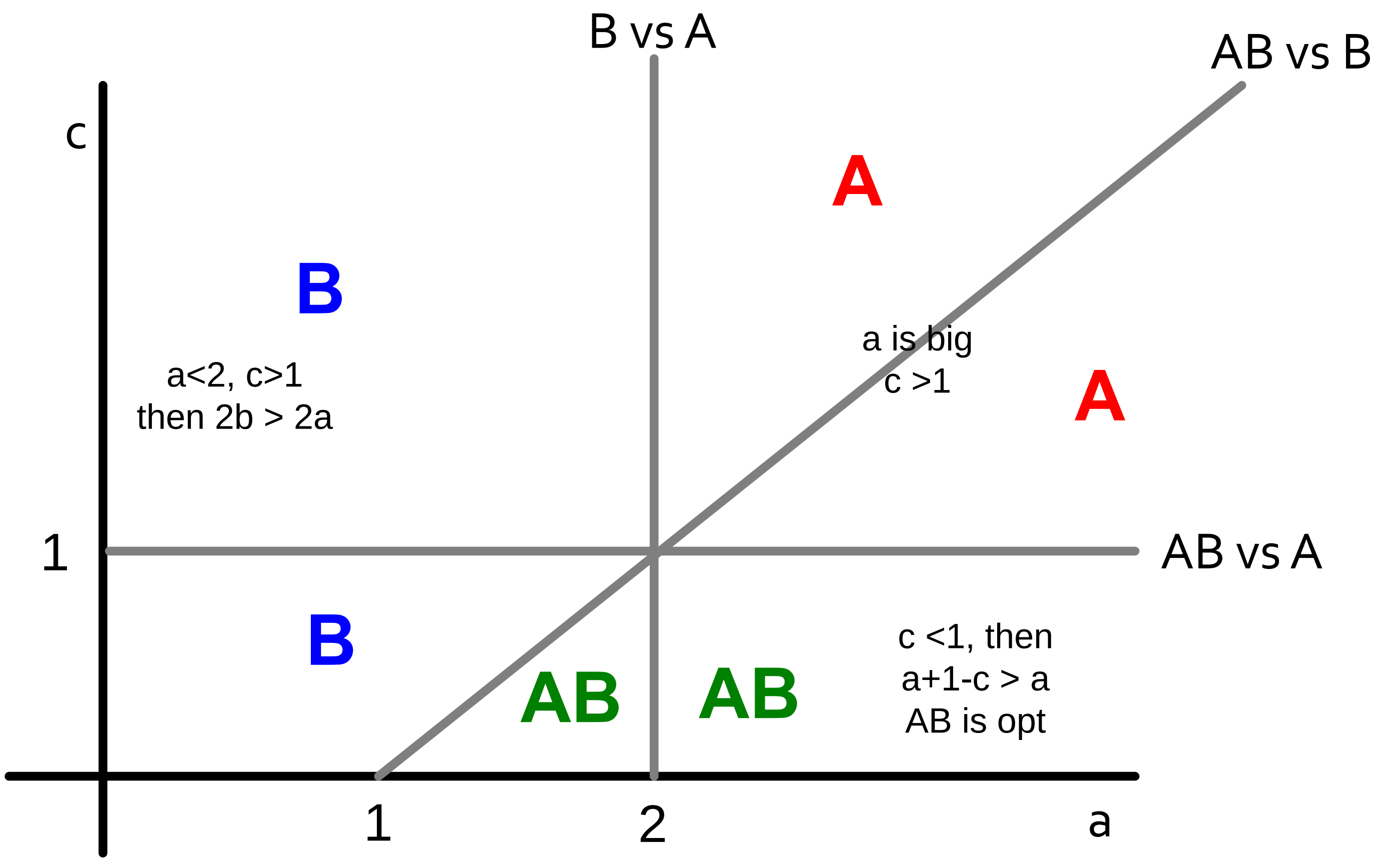
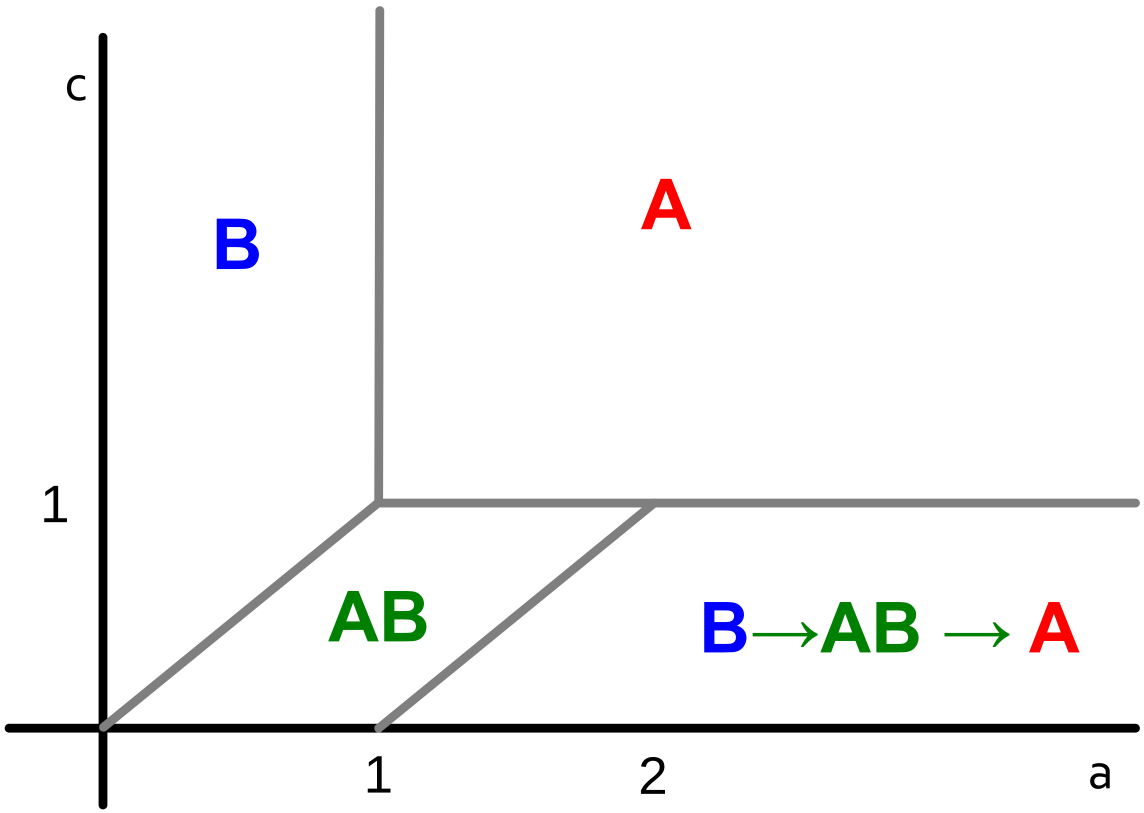




For what pairs (c,a) does A spread?

- Joining the two pictures:



Lesson

- **You manufacture default B and new/better A comes along:**
  - **Infiltration:** If B is too compatible then people will take on both and then drop the worse one (B)
  - **Direct conquest:** If A makes itself not compatible – people on the border must choose. They pick the better one (A)
  - **Buffer zone:** If you choose an optimal level then you keep a static “buffer” between A and B