Signed networks: Theories of Structural Balance and Status

CS224W: Social and Information Network Analysis

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http://cs224w.stanford.edu

Signed networks

- Networks with positive and negative relationships
- Consider an undirected complete graph
- Label each edge as either:
  - Positive: friendship, trust, positive sentiment, …
  - Negative: enemy, distrust, negative sentiment, …
- Examine triples of connected nodes A,B, C

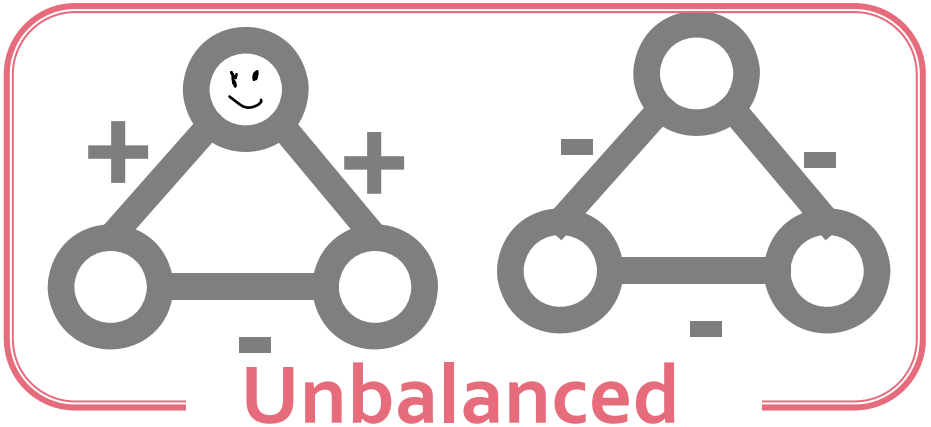
Theory of Structural Balance

Consider edges as undirected

- Start with intuition [Heider '46]:
  - Friend of my friend is my friend
  - Enemy of enemy is my friend
  - Enemy of friend is my enemy
- Look at connected triples of nodes:

Balanced

Consistent with "friend of a friend" or "enemy of the enemy" intuition

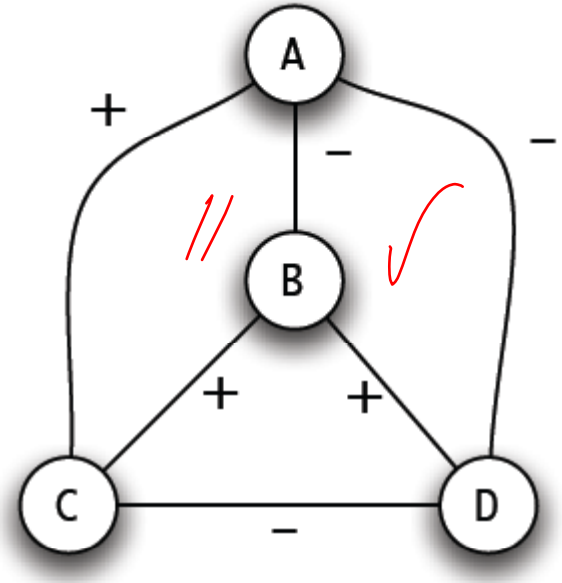


Unbalanced

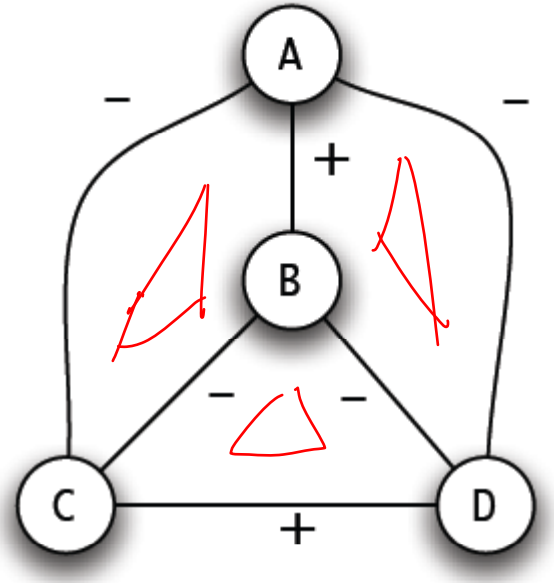
Inconsistent with the "friend of a friend" or "enemy of the enemy" intuition

Balanced/unbalanced networks

- Graph is balanced if every connected triple of nodes has all 3 edges labeled +, or else exactly 1 edge is labeled +.



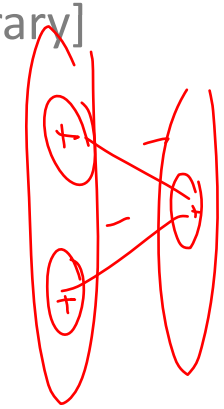
Unbalanced

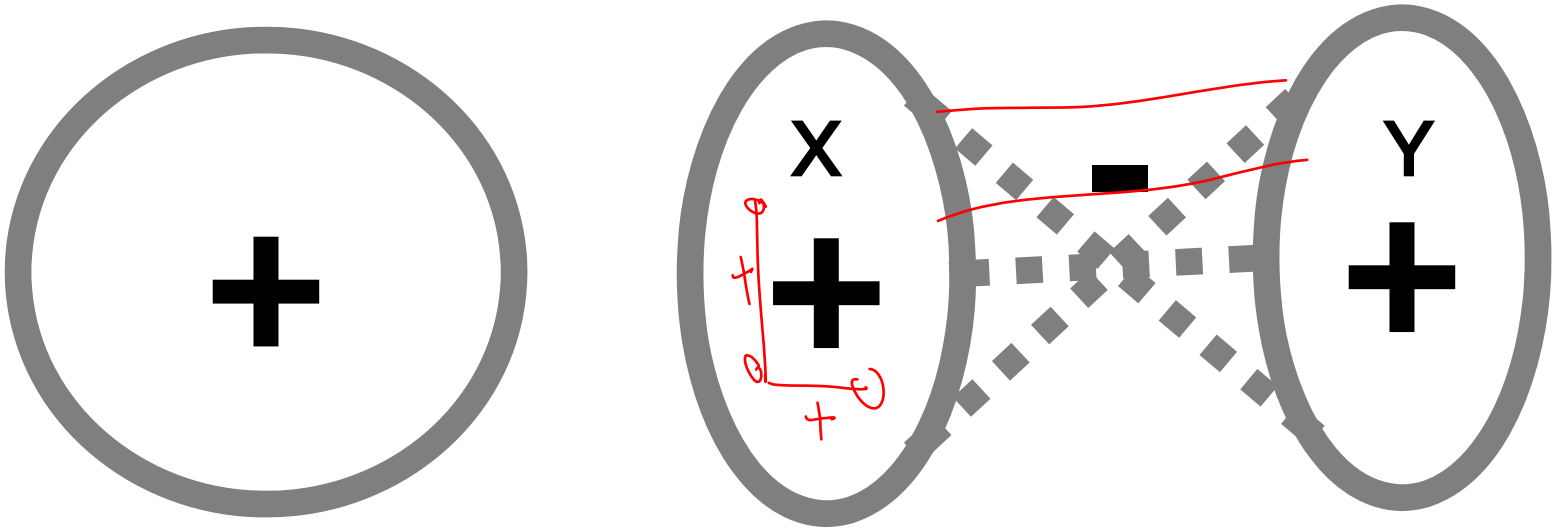


Balanced

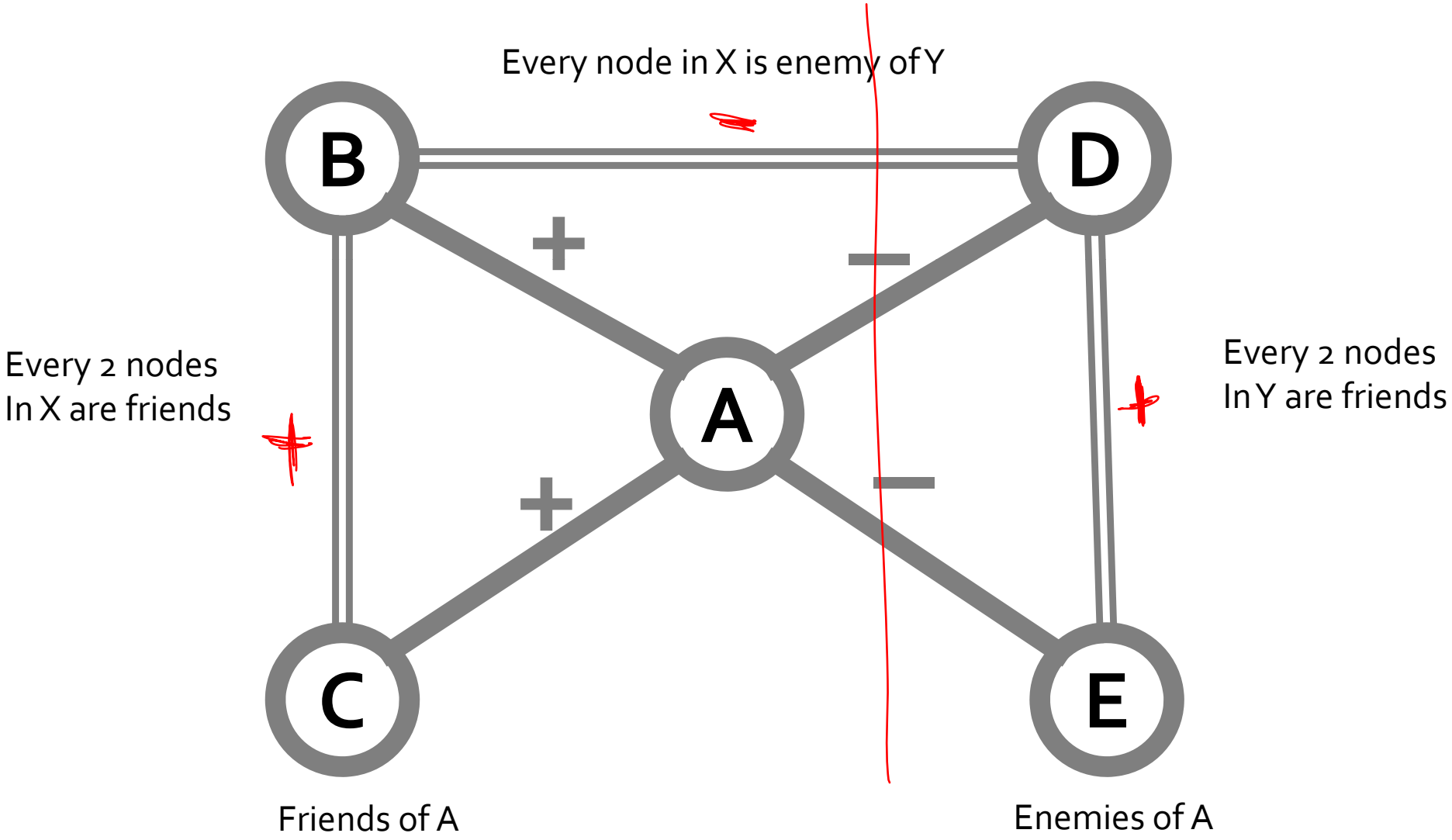
Local balance → Global factions

- Balance implies global coalitions [Cartwright-Harary]
  - If all triangles are balanced, then either:
    - The network contains only positive edges, or
    - Nodes can be split into 2 factions linked by negative edges



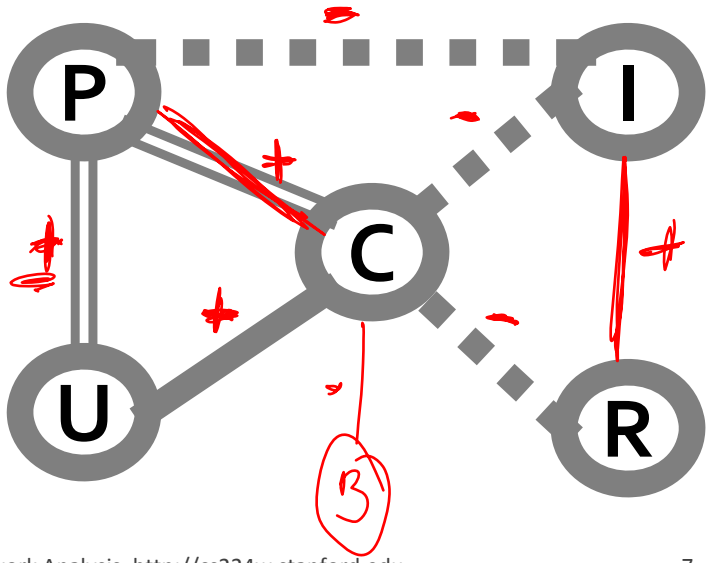


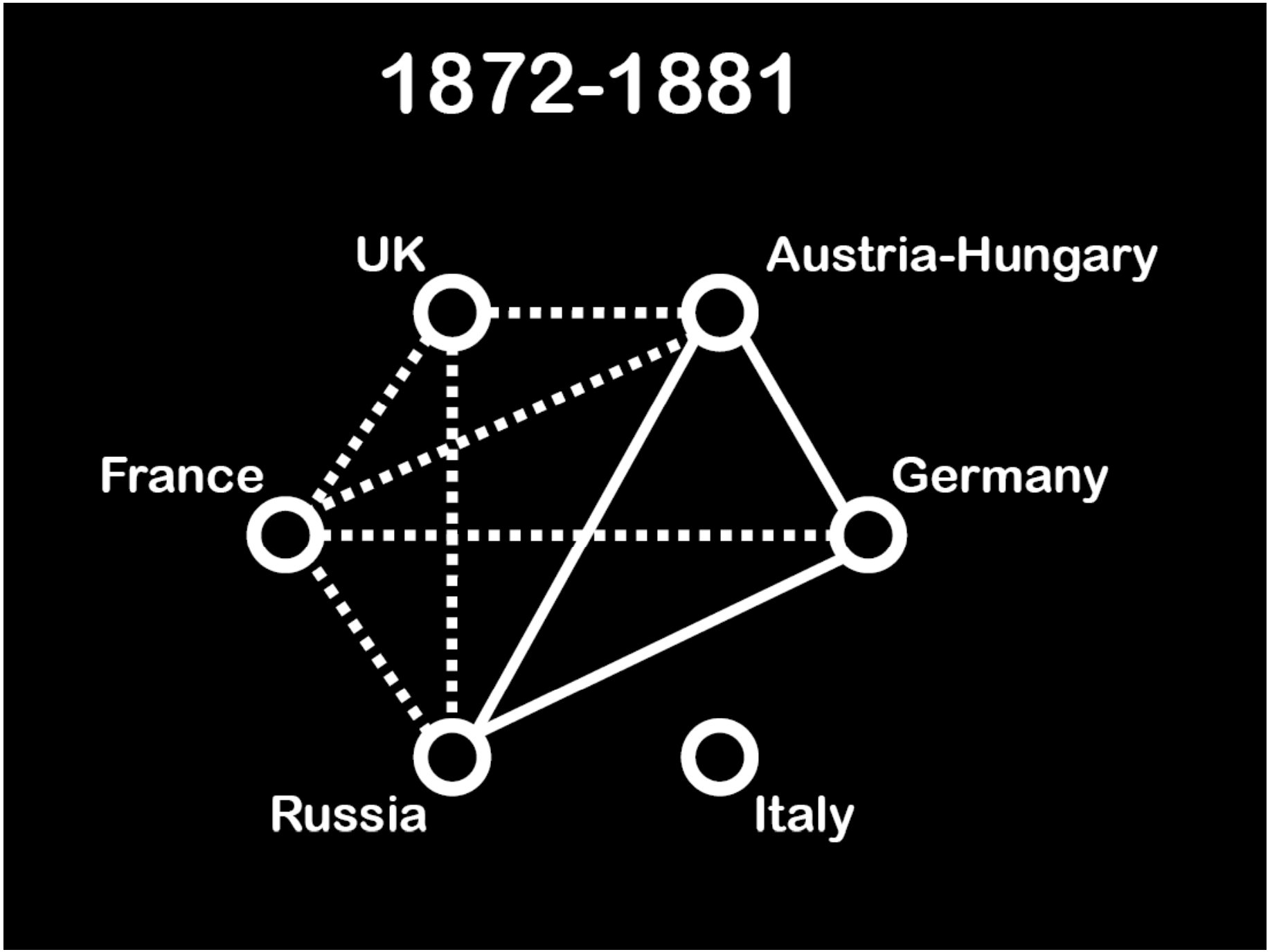
Analysis of balance

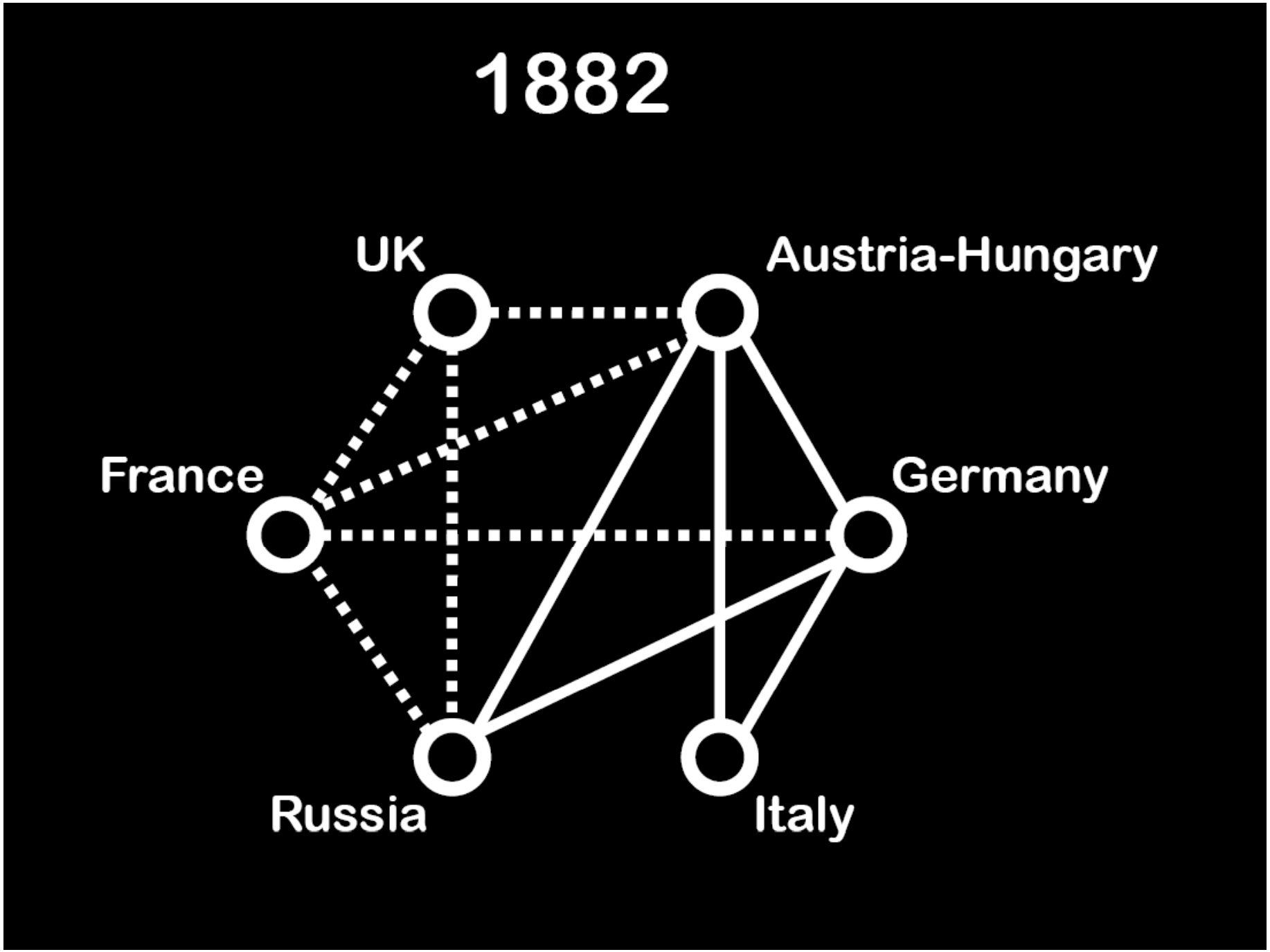


Example

- International relations:
  - Positive edge: alliance
  - Negative edge: animosity
- Separation of Bangladesh from Pakistan in 1971: **U**S supports **P**akistan. Why?
  - USS**R** was enemy of **C**hina
  - **C**hina was enemy of **I**ndia
  - **I**ndia was enemy of **P**akistan
  - **U**S was friendly with **C**hina
  - **C**hina vetoed Bangladesh




1872-1881
UK
Austria-Hungary
France
Germany
Russia
Italy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Russia
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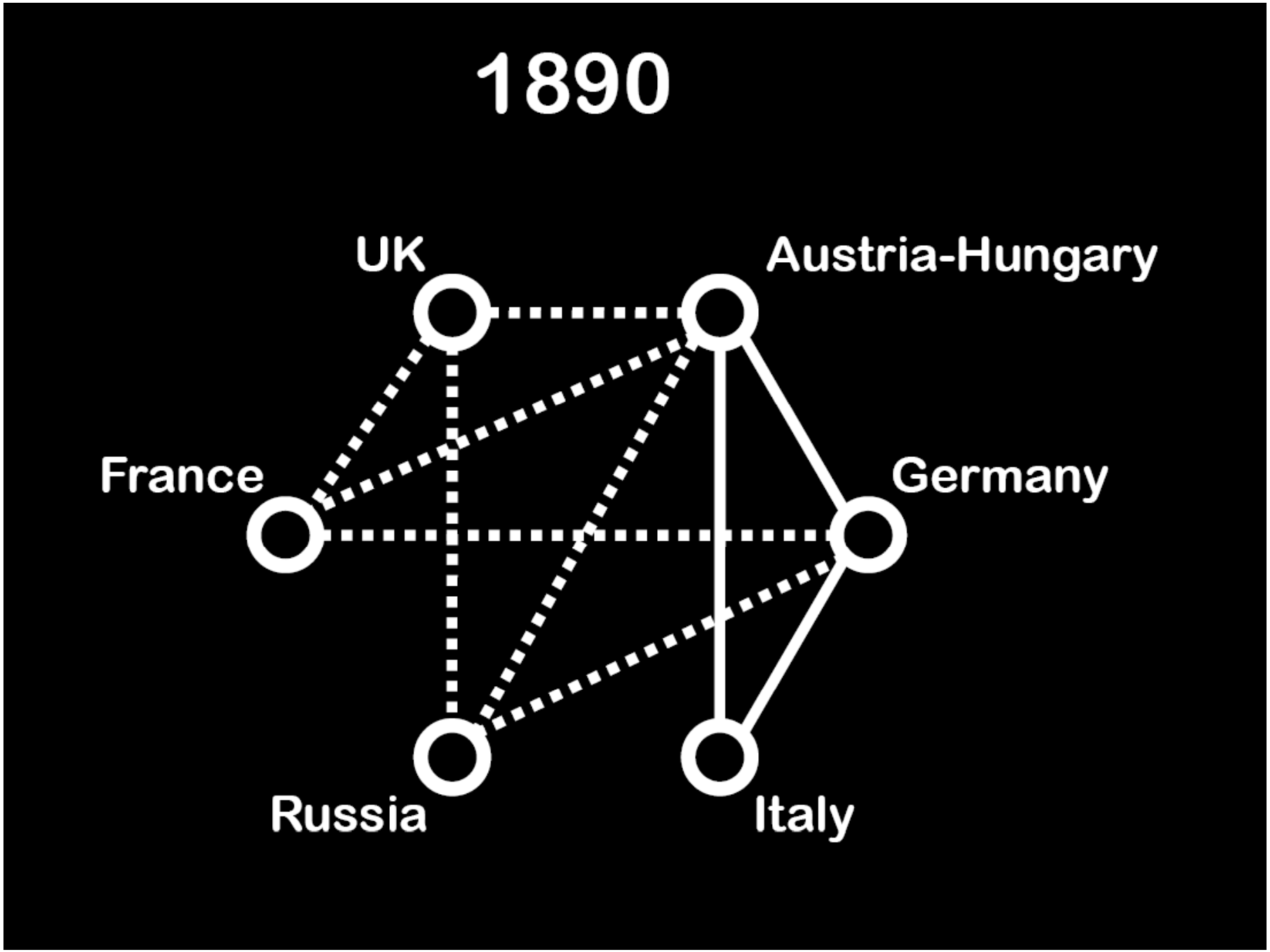
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Germany
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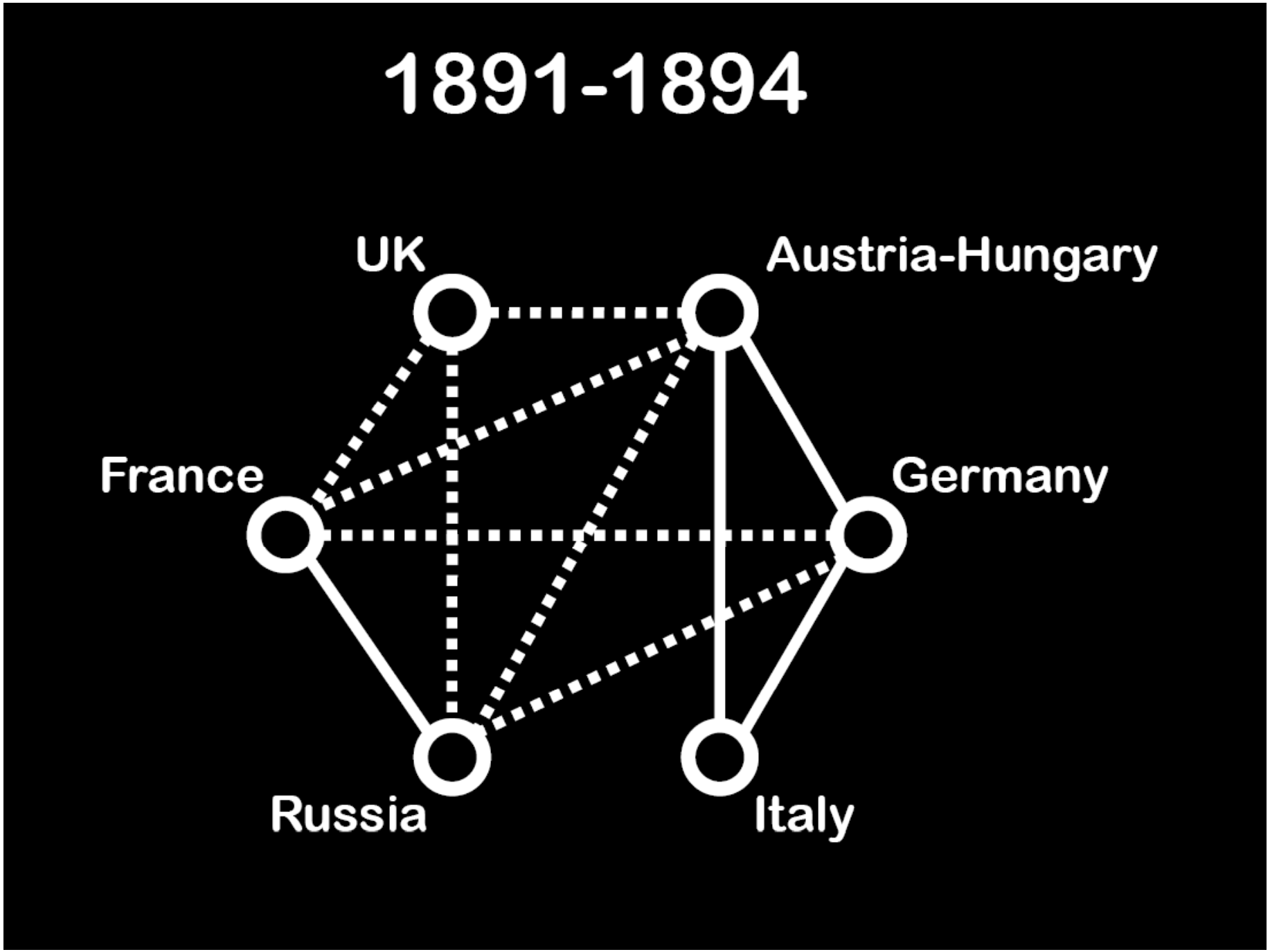
Italy


1891-1894
UK
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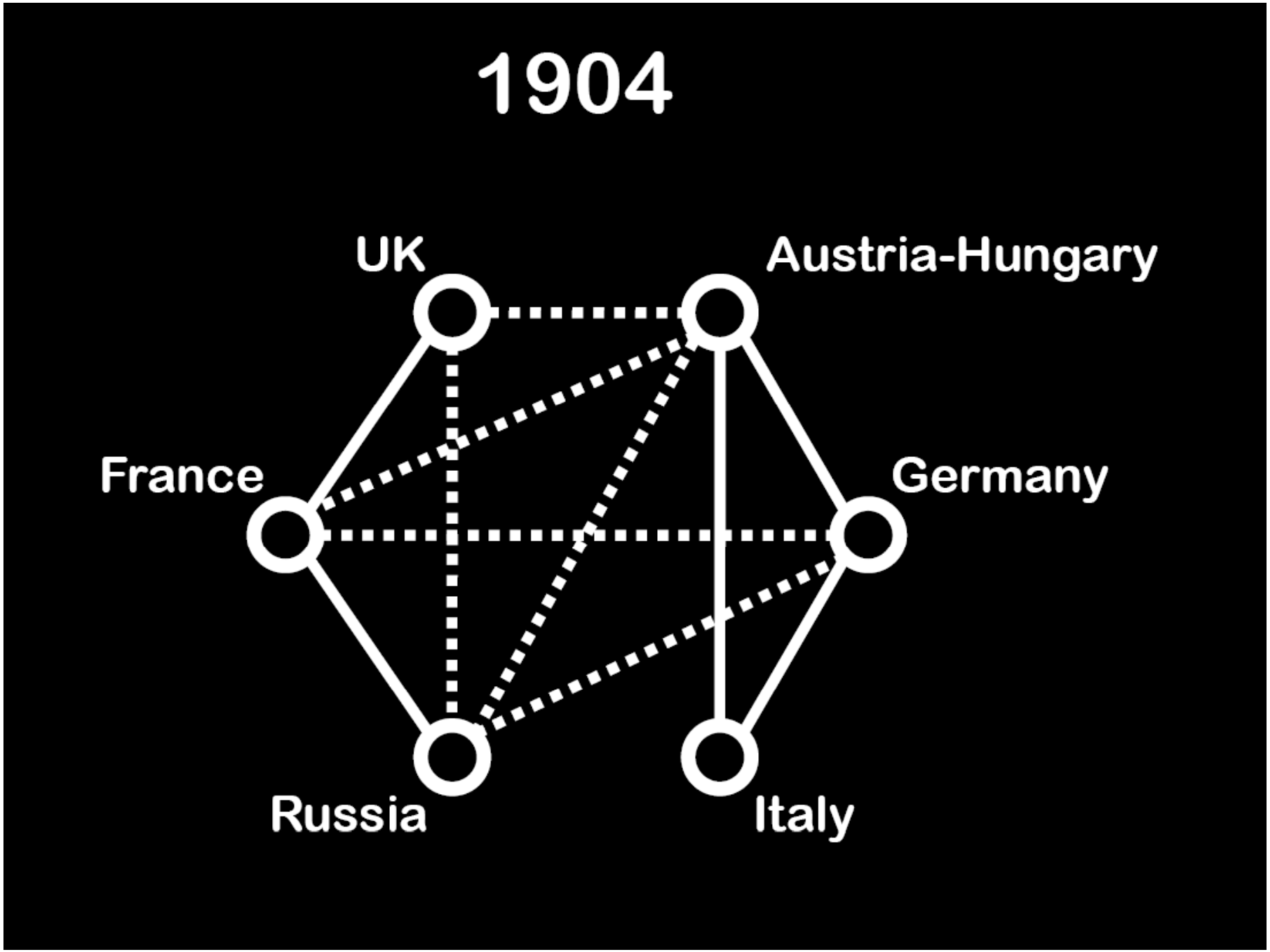
France
Germany
Russia
Italy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UK
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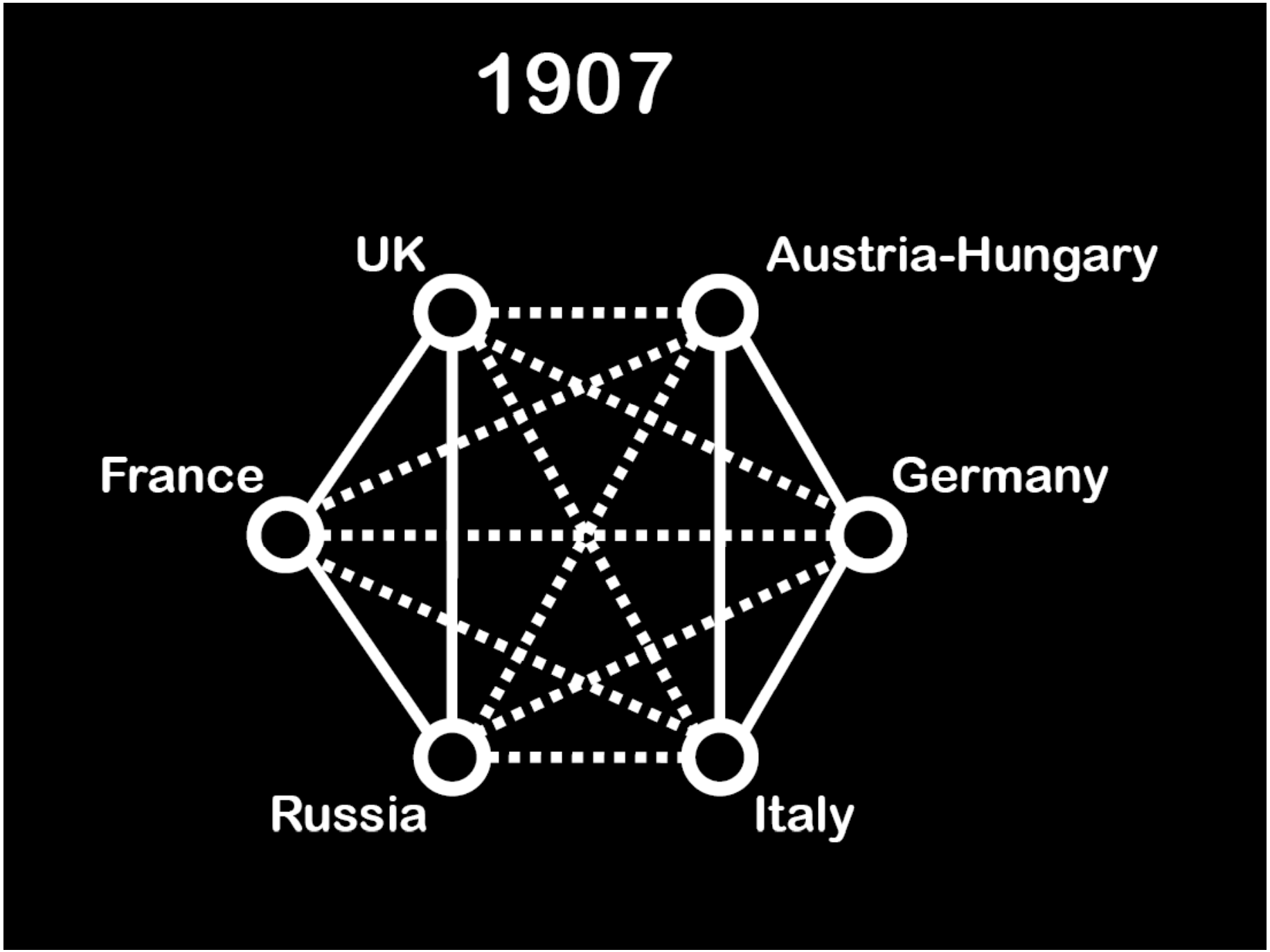
Italy


1907
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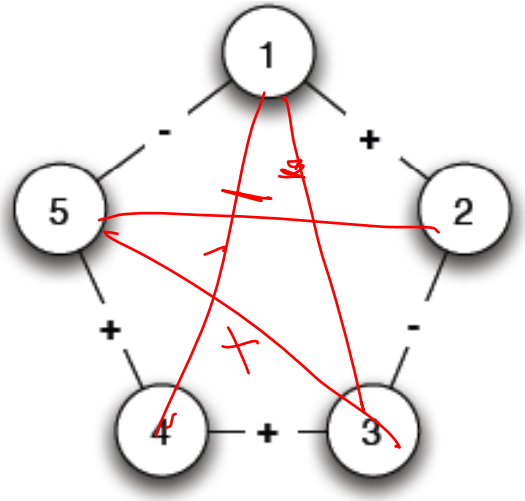
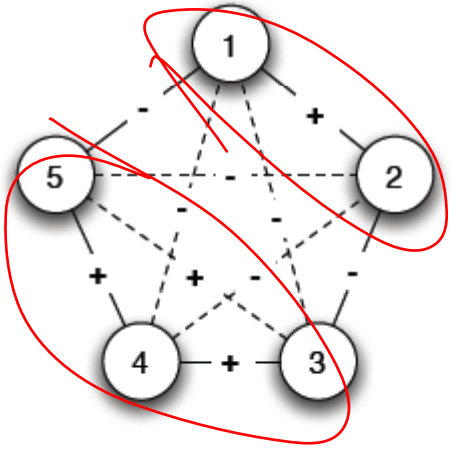
Austria-Hungary
France
Germany
Russia
Italy

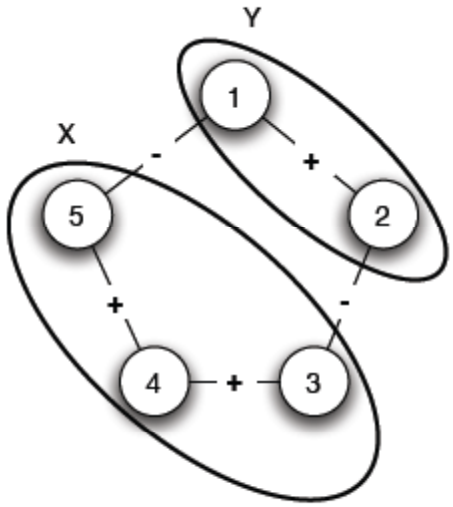
Balance in general networks



Balanced?

- Def 1: Local view
  - Fill in the missing edges to achieve balance
- Def 2: Global view
  - Divide the graph into two coalitions
- Defs are equivalent!

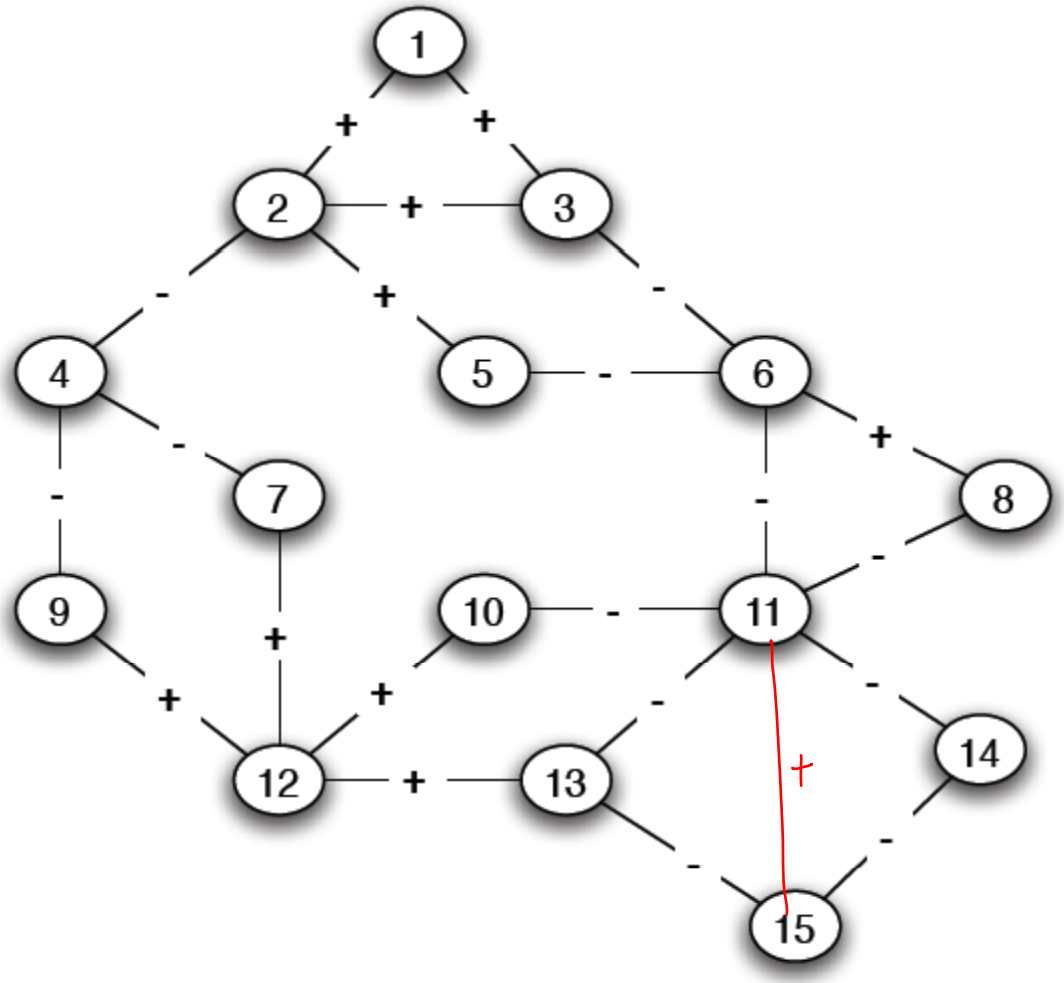




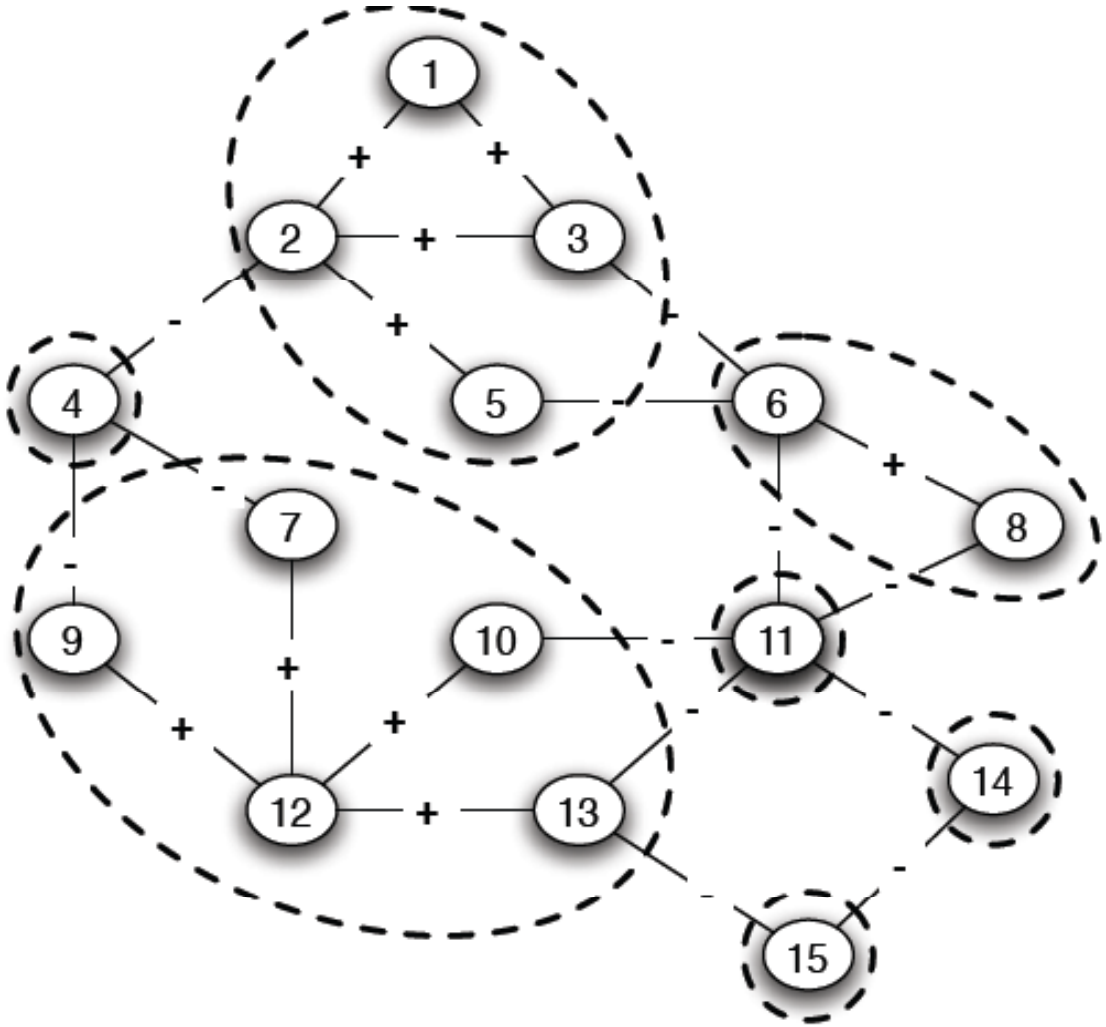
Is a signed network balanced?

- A graph is balanced if and only if it contains **no** cycle with an odd number of negative edges.
- How to compute this?
  - Find connected components on + edges
  - For each component create a super-node
  - Connect components A and B if there is a negative edge between the members
  - Assign super-nodes to sides using BFS

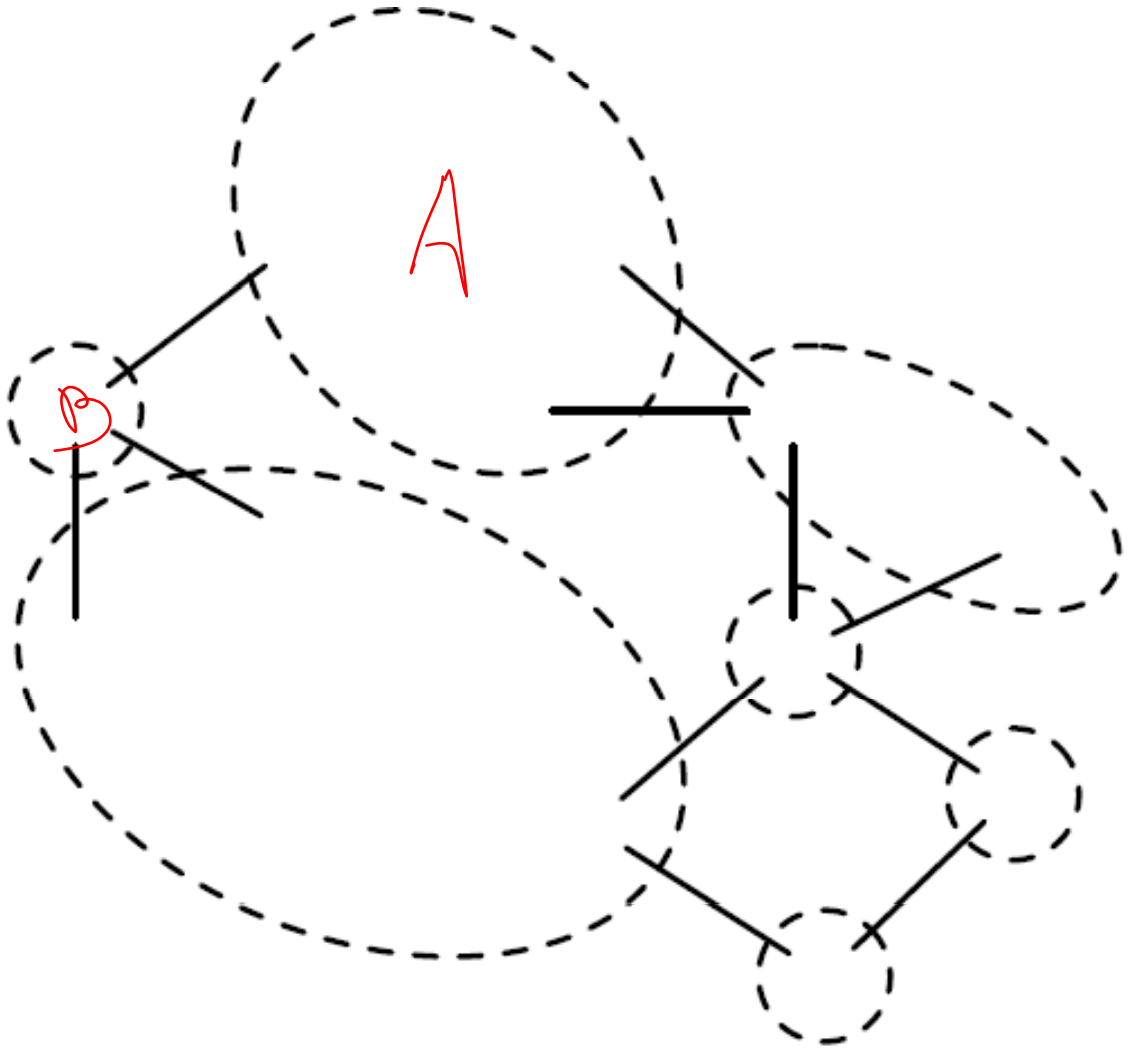
Signed Graph: Is it balanced?



Positive connected components

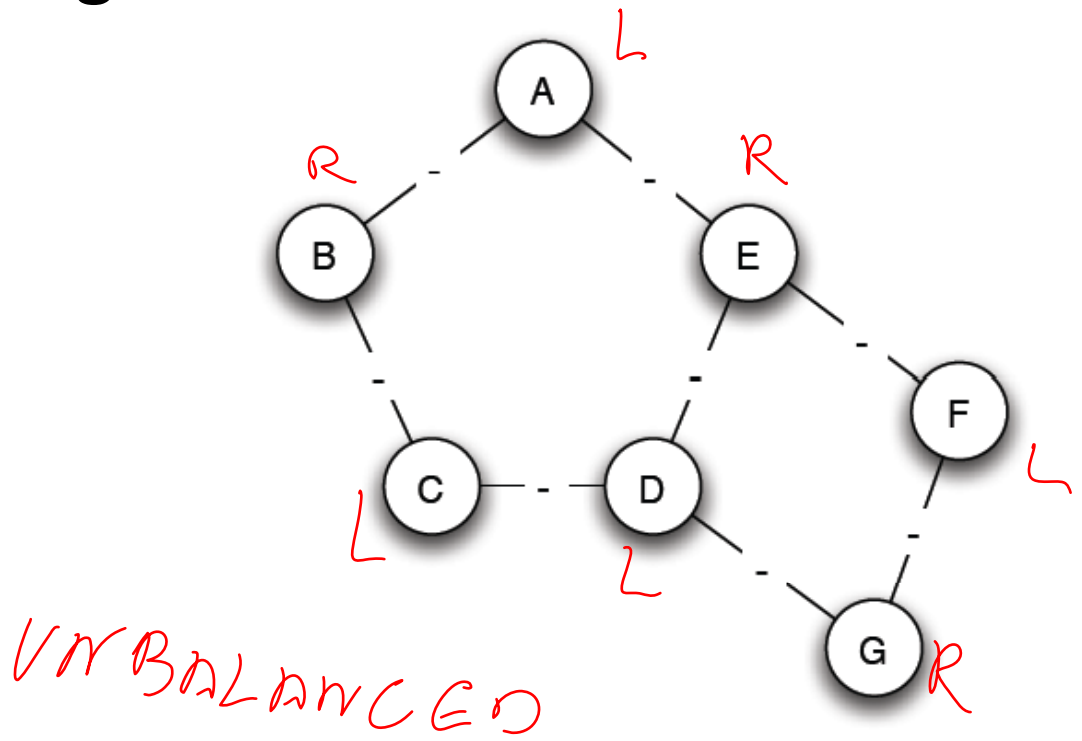


Reduced graph on super nodes



BFS on reduced graph

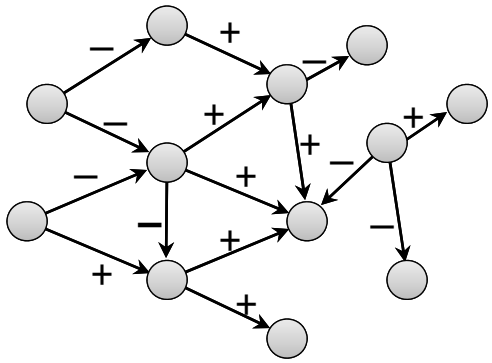
- Using BFS assign each node a side
- Graph is unbalanced if any two super-nodes are assigned the same side



[CHI '10]

Real Large Signed Networks

- Each link A→B is **explicitly** tagged with a sign:
  - Epinions: Trust/Distrust
    - Does A trust B's product reviews?
      (only positive links are visible)
  - Wikipedia: Support/Oppose
    - Does A support B to become Wikipedia administrator?
  - Slashdot: Friend/Foe
    - Does A like B's comments?
  - Other examples:
    - Online multiplayer games [Szell et al. 2010]

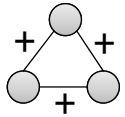
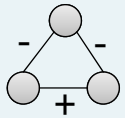
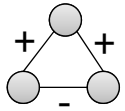
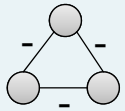


	Epinions	Slashdot	Wikipedia
Nodes	119,217	82,144	7,118
Edges	841,200	549,202	103,747
+ edges	85.0%	77.4%	78.7%
− edges	15.0%	22.6%	21.2%

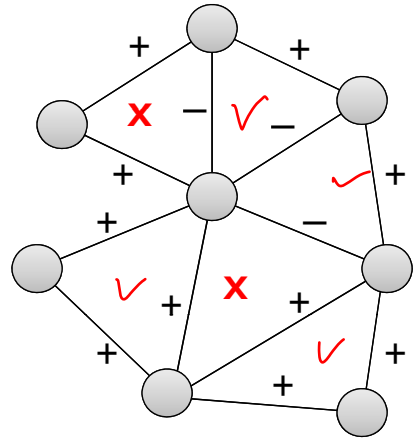
[CHI '10]

Balance in our network data

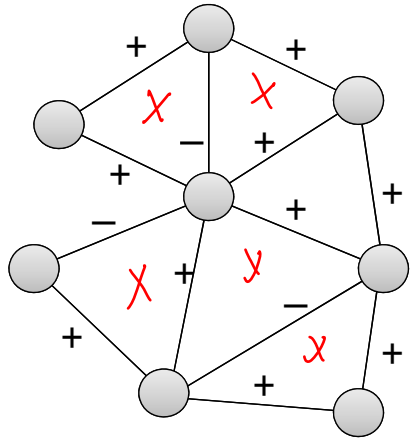
- Does structural balance hold?

Triad	Epinions		Wikipedia		Balance
	P(T)	$P_0(T)$	P(T)	$P_0(T)$	
	0.87	0.62	0.70	0.49	✓
	0.07	0.05	0.21	0.10	✓
	0.05	0.32	0.08	0.49	✓
	0.007	0.003	0.011	0.010	✗

P(T) ... probability of a triad
$P_0(T)$... triad probability if the signs would be random



Real data

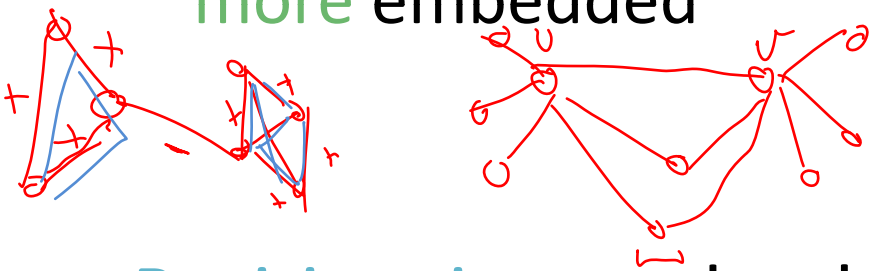


Shuffled data

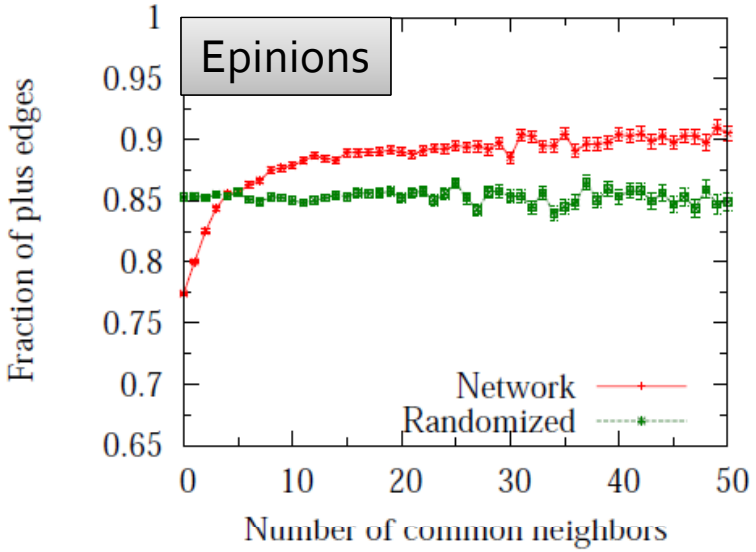
[CHI '10]

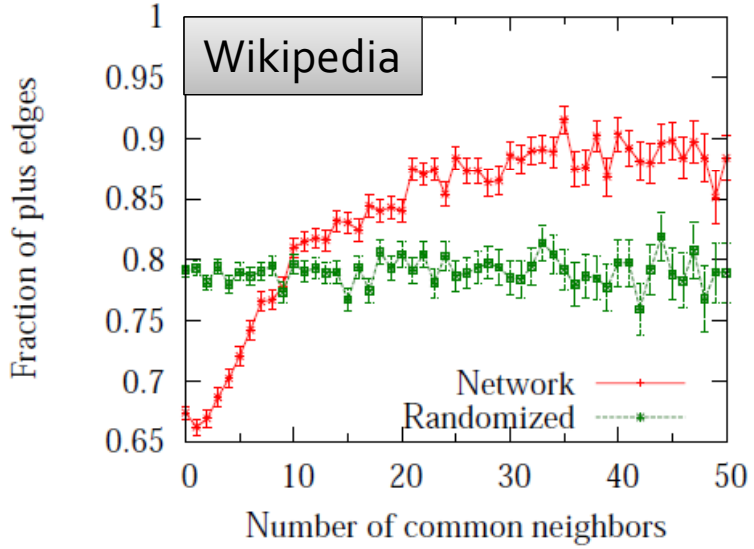
Global factions: Embeddedness

- Embeddedness of ties:
  - Positive ties tend to be more embedded



- Positive ties tend to be more clumped together
- Public display of signs (votes) in Wikipedia further attenuates this



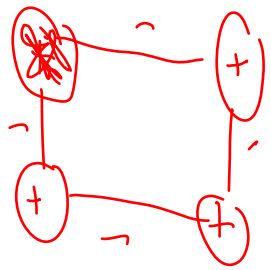


[CHI '10]

Global Structure of Signed Nets

	Size		Clustering		Component	
	Nodes	Edges	Real	Rnd	Real	Rnd
Epinions: −	119,090	123,602	0.012	0.022	0.308	0.334
Epinions: +	119,090	717,027	0.093	0.077	0.815	0.870
Slashdot: −	82,144	124,130	0.005	0.010	0.423	0.524
Slashdot: +	82,144	425,072	0.025	0.022	0.906	0.909
Wikipedia: −	7,115	21,984	0.028	0.031	0.583	0.612
Wikipedia: +	7,115	81,705	0.130	0.103	0.870	0.918

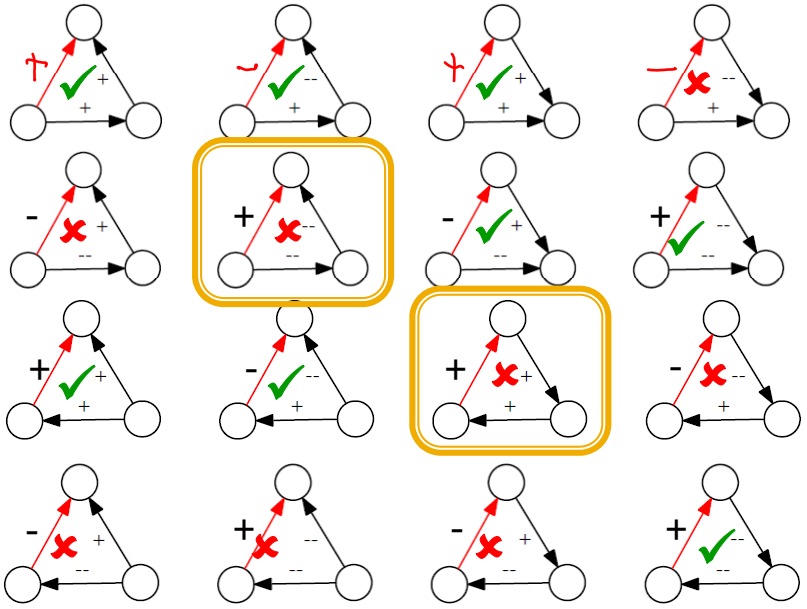
- Clustering:
  - +net: more clustering than baseline
  - –net: less clustering than expected
- Size of connected component:
  - +/–net: smaller than expected



[CHI '10]

Evolving directed networks

- Our networks are really directed
  - trust, opinion (, friendship)
- How many △ are now explained by balance?
  - Half (8 out of 16)
- Is there a better explanantion?
  - Yes. **Theory of Status.**

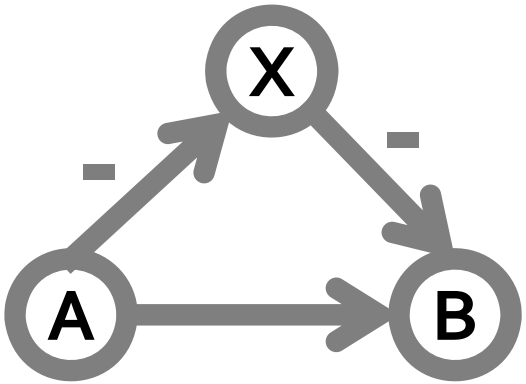


16 *2 signed directed triads

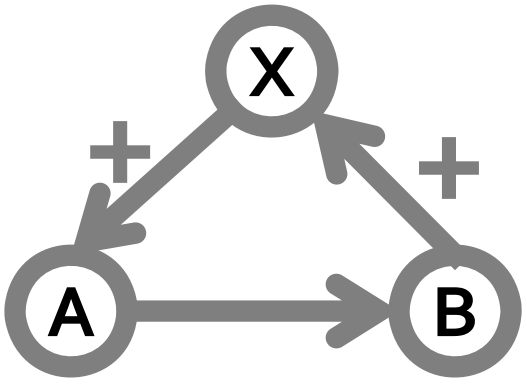
[CHI '10]

Alternate theory: Status

- Links are directed and created over time
- Status theory [Davis-Leinhardt '68, Guha et al. '04, Leskovec et al. '10]
  - Link A $\xrightarrow{+}$ B means: B has higher status than A
  - Link A $\xrightarrow{-}$ B means: B has lower status than A
- Status and balance can give different predictions:



Balance: +
Status: -

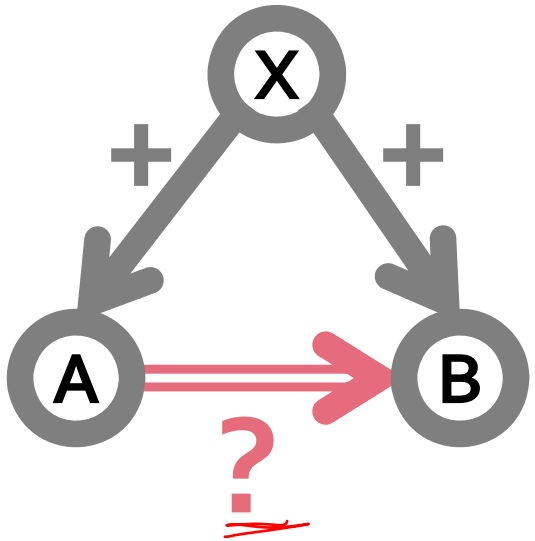


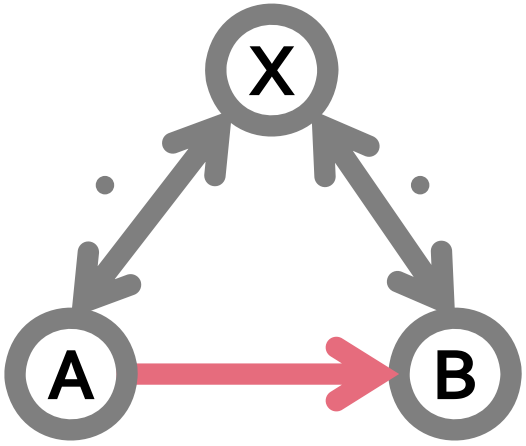
Balance: +
Status: -

Theory of Status

[CHI '10]

- Edges are directed
- Edges are created over time
  - X has links to A and B
  - Now, A links to B (triad A-B-X)
  - How does sign of A-B depend signs of X?
- We need to formalize:
  - Links are embedded in triads – provides context for signs
  - Users are heterogeneous in their linking behavior

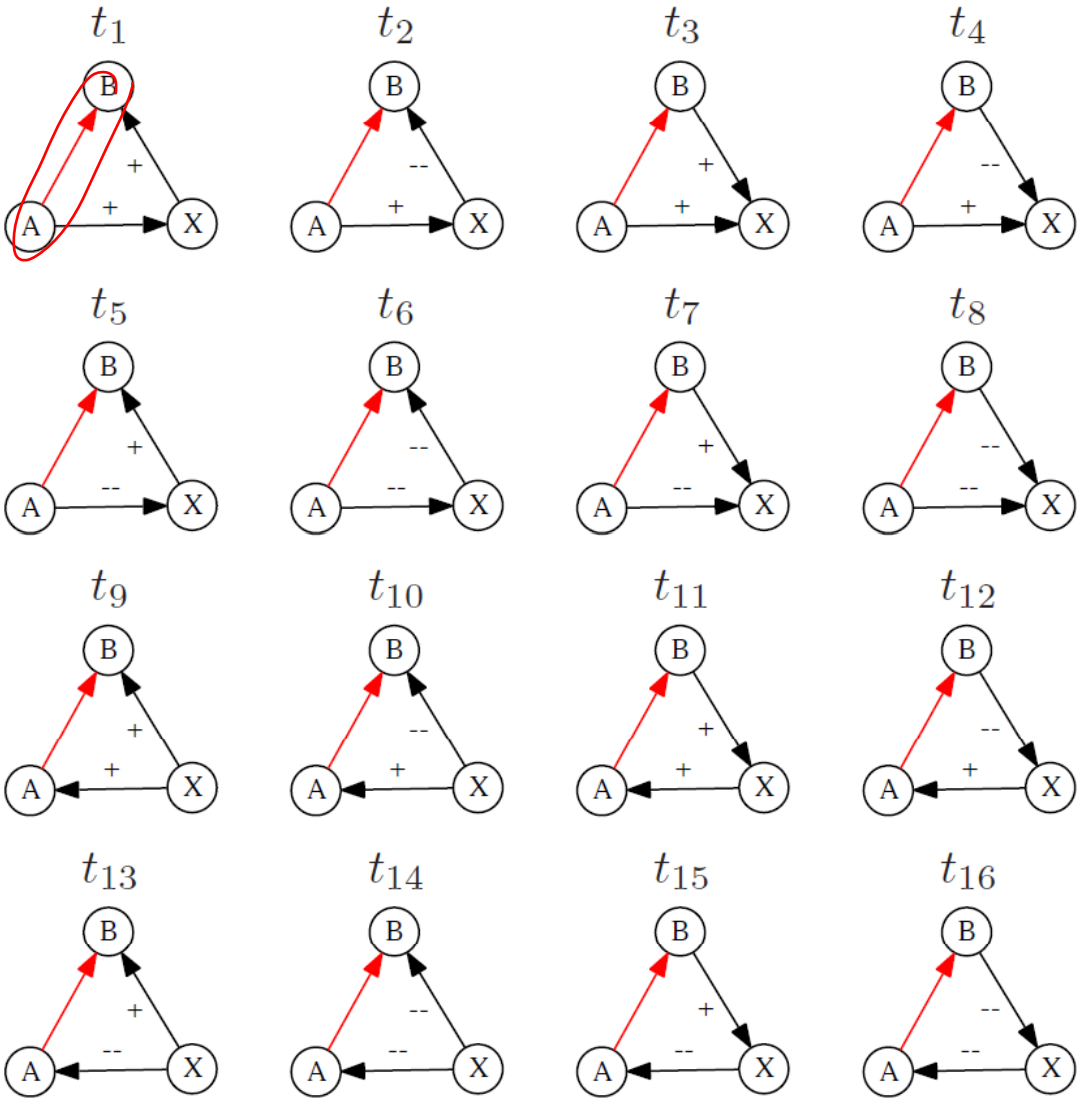




[CHI '10]

16 types of contexs

- Link (A,B) appears in the context (A,B; X)
- 16 different contextualized links:

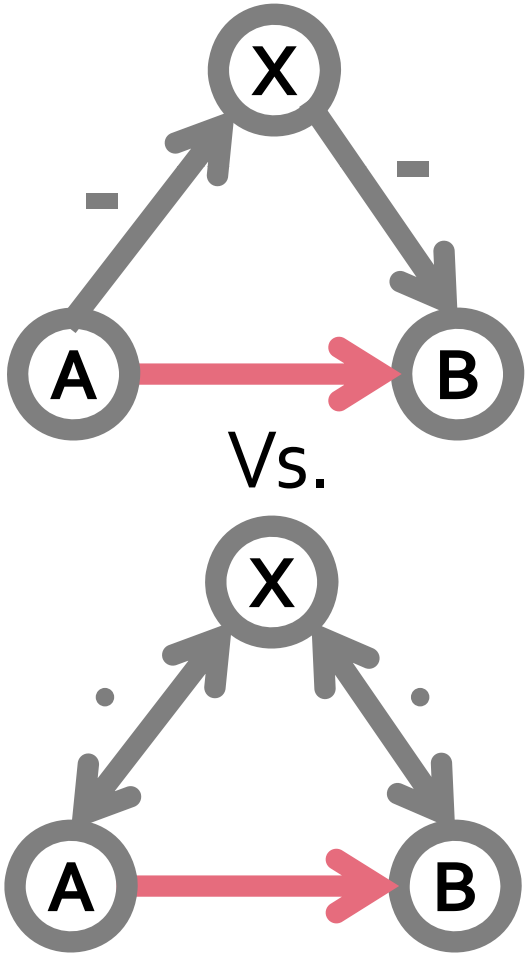


[CHI '10]

Generative (Receptive) Surprise

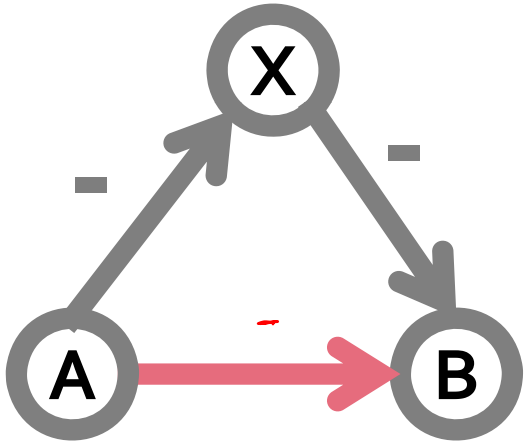
- **Surprise:** How much behavior of user deviates from **baseline** in **context t:**
  - $(A_1, B_1; X_1), \ldots, (A_n, B_n; X_n)$ … instances of contextualized link *t*
  - *k* of them closed with a plus
  - $p_g(A_i)$… generative baseline of A_i
    - empirical prob. of A_i giving a plus
- Then: generative surprise of triad type *t*:

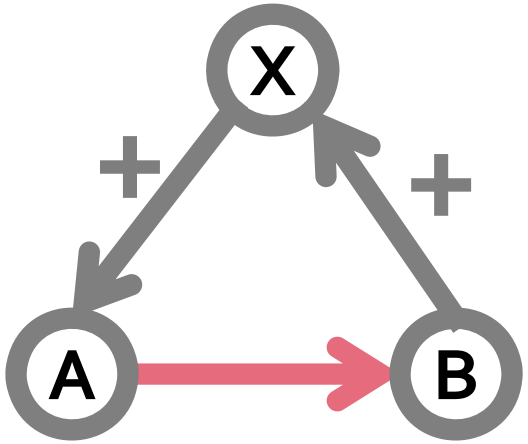
$$s_g(t) = \frac{k - \sum_{i=1}^{n} p_g(A_i)}{\sqrt{\sum_{i}^{n} p_g(A_i)(1 - p_g(A_i))}}$$



Status: Two Examples

- Two basic examples:





Gen. surprise of A:

Rec. surprise of B:

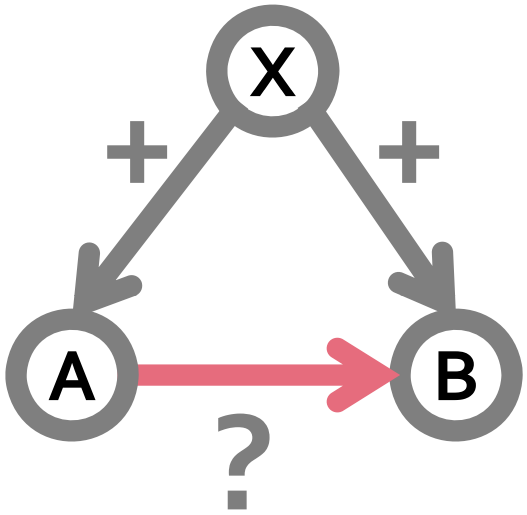
A:

B:

[CHI '10]

Joint positive endorsement

- X positively endorses A and B
- Now A links to B



A puzzle:

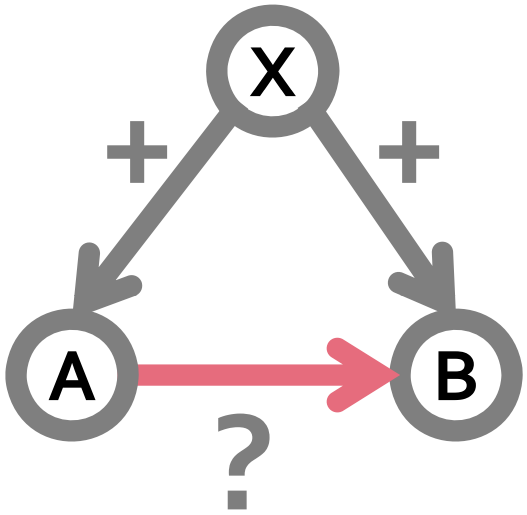
- In our data we observe:
  Fraction of positive links deviates
  - Above generative baseline of A $\delta_g(A) > 0$
  - Below receptive baseline of B $\delta_r(B) < 0$

- Why?

[CHI '10]

A story: Soccer team

- Ask every node: How does skill of B compare to yours?
  - Build a signed directed network

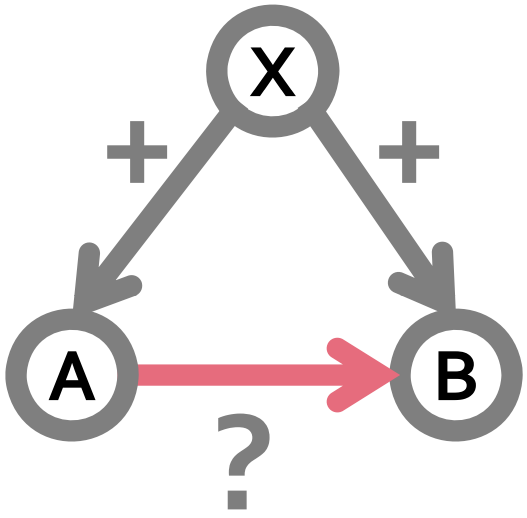


- We haven't asked A about B
- But we know that X thinks A and B are both better than him

- What can we infer about A's answer?

A story: Soccer team

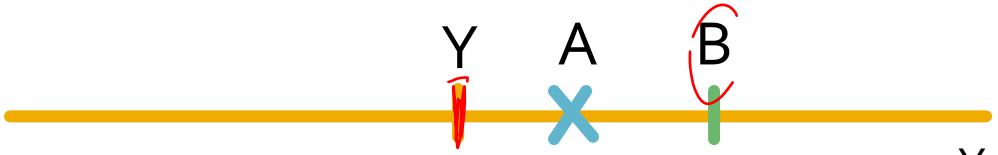
- A's viewpoint:
  - Since B has positive evaluation, B is high status
  - Thus, evaluation A gives is more likely to be positive than the baseline



How does A evaluate B?

A is evaluating someone who is better than avg.

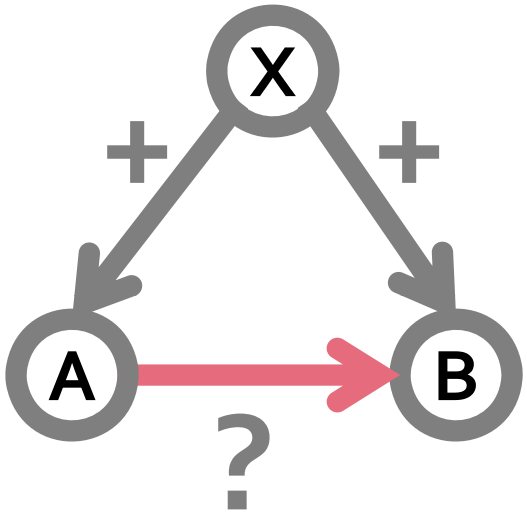
→ A is more positive than average



Y... average node

A story: Soccer team

- B's viewpoint:
  - Since A has positive evaluation, A is high status
  - Thus, evaluation B receives is less likely to be positive than the baseline



How is B evaluated by A?

B is evaluated by someone better than average.
→ They will be more negative to B than average

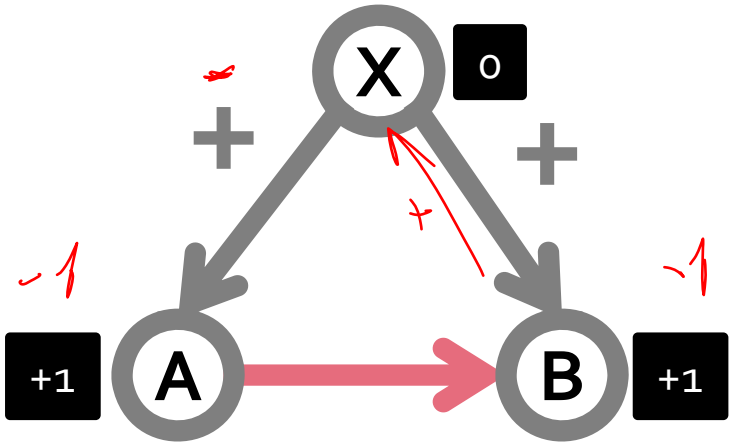
Sign of A→B deviates in different directions depending on the viewpoint!

Y... average node

[CHI '10]

Consistency with status

- Determine node status:
  - Assign X status 0
  - Based on signs and directions of edges set status of A and B



Status-consistent if:
Gen. surprise > 0
Rec. surprise < 0

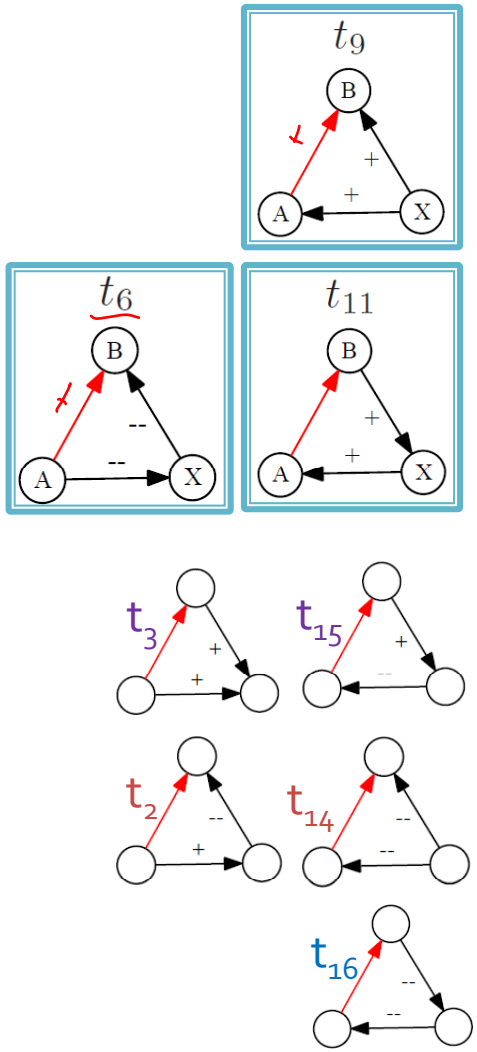
- Surprise is **status**-consistent, if:
  - **G**en. surprise is status-consistent if it has **same** sign as status of B
  - **R**ec. surprise is status-consistent if it has the **opposite** sign from the status of A
- Surprise is **balance**-consistent, if:
  - If it completes a balanced triad

Status vs. Balance (Epinions)

[CHI '10]

- Predictions:

t_i	count	$P(+)$	s_{out}	s_{in}	B_{out}	B_{in}	S_{out}	S_{in}
t_1	178,051	0.97	95.9	197.8	✓	✓	✓	✓
t_2	45,797	0.54	-151.3	-229.9	✓	✓	✓	●
t_3	246,371	0.94	89.9	195.9	✓	✓	●	✓
t_4	25,384	0.89	1.8	44.9	○	○	✓	✓
t_5	45,925	0.30	18.1	-333.7	○	✓	✓	✓
t_6	11,215	0.23	-15.5	-193.6	○	○	✓	✓
t_7	36,184	0.14	-53.1	-357.3	✓	✓	✓	✓
t_8	61,519	0.63	124.1	-225.6	✓	○	✓	✓
t_9	338,238	0.82	207.0	-239.5	✓	○	✓	✓
t_{10}	27,089	0.20	-110.7	-449.6	✓	✓	✓	✓
t_{11}	35,093	0.53	-7.4	-260.1	○	○	✓	✓
t_{12}	20,933	0.71	17.2	-113.4	○	✓	✓	✓
t_{13}	14,305	0.79	23.5	24.0	○	○	✓	✓
t_{14}	30,235	0.69	-12.8	-53.6	○	○	✓	●
t_{15}	17,189	0.76	6.4	24.0	○	○	●	✓
t_{16}	4,133	0.77	11.9	-2.6	✓	○	✓	●
			Number of correct predictions		8	7	14	13



[WWW '10]

From Local to Global Structure

- Fraction of edges of the network that satisfy Balance and Status?
- Observations:
  - No evidence for global balance beyond the random baselines
    - Real data is 80% consistent vs. 80% consistency under random baseline
  - Evidence for global status beyond the random baselines
    - Real data is 80% consistent, but 50% consistency under random baseline

[WWW '10]

From Local to Global Structure

- Both theories make predictions about the global structure of the network
- Structural balance – Factions
  - Find coalitions
- Status theory – Global Status
  - Flip direction and sign of minus edges
  - Assign each node a unique status so that edges point from low to high

[WWW '10]

From Local to Global Structure

- Fraction of edges of the network that satisfy Balance and Status?
- Observations:
  - No evidence for global balance beyond the random baselines
    - Real data is 80% consistent vs. 80% consistency under random baseline
  - Evidence for global status beyond the random baselines
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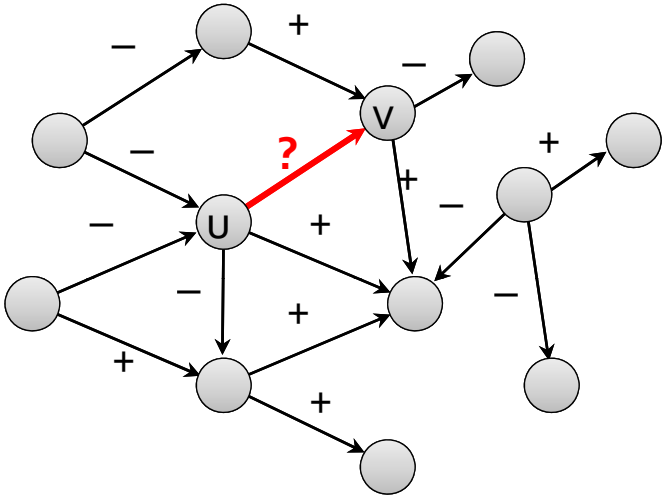
[WWW '10]

Predicting edge signs

Edge sign prediction problem

- Given a network and signs on all but one edge, predict the missing sign



Machine Learning formulation:

- Predict sign of edge (u,v)
- Class label:
  - +1: positive edge
  - -1: negative edge
- Learning method:
  - Logistic regression

$$P(+|x) = \frac{1}{1 + e^{-(b_0 + \sum_i^n b_i x_i)}}$$

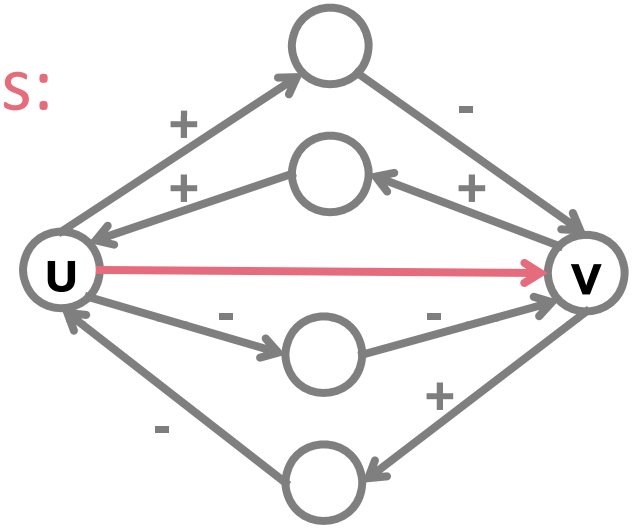
- Dataset:
  - Original: 80% +edges
  - Balanced: 50% +edges
- Evaluation:
  - Accuracy and ROC curves
- Features for learning:
  - Next slide

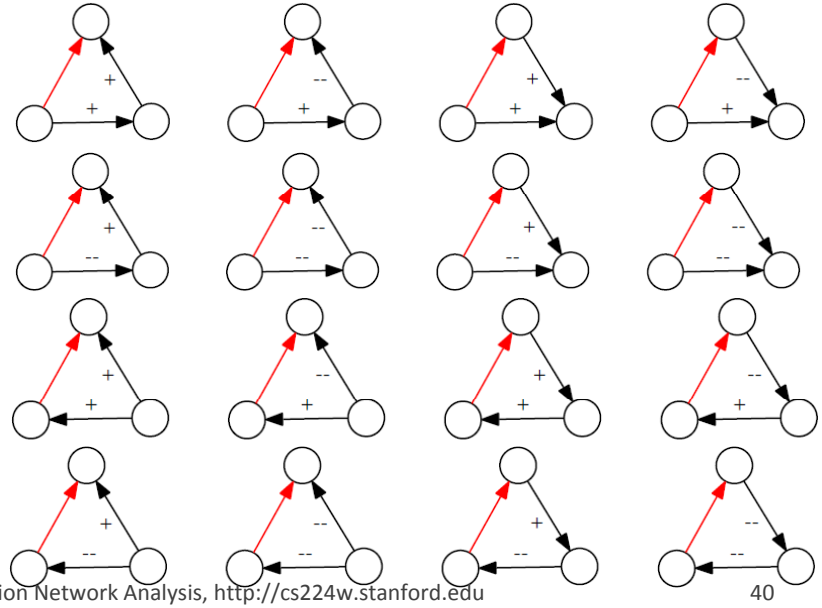
[WWW '10]

Features for learning

For each edge (u,v) create features:

- Triad counts (16):
  - Counts of signed triads edge u→v takes part in
- Node degree (7 features):
  - Signed degree:
    - $d^+_{out}(u)$, $d^-_{out}(u)$, $d^+_{in}(v)$, $d^-_{in}(v)$
  - Total degree:
    - $d_{out}(u)$, $d_{in}(v)$
  - Embeddedness of edge (u,v)

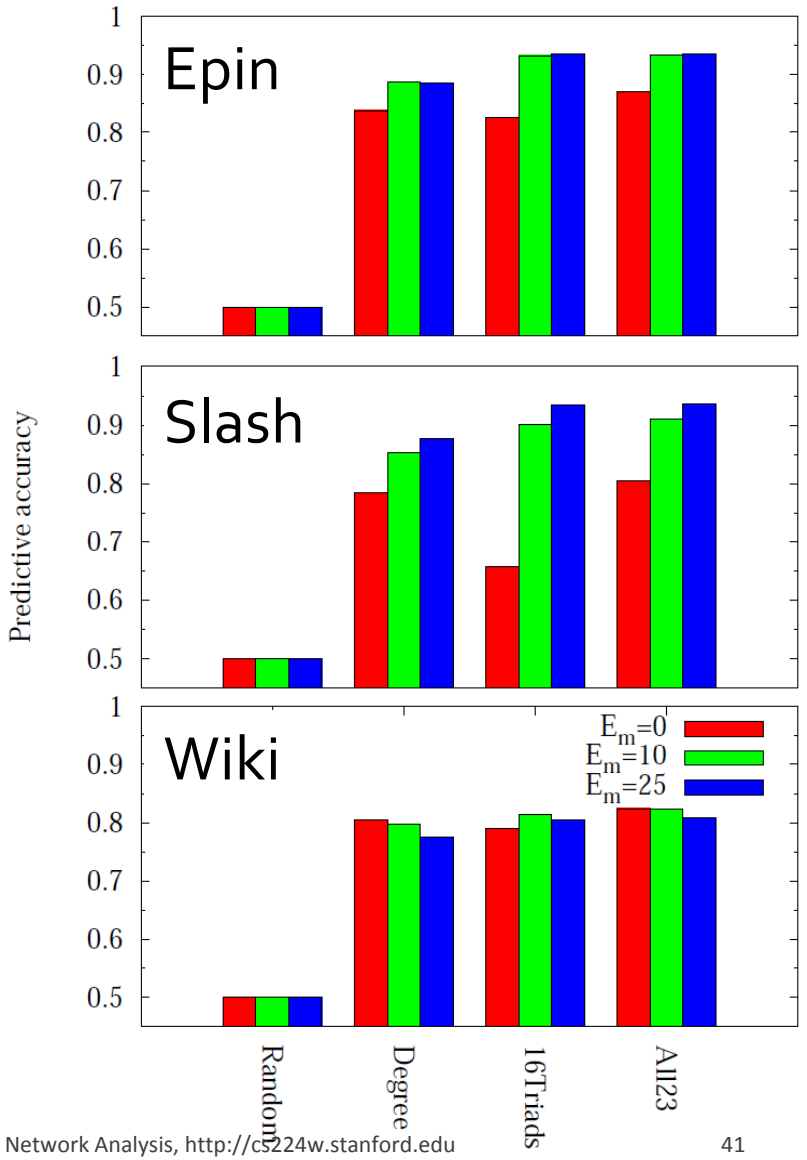




Edge sign prediction

[WWW '10]

- Classification accuracy:
  - Epinions: 93.5%
  - Slashdot: 94.4%
  - Wikipedia: 81%
- Signs can be modeled from local network structure alone
  - Trust propagation model of [Guha et al. '04] has 14% error on Epinions
- Triad features perform less well for less embedded edges
- Wikipedia is harder to model:
  - Votes are publicly visible



Generalization

- Do people use these very different linking systems by obeying the same principles?
  - How generalizable are the results across the datasets?
    - Train on row "dataset", predict on "column"

All23	Epinions	Slashdot	Wikipedia
Epinions	0.9342	0.9289	0.7722
Slashdot	0.9249	0.9351	0.7717
Wikipedia	0.9272	0.9260	0.8021

- Almost perfect generalization of the models even though networks come from very different applications

[ICWSM '10]

Conclusions

- Status vs. Balance, Consistent use of signs