Aggregate Volatility in Complex Power Grid Networks Under Uncertain Market Conditions

Working Draft

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Abstract

The power grid is of crucial importance to the modern economy and in recent years it has drawn also the attention of researchers in the nascent field of complex network science. Most previous works in this field have been descriptive and mainly concerned with highlighting the structure that power grids share with other types of networked (e.g., small world). We propose and develop the first steps of a *prescriptive* study that investigates the situation in which a *continuous* shock (either in demand or supply) propagates on a power grid. We are interested obtaining a tractable, analytic expression of the aggregate volatility resulted from the shock, yet preserving the characteristcs of an engineering network that obeys certain physical laws (i.e., Kirchhoff's theorems). This framework can be then used to set-up problems of economic interest in this context, such as an optimum investment in network elements to achieve minimum disruptions due to a shock. Here we report on identifying the physics-motivated interaction mechanisms between elements in the network. Preliminary results indicate that under certain assumptions and simplifications certain quantities (power flow into a node and likely an "investment" cost associated with that node) may be expressed via a typical network centrality measure - the Bonacich centrality (related to Google's PageRank). Moreover, we plan to use our framework to study empirically shocks on a power grid. For this we implemented algorithms from the complex power networks literature to generate random test networks with realistic characteristics that may be tuned to highlight certain properties or results.

As instructed by Prof. Leskovec, I mention that the week during which the project was due, I had an accident in which I have broken my right arm (which I use to write). This resulted in me not being able to go through a part of the derivations and implementations that I proposed in the milestone.

1 Introduction

The U.S. electric power generation and distribution network has been hailed as "the largest and most complex machine in the world" [Ami04] and the "20^{th} century's engineering innovation most beneficial to our civilization" [Ami11a]. It is a greatly complex network that extends over a vast geographical area whose operation poses both technical (e.g., robustness under failure) and human-related (e.g., consumption behaviors) challenges. In recent years the concept of *smart grid* has received much attention as a way to reduce energy use and the associated environmental costs, and to increase efficiency and reliability of power generation and delivery through better monitoring and control. In the Energy Independence and Security Act of 2007 (Section 1301), it was stated that the Smart Grid should allow for "increased use of digital information and controls to improve reliability, security, and efficiency of the electric grid", as well as "dynamic optimization of grid operations and resources, with full cyber-security..." [Ami11b]. This and similar legislation in the U.S. and other countries [Ami11a] has been adopted to allow a framework for investment in grid technologies that would counterbalance the steady rise in demand for electricity (in the U.S., electricity amounted for 10% of total energy production in 1940, 25% 1970, and 40% in 2002; China's electricity use is projected to double by 2020 [Ami11b]). The smart grid of the 21^{st} century is also expected to be more robust to outages and failures caused by disturbances in supply or demand, which can cause significant financial and other losses (around $80 billion annually in the U.S., and $188 billion worldwide [Ami11b]). Moreover, cascading failures can affect a large number of customers and cause significant economic disruptions (e.g., the August 2003 Northeastern blackout, with over 50 million customers affected and $6 billion in losses). Estimates

[Ami11b] show that both financial and environmental benefits can be substantial - ∼\$70 billion reductions in costs associated with blackouts and ∼18% reductions in GHG emissions *yearly* - for projected investment costs of around \$1.5 trillion.

In this context there has been a sustained effort by researchers to understand the large-scale structure of this type of complex systems, especially for the grids in the U.S. and in Europe. The several areas that have received particular attention in the complex systems literature include the structure of the power network (especially whether the grid has small-world [WS98] or scale-free properties [CP05]) and the reliability and robustness of the grid under different type of attacks or failures [AAN04, SRCCMV08, HOCS+10]. Earlier studies (e.g., [WS98], [CP05] or [AAN04]) abstract away from the actual electrical properties of the network and use a simplified, connection-based topology, in which power generators and distribution stations are nodes, and transmission lines are edges. Newer work has addressed this shortcoming by incorporating into the network definition the some of the underlying physical characteristics that govern the distribution of electricity [WST10d, WST10b, HB08]. It has been shown [WST10c] that the node degree distribution in real power networks does not follow a power law (as in scale-free networks), but is the convolution of a geometric random variable with a discrete random variable.

Traditional engineering literature on power grids focuses on solving (approximately or exactly in some particular cases) optimization problems that make use of detailed physical characteristics and couplings between parts of the system. Examples include the *optimal power flow* problem (OPF, which calculates the optimum generation at buses in the power grid given engineering constraints and a generation cost function) [GIK+11] or the *transmission planning* problem (which calculates the best placement of generation given transmission constraints and costs) [ERG08]. On the other side, much of the complex systems literature on the electric grid has focused on descriptive problems, as illustrated above. The main thread in existing work has been to describe statistical properties of connectives, electrical topology, and structural resilience to failure. We do not know of any attempts to model or quantify the global (system-wide) risks and opportunities presented by the network structure with respect to exogenous processes, in particular effects of operational decisions (e.g., infrastructure investments or renewables integration) or market-side events (e.g., demand or supply shocks). In our view this is mainly because of barrier posed by modeling such highly-complex systems that obey specific physical laws (e.g., Kirchhoff's laws). In this study we propose to arrive at a description of demand-induced processes on the grid that is complex enough as to account for the main physical characteristics of this system, yet simple enough to allow tractability and more generalizable statements about the economic functioning of a smart grid.

The rest of this paper is organized as follows. In Section 2 we review several practical applications of interest in the context of the smart grid that motivate our work. In Section 4 we present relevant elements of power network theory that we use in this paper to develop a simplified approximate model of demand shocks propagating on an electrical topology and show how it may be applied to draw conclusions about investment decisions that increase the reliability of the grid under demand shocks. In Section 5 we describe the empirical framework we used to apply our model to real and simulated power grids. We conclude in Section 6.

2 Background and motivation

Below we introduce several aspects of the electricity market that motivate our work. The discussion is focused on the U.S., but the issues presented are generalizable to any other country with significant grid penetration.

Uncertain demand and supply. On the supply side renewable sources (wind, solar) have effectively zero marginal cost of generation; however they are also intermittent - you cannot know in advance how much wind power you will generate. On the demand side each user consumes an amount of energy that is only revealed in real-time. Utilities need to carefully balance power dispatch under variable generation conditions according to their forecasts of generation and consumption, and within a dynamic market to minimize costs of service [BRK+11]. The increased consumption monitoring capabilities of the *smart meters* allows extracting patterns in user demand profiles that may lead to better market segmentation for demand-response applications [ARS11]. Better matching of demand and supply also has important environmental consequences, as utilities fire up polluting coal plants to meet real-time demand that was not accurately forecast [FFPH09]. We are not aware of any studies (either in the complex networks literature, or in the power engineering literature) that attempt to draw higher-level conclusions about the interactions in such a technological system without resorting to solving fully-specified engineering problems (e.g., [GIK+11, ERG08]).

Risks of power delivery failure. Cascading failure of service in power grids (i.e., blackouts) can achieve massive scale, with millions of customers affected and billions of dollars in losses, such as the ones in the Northeastern U.S. and Italy in 2003, or the one in Germany in 2006 [HOCS+10, Ami04]. Studies of the connectivity topology of the electrical grid for the U.S. [WS98, AAN04, CP05] have shown that this system resembles a small-world network, which is most vulnerable to failure of highly-connected nodes. It has been shown that incorporating the electrical characteristics of the power network (e.g., admittance matrix) into the analysis leads to large changes in node centrality [WST10d, WST10b, HBCSB10]. The few analytical models of cascades in complex networks have typically adopted a extremely “binary“ approach to node failure (i.e., a node can be either ”active” or “disabled” with no in-between states), which makes it very hard to carry an analytical approach. Thus most complex network analyses of blackouts, including very highly cited works [ML02] where the authors proposed a model of load redistribution in case of electric bus failure, have been descriptive in nature.

Robust infrastructure investments. There has been much criticism that the level of infrastructure spending in the U.S. is lacking and has not increased at par with the increase in energy demand [Ami04]. Some studies have argued that increased infrastructure spending (e.g., on capacity increase) will not necessarily lead to better service reliability (or alternatively less vulnerability to failure) because of inherent inefficiencies in the electric topology of the grid [WST10a]. Although the complex networks framework is well suited to analyzing the factors that influence the system-wide behavior of large-scale, interconnected systems, we haven’t been able to find any studies that analyze the financial effect of operational decisions that influence network characteristics as to fit a planner’s objective. We would like to study infrastructure investment decisions that contribute to the absorption and integration of variable demand and supply, and that may increase the system robustness under various types of failures.

3 Electric network theory

We first review several concepts relevant for electrical networks (following [Sac03, LL10]). We use this exposure to motivate our proposed simplified model of interactions between components of the electrical network.

Electric topology of the power grid

Consider an AC electric network. It is composed of N *nodes*, i.e., generation and load buses (“terminal nodes“), or transmission buses (“internal nodes“), and L *branches*, i.e., the transmission lines. A special node is the *reference (ground) node* of zero voltage; any branch that connects to that node is a *shunt*

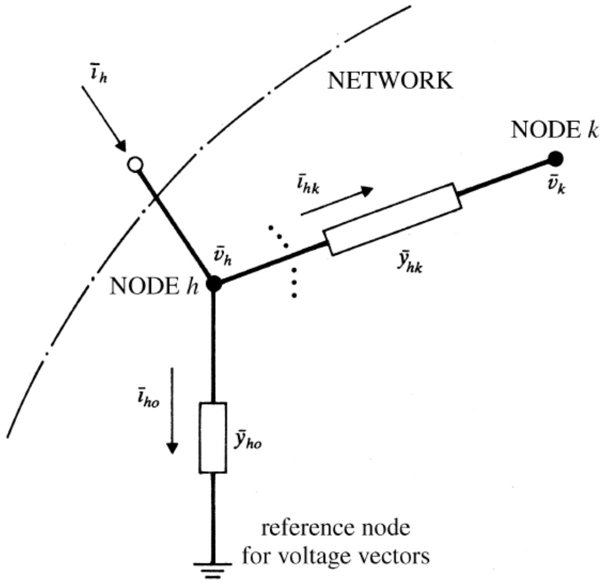

Figure 1: Nodes and branches in an electrical network. Currents follow Kirchhoff's First Theorem. The relationship between currents and voltages is given by Ohm's Law. Source: [Sac03].

branch. The topological properties of the network may be summarized via the $L \times N$ line-node incidence (or "connection") matrix $\mathbf{C}$ (here l is a branch and s, t, and k are nodes):

$$\mathbf{C} : \begin{cases} C(l,s) & = +1 \\ C(l,t) & = -1 \\ C(l,k) & = 0, \text{ with } k \neq s \text{ or } t \end{cases}$$

Each node h is characterized by a (complex) voltage $\bar{v}_h$, while each transmission line l has an impedance $z(l) = r(l) + jx(l)$, with $r(l)$ the *resistance* and $x(l)$ the *reactance*, and an admittance $y(l) = 1/z(l) = g(l) + jb(l)$, where $g(l)$ is the *conductance*, and $b(l)$ is the *susceptance*. Then the branch voltages are given by $\bar{\mathbf{u}} = \mathbf{C}\bar{\mathbf{V}}$, with $\bar{\mathbf{V}} = (\bar{v}_1, \bar{v}_2, \ldots \bar{v}_N)^T$, and by *Ohm's Law* the branch currents are given by

$$\bar{\mathbf{f}} = diag(\bar{\mathbf{Y}}^L)\bar{u} = diag(\bar{\mathbf{Y}}^L)\mathbf{C}\bar{\mathbf{V}}, \tag{1}$$

with $\bar{\mathbf{Y}}^L$ the $L \times 1$ vector of (complex) line admittances.

Current flows and voltages follow (to a first approximation) Kirchhoff's Current Law (KCL, arising from to conservation of charge), and Kirchhoff's Voltage Law (KVL, arising from conservation of energy). For any two nodes h and k, we may consider the situation depicted in Figure 1 where a current $\bar{\imath}_h$ goes into node h, and a current $\bar{\imath}_{hk}$ comes out of node k (a current $\bar{\imath}_{h0}$ goes into the ground). Using Kirchhoff's First (Current) and Second (Voltage) Theorems, one may express the two currents as

$$\begin{aligned} \bar{\imath}_{h0} &= \bar{y}_{h0}\bar{v}_h \text{ and} \\ \bar{\imath}_{hk} &= \bar{y}_{hk}(\bar{v}_h - \bar{v}_k) \end{aligned} \tag{2}$$

This may be extended over all branches hk that go out of node h:

$$\bar{\imath}_h = \bar{\imath}_{h0} + \sum_{\substack{k=1 \\ k \neq h}}^{N} \bar{\imath}_{hk} \tag{3}$$

For the entire network this can be expressed succinctly as

$$\bar{\mathbf{I}} = \bar{\mathbf{Y}}\bar{\mathbf{V}}, \tag{4}$$

where $\bar{\mathbf{Y}}$ is the admittance matrix and $\bar{\mathbf{I}}$ the vector of *externally injected* currents. Then the network admittance can be expressed as

$$\bar{\mathbf{Y}} = \mathbf{C}^T diag(\bar{\mathbf{Y}}^L)\mathbf{C}, \tag{5}$$

where the elements of the *sparse* matrix $\bar{\mathbf{Y}}$ are given by:

$$\bar{\mathbf{Y}} : \begin{cases} \bar{Y}(h,h) & = \bar{y}_{h0} + \sum_{\substack{k=1 \\ k\neq h}}^{N} \bar{y}_{hk} \\ \bar{Y}(h,k) & = -\bar{y}_{hk}, \;\; (k \neq h) \end{cases}$$

Power equations for the electric grid

Power flows along transmission lines from higher to lower voltages (in the direction of branch currents). The complex voltage at node h can be written as $\bar{v}_h = v_h e^{j\alpha_h}$, where α_h is the phase difference between voltage and current at steady state. Then it can be shown from Equations (3) and (2) that the complex power $\bar{P}_h = \bar{v}_h \bar{\imath}_h = P_h + jQ_h$ *injected from the external* at node h can be expressed in terms of the active (P_h) and reactive (Q_h) components as follows:

$$\begin{aligned} P_h &= v_h^2 G_{hh} + v_h \sum_{\substack{k=1 \\ k\neq h}}^{N} v_k (G_{hk} \cos\alpha_{hk} + B_{hk} \sin\alpha_{hk}) \\ Q_h &= -v_h^2 B_{hh} + v_h \sum_{\substack{k=1 \\ k\neq h}}^{N} v_k (G_{hk} \sin\alpha_{hk} - B_{hk} \cos\alpha_{hk}). \end{aligned} \tag{6}$$

If the network is predominately reactive ($G \cong 0$) and the phase shifts α are generally small, so the above expressions may simplify to:

$$\begin{aligned} P_h &= v_h \sum_{\substack{k=1 \\ k\neq h}}^{N} v_k B_{hk} \alpha_{hk} \\ Q_h &= -v_h \sum_{k=1}^{N} v_k B_{hk}. \end{aligned} \tag{7}$$

As such, under the above assumption of zero transmission losses, the power entering node h coming from node k can be expressed as:

$$\begin{aligned} P_{kh} &= v_h v_k B_{hk} \alpha_{hk} \\ Q_{kh} &= -v_h^2 (B_{h0} + B_{hk}) + B_{hk} v_h v_k. \end{aligned} \tag{8}$$

Note that the active power P_h depends on both electrical properties of the network (the branch susceptances B_{hk}) and the phase angles α_{hk}, whereas the reactive power Q_h does not depend on the phase angles.

From the first part of Equation (7) we may also express the phase angles as

$$\alpha^{\mathbf{0}} = \mathbf{TP}, \tag{9}$$

with $\mathbf{P}$ the vector of powers injected at each of the nodes, $\alpha_h^0 = \alpha_h - \alpha_0$ the phase angle with respect to the node 0 chosen as reference, and T the matrix defined by $T_{hh} = \sum_r B_{hr} v_h v_r$ and $T_{hk} = -B_{hk} v_h v_k$. Now if we let the power at node i vary bt δP_i, the above Equation and Equation (8) imply for the flow power between nodes h and k:

$$\delta P_{hk} = B'_{kh}(T_{hi} - T_{ki})\delta P_i \equiv \zeta_{hk}^i \delta P_i, \tag{10}$$

where $B'_{kh} = B_{kh} v_k v_h$.

4 Model of shock propagation on the grid

In this Section we describe the simplifying assumptions that we make to arrive at a description of the interactions between nodes in a power grid that is locally linear. We argue that if assuming small perturbations from an initial steady state and operation below the capacity threshold for network elements one may arrive at results similar to complex network theory treatments of other problems that study the flow of other quantities over a network medium.

Simplified power flow model

Suppose we are interested in a situation where node voltages are fixed (e.g., have been set to optimum operating values by grid operators) for a lossless network ($G \cong 0$). The steady-state phase angles α_{hk} and node voltages v_h may be obtained by e.g., solving an optimum power flow problem [GIK$^+$11] or an optimum transmission planning problem [ERG08]. Here we ask how surges in power either on the demand or supply side propagate through this network operating at an engineering equilibrium point.

We start by noting that it is the active power that it is transmitted to the load nodes through the electric network. Moreover, taking derivatives of the quantities in Equation (8) shows that the active power is most sensitive to a change in the phase angles, whereas reactive power is mainly influenced by changes in voltage magnitudes. Thus we may reasonably assume a situation where node voltages are kept constant. In a simplified approach we consider the phase angles α_{hk} as jointly characterizing network electrical topology along with the line impedances B_{hk} and node voltages v_h. As such, in Equations (7 and 8) above (the active power part) we combine the branch-specific coefficients into $W_{kh} = v_k B_{kh} \alpha_{kh}$ and define $\hat{W}_{kh} = \frac{W_{kh}}{\sum_k W_{kh}}$. Note that the matrix $\{\hat{W}\}_{kh}$ is *not* symmetric because of the v_k term. Then the expression for the active power from Equation (7) becomes:

$$P_{kh} = P_h \hat{W}_{kh}, \tag{11}$$

i.e., the power entering node k coming from node h is a fraction $\hat{W}_{kh}$ of the power entering through node h.

Consider now the situation in Figure 2. There the superscripts s and d indicate, in that order, generated power ("supply") and consumed power ("demand"). We thus define the net power produced at a node h as

$$\Delta P_h = P_h^s - P_h^d, \tag{12}$$

with P_h^d the load at bus h and P_h^s the generation at bus h. Note that ΔP_h can be either positive (net power generation) or negative (net power consumer). Depending on the sign of ΔP_h, nodes can be either generators ($\Delta P_h > 0$), consumer loads ($\Delta P < 0$), or transmission nodes ($\Delta P_h = 0$).

Energy conservation requires that, for each node h, power flowing out of a node balances the power flowing in and the (net) power produced (or consumed) at that node:

$$P_j^{out} = \Delta P_j + P_j^{in}. \tag{13}$$

Denoting $\Pi_j \equiv P_j^{in}$ we recall from Equation (11) that $\Pi_h = \sum_j P_j^{out} \hat{W}_{jh}$, and thus we may use Equation (13) to write

$$\begin{aligned} \Pi_h &= \sum_j \hat{W}_{jh}(\Delta P_j + \Pi_j) \\ &= \sum_j \hat{W}_{jh} \Delta P_j + \sum_j \hat{W}_{jh} \Pi_j, \end{aligned} \tag{14}$$

which in vector form amounts to writing

$$\mathbf{\Pi} = \hat{\mathbf{W}} \mathbf{\Delta P} + \hat{\mathbf{W}} \mathbf{\Pi}. \tag{15}$$

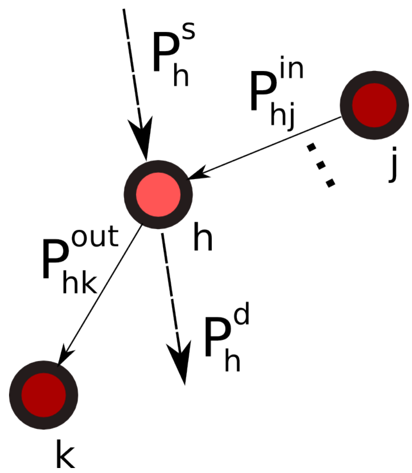

Figure 2: Energy conservation at each node: net power flowing into any node h equals net power flowing out.

Solving the above equation for the case $\Delta P_h \neq 0$ we arrive at an expression for the power flow entering nodes in the network:

$$\mathbf{\Pi} = (\mathbf{I} - \hat{\mathbf{W}})^{-1}\hat{\mathbf{W}}\mathbf{\Delta P}. \tag{16}$$

When $\Delta P_h = 0$ the above Equation (15) suggests:

$$\mathbf{\Pi} = \hat{\mathbf{W}}\mathbf{\Pi}, \tag{17}$$

i.e., for transmission nodes, the power flowing into each node is given by the entries of the eigenvector corresponding to a unit eigenvalue of the steady-state weights matrix $\hat{\mathbf{W}}$. Note that power entering each node in the network in Equation (16) is related to the Bonacich centrality of that node. This result is similar to other conclusions about centrality in the complex networks literature, e.g., [VBS11], which studies the flow of of "corporate control" over the global network of trans-national companies. Here we however highlighted the physics behind this conclusion for the case of a power network.

Optimum power injection model

Now suppose, on the contrary, that we are interested in the response of a grid operator when faced with an external event affecting the economic and engineering operational state of the network, e.g., the sudden loss of *continuous* amounts of power at some nodes. Here we consider a *lossy* line $G > 0$ in the case of small phase angles α_{hk} ($\sin\alpha_{hk} \cong \alpha_{hk}$ and $\cos\alpha_{hk} \cong 1$) and ask what the best response of the grid operator is, given external (market) conditions.

Recall the expression for the active power in Equation (6), and denote $H_{hk} = G_{hk} + B_{hk}\alpha_{hk}$. An external agent acts on node h (operating at voltage v_h) by drawing a current ι_h, which amounts to a power $\pi_h = \iota_h v_h$. Then we may write for the power injected at node h:

$$P_h = v_h^2 G_h h + v_h \sum_{\substack{k=1\\k\neq h}}^{N} v_k H_{hk} - \iota_h v_h. \tag{18}$$

We are interested in generating the minimum amount of power such that some financial measure is optimized. The best response to the external action ι_h is then linear in the voltages (we take $\partial P_h/\partial v_h = 0$):

$$2G_{hh} + \sum_{\substack{k=1\\k\neq h}}^{N} v_k H_{hk} - \iota_h = 0 \tag{19}$$

Then in vector form we may write (with $\gamma_h = \frac{1}{2G_{hh}}$)

$$\mathbf{v} = \boldsymbol{\iota} - diag(\boldsymbol{\gamma})\mathbf{H}\mathbf{v}, \tag{20}$$

from which we may solve for $\mathbf{v}$:

$$\mathbf{v} = (\mathbf{I} - diag(\boldsymbol{\gamma})\mathbf{H})^{-1}\,\boldsymbol{\iota}. \tag{21}$$

This model is most closely related to [CBO11], where the authors formulate the optimal consumption of a network agent in response to an externally-specified price signal and perfect knowledge of the consumption levels of neighboring nodes in the network. The best response voltage at a given node is related, as before, to the Bonacich centrality of that node.

Stochastic demand and supply and associated operational decisions

As future work we plan to investigate analytically how a shock in demand or supply propagates on a network for which the simplified interaction mechanisms are as illustrated above. In the first setting (simplified power flow model) we may explore several scenarios:

- Demand is fixed; we let supply be stochastic, i.e., $P_h^s \to P_h^s + \epsilon$, with some shock $\epsilon \sim N(0, \sigma^2)$. We may then calculate how much of σ^2 is present at some other node k in the network as power inflow. Then we could define an optimal investment problem which associates a (linear, quadratic...) cost with the perturbance at each node, then minimizes that cost;
- Supply is fixed; we let demand be stochastic similarly as before. In the current formulation it seems that the two cases are equivalent when either is fixed. Perhaps some regularity conditions set in that we need to be careful about, e.g., power flow sign changes may affect the weights $\hat{W}_{hk}$ through the phase angles?
- Either supply or demand receives shocks at more than one node; Are the effects superposable, or are there correlations terms arising due to network interactions?
- Let both supply and demand be variable. Study implications of “forecastability” of supply vs demand (i.e., which one has smaller variance). That is, let some demand and supply shocks appear on the network; is it better in terms of investment cost to know the demand more accurately (lower variance), or is it more cost-effective to have more information about the supply?

In the second setting (best-response injection) we could similarly study a stochastic external action $\boldsymbol{\iota} \sim N(0, \sigma^2)$, and carry analyses as suggested above.

5 Empirical framework

As previously mentioned, we would like to be able to illustrate our model on a variety of power networks that are both adjustable to fit our particular experiments, but at the same time retain the statistical characteristics empirically observed in real grids. We first present measures of node centrality proposed in the complex power grids literature that take into account some of the physical characteristics of the underlying network [WST10b]. We then describe a method proposed in the literature [WST10b] for generating random networks that follow the observed statistical properties of real power grids. We shall use the random electric topologies to illustrate our economic model for electric networks of varying sizes and other characteristics. This will enable us to perform simulations that would generalize some of the conclusions of the model.

Measures of electrical centrality

Using the admittance matrix and its correspondence to the usual graph *Laplacian* one can extend the usual notions of centrality - betweenness, degree, closeness, or eigenvector (i.e., PageRank) - to the electric grid. In [WST10b] several such measures are described in close correspondence to the same quantities derived from unweighted graphs. As such, *degree centrality* of a vertex v, $C_d(v) = \frac{\deg(v)}{n-1} = \frac{L(v,v)}{n-1}$ becomes the *electrical degree centrality*

$$C_{d_Y}(v) = \frac{||Y(v,v)||}{n-1}. \tag{22}$$

Similarly, the eigenvector centrality (or Bonacich centrality [Jac08]) $C_e(v) = \frac{1}{\lambda_{\max}} \sum_{j=1}^{n} A(v,j)x_j$ becomes

$$C_e(v) = ||\frac{1}{\lambda_{\max}} \sum_{j=1}^{n} A_Y(v,j)x_j||,$$

with $A_Y = -Y + D(Y)$.
Another centrality measure that we are interested in is related to *closeness centrality*, which in a non-weighted network is the mean geodesic distance between two nodes:

$$C_c(v) = \frac{n-1}{\sum_{t \in V n v} d_G(v,t)}. \tag{23}$$

The geodesic distance d_G above can be replaced by the "shortest electrical distance" between any vertices v and t, $d_Z(v,t) = || \sum_{(i,j) \in E \cap \mathrm{path}(v \to t)} Z_{pr}(i,j)||$.

While the measures described above do incorporate certain aspects of the physics of a complex grid (that is, the fact that transmission lines are heterogeneous in their intrinsic characteristics, summarized by the complex impedance/admittance value), they actually fail to account for two fundamental physical phenomena in power grids, namely:

- Physical power networks are directed in steady state [DY11], i.e., power flows in well-specified directions along transmission lines. As such, the actual network is asymmetric (because of directedness), which is not reflected in formulations in the literature.
- In physical grids, power doesn't flow on shortest paths, or "shortest electrical paths"; rather, it obeys Kirchhoff's Laws 3. As such there are specific interactions between each node and its' neighbors that are not captured in the simple formulations above.

Some of these concerns have been addressed to some extent in the complex power grids literature, e.g., in [DY11], where the authors have proposed a measure of *link* centrality based on the fraction of the total current that flows through it from the source node to the sink node. There an approach based on the graph-theoretic Min-Cut problem is proposed to calculate this centrality measure practically. However we feel that our contribution will better address the above points as our treatment is built up from basic principles.

Generation of random electric topologies

In order to apply our model in a variety of cases, we need to be able to simulate power grids that are realistic in their statistical properties. As noted before, real power grids have been found to be sparse networks that retain some of the characteristics of small-world neworks [WST10d]. In particular, real grids have better topological connectivity than small-world networks, as they are formed of interconnected small-world "communities" [WST10a]. The node degree distribution **X** in a power grid follows a distribution that has a geometric part **G** (the long tail) and a discrete part **D** (that models the irregularities that deviate from the geometric distribution) [WST10c]:

$$p_{\mathbf{X}} = p_{\mathbf{G}} \circ p_{\mathbf{D}}, \tag{24}$$

where the $\circ$ operator above indicates a convolution between the respective PMFs. The random variable **G** is a truncated geometric with a threshold of $k_{\max}$,

$$\Pr(\mathbf{G} = k) = \frac{(1-p)^k p}{\sum_{i=0}^{k_{\max}} (1-p)^i p}, \tag{25}$$

while the random variable **D** is discrete and can take a small number of values:

$$\Pr(\mathbf{D} = k) = p_k, \quad k = 1, 2, \ldots K \tag{26}$$

Electrical characteristics of transmission lines (admittances) may follow one of several types of distributions, including clipped lognormal, lognormal, and generalized Pareto distributions. An original random variable $\mathbf{Y} \sim f_Y(y)$ that is "clipped" by an exponential cutoff at $Z_{\max}$ results in the variable

$$X \sim f_X(x) = \frac{Z_{\max}}{Z_{\max} - x} f_Y \left(-Z_{\max} \log \left(1 - \frac{x}{Z_{\max}} \right) \right). \tag{27}$$

The lognormal distribution is given by:

$$\mathrm{logn}(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-\log x - \mu}{2\sigma^2}}. \tag{28}$$

The observations above are used in [WST10c] to formulate a model of a realistic electrical network which we have implemented for this study. The main steps of this model are:

1. Generate several regular lattices (much like in the Watts-Strogatz small-world model), but with a geometric distribution of the local neighborhood. These lattices are disconnected initially.

2. For each lattice, rewire several links such that there is non-zero correlation between rewired links. [WST10c] uses a Markov process to ensure this property.

3. Generate several long-range lattice links to connect the previously-generated regular lattices

4. Generate impedances as described above and assign them to the different link types such that local links have lowest impedance, and lattice connections have largest impedance. This accounts for the fact that transmission line impedance depends almost linearly with physical line length.

Real power grids data

For this study we use several public datasets created by the `IEEE` that have been described in the literature and are available online[1]. These test-case networks of 14, 30, 57, 118, and 300 nodes (i.e., either generators or distributor buses) are weighted graphs in which edges weights correspond to transmission line impedances. The nodes have many electrical attributes that are relevant for thees power systems. For example, the `IEEE-300` system contains 300 nodes and 409 edges. These systems are referenced in many power grid network analysis studies (e.g., [WST10b, HB08]). The schematic of the `IEEE-30` bus network is presented in Figure 3.
The literature references several other datasets, which unfortunately are either not detailed enough for our analysis (i.e., do not contain physical characteristics of the network), or prove very hard to obtain or are not in the public domain. Examples from the former category include the dataset on the power grid in the Western U.S. that is used in, e.g., [WS98]. In the latter category we may list the `NYISO-2935` dataset (of the New York power grid) and commercial databases such as the one maintained by Platts, Inc.[2].

[1] http://www.ee.washington.edu/research/pstca/
[2] http://www.platts.com/

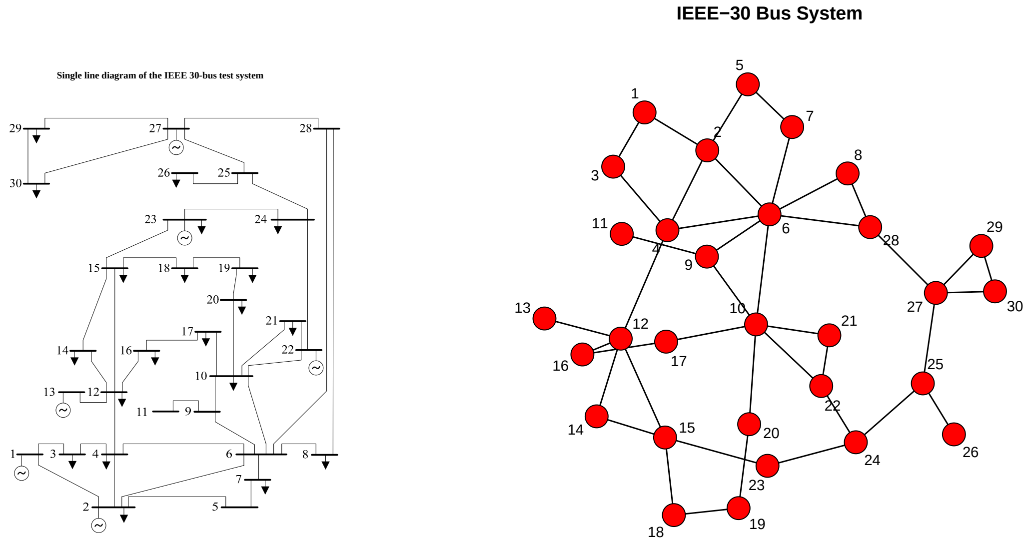

Figure 3: A real power network on 30 nodes (the `IEEE-30` test case). Left: electric schematic; Right: node-link topological diagram.

Empirical analysis

We implemented the algorithm for generating random electric topologies as described above. One such random topology is presented in Figure 4 for a network with $N = 300$ buses and approximately $m = 420$ links (right panel). In the same figure (left panel) we present the a real dataset (the `IEEE-300` test case) of a network with roughly the same number of nodes and links. We compute the three different centrality measures for the two cases (real and random networks) to highlight the difference between the topological and electrical characteristics as defined above. For the random case, we took the average of $N_{\text{Trials}} = 100$ test electrical networks generated at random. To generate the values of the line impedances for the random case we used the clipped lognormal distribution discussed above. The results of this exercise are presented in Figure 4. We observe that while the degree centralities (topological and electrical) are relatively well matched for both the real and random networks, there are large differences in the eigenvector and closeness centralities results for the topological and electrical cases for both the random and real networks.

Another experiment that we performed was to study the behavior of distribution of the normalized centrality indices with increasing network size. We generated networks of sizes N between 14 and 1000 nodes, in which the size of a small-world sub-network scaled as $\sqrt{N}$. For each value of N we retained the average statistics over $N_{\text{Trials}} = 10$ test electrical networks generated at random. We observe that the closeness centrality is consistently higher for small network sizes, whereas the other two measures slowly, but consistently decrease in value with increased number of nodes.

6 Conclusions

We have introduced and motivated an application of complex network theory for studying exogenous processes on a power grid. We studied physical interactions between nodes on a power grid, and show that under certain simplifying assumptions power entering a given node may be related to the Bonacich centrality of that node. Also we show that under a scenario where the voltage at each node is the decision variable, that quantity may also be expressed using a Bonacich centrality term. We

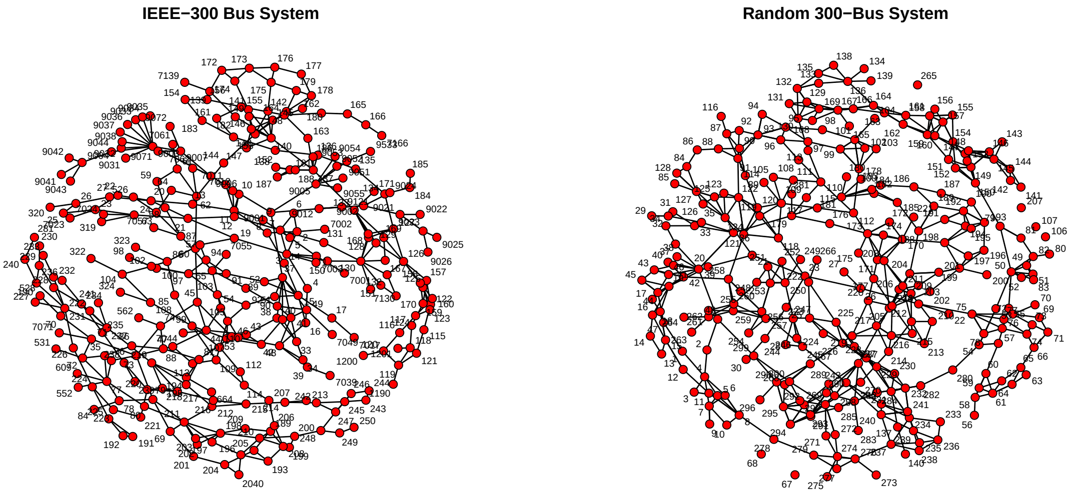

Figure 4: Example power networks. Left: real power network on 300 nodes (the `IEEE-300` test case); Right: simulated power network on 300 nodes using the algorithm in [WST10c].

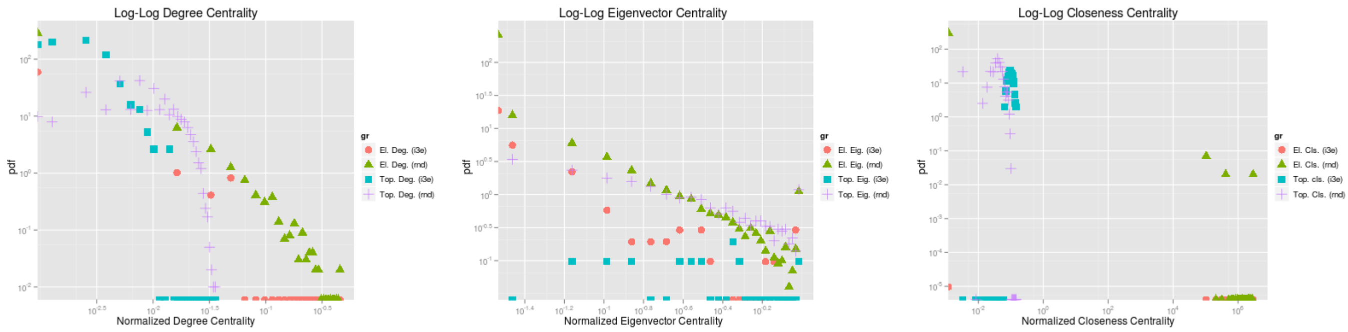

Figure 5: Centrality measures for real and random networks of $N = 300$ nodes and about $m = 420$ links (doubly logarithmic plots). Left: degree centrality; Middle: eigenvector centrality; Left: closeness centrality.

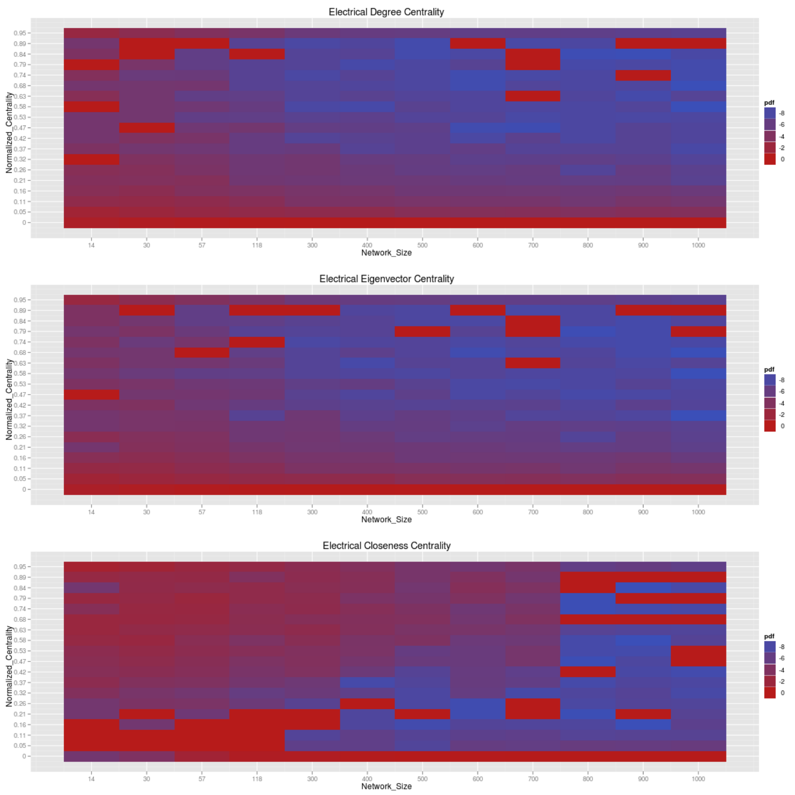

Figure 6: Centrality measures statistics vs random network size. Each column color-codes the distribution of the centrality measure (in steps of 0.05 percentiles between 0 and 1) for a given network size. Top: degree centrality; Middle: eigenvector centrality; Bottom: closeness centrality. Values are logarithmic.

thus set up a preliminary framework that may be used to investigate externally-induced shocks and associated financial aspects.

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